Audits and Tax Evasion: An Application to Labor Taxes in Italy

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Abstract

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1. Introduction

Tax evasion is a big fiscal problem: large amounts of taxes go unpaid, and even more would not be paid were it not for auditing.[1] Do the tax authorities efficiently deter tax evasion, that is, do they use their limited resources in the most effective way? Theoretical papers on optimal auditing (see ***) address precisely this question, by computing the optimal auditing strategy within sophisticated game-theoretic models. There is a wealth of such theoretical papers, and they differ according to what is assumed the auditor maximizes, as well as whether the auditor can commit to a strategy. Not much is known empirically about whether, or which of these models fit the data, largely because of lack of tax auditing data. This paper can make progress because we have access to a novel data set, the administrative data of INPS, the Italian labor tax auditors. We can therefore leverage the wealth of insights contained in the theoretical literature on optimal auditing and bring them to the data. Our analysis suggests that the auditing strategy of INPS is not consistent with optimal *deterrence* of tax evasion. Rather, the strategy appears to be geared towards *detection of underreporters* (which, incidentally, is not an objective function which has received any attention in the optimal auditing literature).

To see the difference between these two objective functions, consider an auditor facing a mass 1 of firm, each of which has a high income H with probability 1/2, and a low income L with complementary probability. The auditor has enough resources to audit half the firms. Assume each firm will report truthfully if the probability of being audited exceeds 1/3, and otherwise it reports L. Now, if the auditor commits to auditing all the firms who report L then it is an equilibrium for all firms to report truthfully. (Indeed, a firm with true income H would face probability 1 of being audited if it misreports.) This is therefore an optimal strategy for an auditor who aims to minimize tax evasion. Notice, however, that the success rate on audits is zero. This cannot please an auditor whose aim is to detect underreporters. Such an auditor should decrease the probability of auditing low reports to just under 1/3, so as to engineer some underreporting in equilibrium. The general point is that an auditor who aims to detect underreporting may potentially engage in some pretty inefficient strategies, at least from the point of view of minimizing tax evasion.

[1] The tax gap in the United States for income taxe is evaluated to around 17% of total taxes owed. This represents more than 95 billion dollars in 1993 (see Andreoni et al. - **should find a more recent figure**).

Why do we believe that the objective of INPS is detection of underreporters? We find that the success rate of INPS—whether an audit finds underreporting by *any* amount—is constant across audit classes. Moreover, this success rate is large, roughly 50% of audits are successful in this sense. Finally, no class of firm is audited with probability 1 as far as we can tell. These basic facts are inconsistent with the equilibrium of the standard model in which the auditor minimizes tax evasion. In the baseline version of this model, the auditor optimally commits to a strategy which audits with probability 1 only those firms which report less than a threshold. Naturally, in equilibrium no firm would underreport at or below the threshold, and so the success rate of audits in equilibrium is zero.[2] Clearly, the predictions of the standard model do not match our empirical evidence. Rather, the empirical evidence is consistent with a no-commitment model in which the auditor maximizes successful audits. We develop such a model and characterize its equilibrium. We show that in this model the auditor "arbitrages away" any differences in success probability across audit classes, leading to the observed equalization in success rates, just as observed in the INPS data.

This paper makes several contributions:

1. It provides an identification result for auditing games. This result allows us to falsify the joint hypotheses that (a) the auditor has no commitment powers, and (b) that the auditor has a specific objective function. For example, we provide a test which would allow us to reject the hypothesis that the auditor pursues the objective of reducing the amount of taxes evaded, without being able to publicly commit to any auditing strategy.

2. We apply this identification result to the INPS data set. We find that we cannot reject the joint hypotheses that: (a) INPS has a no-commitment strategy; and (b) INPS maximizes the success rate of audits, that is, the probability that the audited is found underreporting by any amount.

3. We then develop a new strategic model of tax compliance and enforcement, one where the auditor maximizes the probability of a successful audit in the absence of commitment. These are, of course, exactly the joint hypotheses about INPS's behavior that we could not reject. The equilibrium strategies for firms and auditor are solved for in closed form. Under standard assumptions about the distribution of firm size, the equilibrium can reproduce an

[2] See Sanchez and Sobel (1987).

additional finding from INPS data: namely, that the monetary value of tax evaded is increasing in the size of the firm.

1.1. Related literature

CITE SOME LITERATURE ON IDENTIFICATION

Measuring tax evasion however is very difficult. First, tax evasion is by nature concealed, and the data about compliance is not widely available. Second, compliance goes hand in hand with enforcement, and the observed compliance is the result of a game between tax-payers and enforcement agencies.

There are many theoretical models of auditing game.[3] The first model is Border and Sobel (1978), and many variants have followed. See Andreoni and Miller (???) for a survey of work in this area. The main divide between these models goes through the assumption of commitment of the tax agency. Some models assume (in the contract theory tradition) that the tax agency can announce and commit to an audit policy that is known to taxpayers before they file their returns. Other models assume that the tax agency cannot commit to its audit policy and decides after the returns are file which taxpayer to audit. These two assumptions are reasonable but yield very different predictions about tax evasion and auditing results.[4] To our knowledge, our paper is the first to bring any of these theoretical models to data.

2. Tax Evasion and Auditing: A General Framework

We now present the framework we will use. The goal is to have a framework general enough to encompass virtually all of the different assumptions made in existing auditing models. This requires allowing for a very general objective function for the auditor and the firms, for different assumptions about commitment, and for other frictions. The "identification agenda" is the search for a methodology that would allow us to select among these different assumptions, within the context of specific empirical applications.

[3] We define an auditing game as a game of incomplete in which firms choose how much of their income to report, and an auditor decides which firms to audit based partly on the reports. If the firm is not audited then the report determines the taxes paid. The report also determines a penalty, which is levied only if the firm is audited and did not report honestly.

[4] Other modelling assumptions have not been settled in the litterature and vary from models to models. In particular, the exact objectives of the taxpayers and the audting agencies remain an open question.

The players are an auditor and a mass of firms with measure 1.[5] The auditor classifies firms into different audit classes according to any number of firm characteristics which are observable to the auditor (sector, geographic location, etc.). For expositional convenience we start by describing the game with only one audit class, then extend it to the case of several audit classes.

Denote by x the true tax base, which is unobserved by the auditor until an audit is made. The auditor thinks the true tax base is distributed with density f. The function $\pi(x,r,p)$ denotes the auditor's expected payoff from auditing with probability p a firm which reports r and has true tax base x.

Example 1. *If the auditor maximizes the total returns (taxes paid plus revenue from audits) then* $\pi(x,r,p) = tr + p\,(\theta + t)\max(x - r, 0)$.

Maximization of total returns is the conventional assumption in the existing theoretical literature on strategic auditing. Much less common is the following alternative assumption.

Example 2. *If the auditor maximizes the success rate from audits then* $\pi(x,r,p) = pI_{(x-r)>0}$.

Turning to the firms, the function $\kappa(x,r,p)$ represents the expected payoff of a firms with true tax base x who reports r and is audited with probability p. In practice, it can happen that the firm is subject to auditing in different arenas which may, to an extent, indirectly bring to light the same irregularities that our auditor is concerned with. For example, an Italian firm is not only subject to INPS audits, but also to income tax audits carried out by a different auditor.[6] Similarly, it could be that some of the auditor makes are not generated by the game we model here—for example, they could be compelled by law and thus not functional to maximizing π. In these cases we interpret the function κ as expressing the firm's incentives to misrepresent its income after taking into account all the other "extraneous" audits.

We denote by $r(x)$ the report of a firm with true tax base x, and by $p(r)$ the auditing probability chosen for a firm which reports r. B represents the budget constraint on audits. Formally, the equilibrium of the auditing game is defined by the following constrained maximization problem.

[5] Our analysis also applies to the case of auditors and individuals filing income taxes.

[6] In this case the Guardia di Finanza.

$$
\begin{aligned}
p^*(\cdot) &\in \arg\max_{p(\cdot)} \int_a^b \pi(x, r(x), p(r(x)))\, f(x)\, dx \\
\text{subject to: } & \int_a^b p(r(x))\, f(x)\, dx \leq B \\
\text{and, either } & r(x) \in \arg\max_r \kappa(x, r, p^*(r)) && \text{(NOCOMM)} \\
\text{or } & r(x) \in \arg\max_r \kappa(x, r, p(r)). && \text{(COMM)}
\end{aligned}
$$

The first equation represents the auditor's payoff, the second equation the budget constraint. The third and fourth equations are mutually exclusive; they represent the firm's problem under two alternative formulations. Equation (NOCOMM) represents the case in which the auditor cannot commit to an auditing schedule, whereas equation (COMM) represents the commitment case. The majority of the theoretical literature on strategic auditing proceeds under assumption (COMM), but Erard and Feinstein (1994) use (NOCOMM).

Introducing several audit classes The model above is sufficiently general to embed most of the theoretical models of strategic auditing, which generally abstract from the presence of auditing classes. For empirical purposes, however, it is important to allow for the presence of several audit classes in which the auditor classifies firms according to observable (to the auditor) characteristics. We will also have to worry that we, the researchers, may not be able fully to distinguish these audit classes. (More on this later.) Scotchmer (1994) is the first to point out the concerns raised by the presence of latent audit classes.

An audit class is simply a distinct group of firms with some distinguishable characteristics. Let k index the set of all audit classes that are distinguishable by the auditor. Their relative frequency in the population is given by $G(k)$, with $\sum_k G(k) = 1$. Firms that belong to a given audit class are like all other firms in that they have a privately known taxable income and seek to minimize their tax bill. Also, they face the same penalties if found cheating. What makes them different in the eye of the auditor is the distribution of their taxable income, which the auditor uses to make inference. Conditional on being in class k, the tax base of firms is distributed according to the probability density $f_k(x)$. A firm from audit class k faces a class-specific audit schedule $p_k(\cdot)$.

Introducing inaccurate audits We allow for audits to produce imperfect signals of a firm's true tax base. Formally, we assume that the auditor does not observe a firm's x, but rather a number ξ which is correlated with x and that we call *detected income*. We assume that the auditor maximizes

$$\pi(\xi, r, p).$$

This allows for the possibility that the auditor might not detect underreporting (in which case $\xi \leq r$ even though $x > \xi$) or that the auditor may in fact mistakenly "overdetect" (and in this case $\xi > x$). We model ξ as the realization of a random variable Ξ_k with distribution $\upsilon_k(\xi|x, r)$. Note that we allow the distribution of ξ to depend on the audit class k. This dependence allows for the possibility that it might be more difficult to detect fraud in certain occupations (for example, industries that use part-time labor such as the restaurant industry, construction, agriculture).

In the presence of inaccurate audits, the firm's payoff is potentially a function of ξ, so we will write

$$\kappa(\xi, x, r, p).$$

The assumption of class-specific inaccuracies in audits is made by Macho-Stadler and Perez-Castrillo (1997).

Introducing residual heterogeneity of firms In the base model the only unobserved heterogeneity of firms is x, their true income. A (somewhat unrealistic) implication is that all firms with the same true income report in the same way. We can relax this assumption by simply assuming that the firm's unobserved characteristics are expressed by an N-dimensional vector $\mathbf{x} = (x_1, ..., x_N)$ with density $f_k(\mathbf{x})$. For ease of interpretation we assume that x_1, the first dimension of the vector, represents the tax base, while the other dimensions capture additional heterogeneity which impacts the firm's choice of reporting. The firm's payoff will then be given by $\kappa(\xi, \mathbf{x}, r, p)$, and the firm's equilibrium strategy by $r(\mathbf{x})$. The increased dimensionality permitted in this general formulation allows us to capture, as a special case, the case in which a fraction of the firms is honest and never underreports, while the rest is "normal" and behaves as in the baseline model. A model with these features is analyzed in Section 5.

The framework After introducing all these extensions, the equilibrium of the auditing game is defined by the following constrained maximization problem.

$$\begin{aligned} \{p_k^*(\cdot)\}_k \in \arg\max_{\{p_k(\cdot)\}_k} \sum_k G(k) \int E\left[\pi(\Xi_k, r_k(\mathbf{x}), p_k(r_k(\mathbf{x})))|\mathbf{x}, r_k(\mathbf{x})\right] \; f_k(\mathbf{x})\, d\mathbf{x} \\ \text{st: } \sum_k G(k) \int p_k(r_k(\mathbf{x}))\, f_k(\mathbf{x})\, d\mathbf{x} \le B \\ r_k(\mathbf{x}) \in \arg\max_r E\left[\kappa(\Xi_k, \mathbf{x}, r, p_k^*(r))|\mathbf{x}, r\right] \text{ for each } k, \mathbf{x}, \end{aligned}$$

where the integrals are understood to be of multiple variables, over the N dimensions of $\mathbf{x}$. The last line captures the no-commitment case. To capture the commitment case it suffices to replace $p_k^*(r)$ with $p_k(r)$.

Assumptions We henceforth maintain the following two assumptions. Taken together, these assumptions characterize this hitherto abstract setting as an auditing game.

Assumption 1. ***(Deterrability)*** *For all k, if $p_k(r) = 1$ then no firm in class k with $x_1 > r$ reports r.*

Assumption 1 says that, if r is audited with sufficient frequency, then the firm's payoff function is such that no type will underreport r. This assumption means that every type can be deterred from misreporting, if the probability of auditing is sufficiently large. The next assumption says that the auditor's expected payoff from auditing someone who reports correctly (or even overreports) cannot be positive.

Assumption 2. ***(Unprofitability of auditing firms who report correctly)*** *For all $k, \mathbf{x}$ we have $E[\pi(\Xi_k, r, p)|\mathbf{x}, r] \le 0$ when $r \ge x_1$.*

Even if ξ systematically "exagerates" relative to x_1, Assumption 2 can hold if there is a cost of auditing.

3. Identification

We would like to use the data to, inasmuch as possible, learn the nature of the game that auditor and firms play. We would like to know whether the auditor has the ability to commit to an auditing strategy (i.e., whether the (NOCOMM) or the (COMM) version of the game is being played). We would also like to know as much as possible about the objective functions that auditor and firms are maximizing, i.e., the functional forms π and κ. Our job as econometricians is to use the available data to answer these questions. We call this the *identification problem*.

Ideally, we would like the identification strategy to not depend on fine details of the problem (i.e., knowledge of, or assumptions about the distributions $G(k)$ and f_k, for example). We would, also, prefer not to have to solve for the full equilibrium of the game between auditor and firms, because that is often complicated. Most importantly, we would like our methods to be robust to unobservables, that is, we want to allow for the possibility that we, the researcher, may not know as much as the auditor and firms know when they set their strategies. This is an important robustness property, because we often lack access to the full data that the auditor can see. In this spirit of "informational parsimony," we proceed to lay out our assumptions as to what features of the data we can and cannot observe.

In this section we proceed, for expositional ease, as if r can only take integer values (dollars, or cents, in our auditing setup) and ξ also can only take integer values. [7]

What we cannot observe: latent audit classes We assume that we, the researcher, are only able to observe coarse partitions encompassing several auditing classes. We will denote these partitions by K_i. For example, the set of auditing classes observed by the auditor may be $k_1, ..., k_5$, but we, the researcher, are only able to ascertain whether a particular observation belongs to $K_1 = \{k_1, k_2, k_3\}$ or $K_2 = \{k_4, k_5\}$.

What we can observe: empirical averages We assume that we, the researcher, observe individual data on each audit. Audits are indexed by d. For each audit d we observe the reported income r_d, the detected income ξ_d, and what partition $K(d)$ the audited firm belongs to.

Take any function $h(\xi, r)$. Think of it provisionally as the return from auditing a firm who reports r and is found to have a tax base ξ. For each r and each K_i, we want to form the sample average of $h(\xi, r)$ conditional on r and on K_i, which

[7] Thus the probability $f_k(\cdot)$ must be understood as having a support that is countable, rather than a continuum.

is defined as follows. Let the set of all audits of firms who report r and belong to partition K_i be denoted by

$$D(r, K_i) = \{d : r_d = r, K(d) = K_i\}.$$

Then the average h conditional on r and on K_i is the statistic defined as

$$\overline{h}(r, K_i) = \frac{\sum_{d \in D(r,K_i)} h(\xi_d, r_d)}{\sum_{d \in D(r,K_i)} 1}, \tag{1}$$

and we set $\overline{h}(r, K_i) = 0$ when its denominator is zero. The quantity $\overline{h}(r, K_i)$ is to be interpreted as the average return, as computed from the data, from auditing a firm in partition K_i who reports income r. Our identification strategy will be based on studying the properties of $\overline{h}(r, K_i)$.

Of note, $\overline{h}(r, K_i)$ can be computed using solely informations about audits. It is not necessary to have information about the distributions $G(k)$ and f_k, nor even about the probability of being audited p_k. This parsimony is convenient because in order to form p_k, for example, it would be necessary to have information on the universe of all the firms, including those which are not audited. Such information is not necessary for our analysis. Nevertheless, the expected value of $\overline{h}(r, K_i)$ does depend on all these quantities in a way which we describe next.

We think of each point in our data as an i.i.d. realization of a random vector generated by the equilibrium behavior of firms and auditor. Thus, the probability that a random element of our sample $(\xi_d, r_d, K(d))$ is equal to (ξ, r, K_i) is given by

$$\sum_{\substack{k \in K_i \\ \mathbf{x} \in X_k^*(r)}} G(k) f_k(\mathbf{x}) p_k^*(r) \upsilon_k(\xi | \mathbf{x}, r), \tag{2}$$

where $p_k^*(r)$ represents the equilibrium probability that a firm in audit class k who reports r is audited, and $X_k^*(r)$ represents the set of $\mathbf{x}$'s which in equilibrium lead a firm in audit class k to report r. The term $G(k) f_k(\mathbf{x}) p_k^*(r)$ represents the probability that a firm belongs to audit class k and has a true tax base $\mathbf{x}$ which in equilibrium leads the firm to report r, and is audited. Using formula (2), the expected value of $\overline{h}(r; K_i)$ is given by

$$\frac{\sum_{\substack{k \in K_i \\ \mathbf{x} \in X_k^*(r)}} E[h(\Xi_k, r) | \mathbf{x}, r] \; G(k) f_k(\mathbf{x}) p_k^*(r)}{\sum_{\substack{k \in K_i \\ \mathbf{x} \in X_k^*(r)}} G(k) f_k(\mathbf{x}) p_k^*(r)} \tag{3}$$

This formula contains all the functions $G(k), f_k(\cdot), p_k^*(\cdot)$ about which we, the parsimonious researcher, prefer not to make assumptions. Expression (3) is the limit in probability of $\overline{h}(r, K_i)$ as the sample size grows large.

3.1. Identification without commitment

We are now ready to present our identification result. In this section we deal with the case in which the auditor can/does not publicly commit to an auditing schedule. The main result in this section deals with the case in which the auditor's payoff function is linear in the audit probabilities.

Assumption 3. *$\pi(\xi, r, p)$ is a linear affine function of p, that is,*

$$\pi(\xi, r, p) = A(\xi, r) + pC(\xi, r),$$

with $A(\xi, r) \geq 0$.

Assumption 3 holds in Examples 1 and 2 above, and therefore in most of the theoretical literature on strategic auditing. The term $C(\xi, r)$ represents the perceived return from audits, including any costs of auditing, whereas the term $A(\xi, r)$ can be interpreted as the contribution to the auditor's payoff of a firm who reports r and is not audited; typically, this would be the tax paid before the audit, so it makes sense to assume that it is nonnegative. We view Assumption 3 as not overly restrictive. In any case, this assumption can be tested with data, as discussed in the next Proposition.

The next proposition is our identification result. It says, roughly, that if we find some statistic of the data that is equalized across audit classes, then this statistic could well be part of what the auditor is maximizing, provided the auditor has no commitment. Intuitively, an auditor with no commitment will arbitrage his audits across audit classes, i.e., will direct his audits on the classes that promise the highest return from the audit—whatever that return might be. This arbitraging behavior leads, in an equilibrium where firms respond to auditing, to an equalization of the auditor's margins across all audited classes.

Proposition 3. *If one can reject the hypothesis that $E\left[\overline{h}(r, K_i)\right]$ is independent of r and K_i, then one can reject the joint hypotheses that (a) the auditor can/does not commit to an auditing schedule, and (b) Assumption 3 holds with $C(\xi, r) = h(\xi, r)$. Conversely, if a function $h(\xi, r)$ can be found such that one cannot reject the hypothesis that $E\left[\overline{h}(r, K_i)\right]$ is independent of r and K_i, then one cannot* *reject the hypotheses that (a) the auditor can/does not commit to an auditing schedule, and (b) Assumption 3 holds with $C(\xi, r) = h(\xi, r)$.*

Proof. The proof is made by showing that, if assumption (a) and (b) hold then $E\left[\overline{h}\left(r, K_i\right)\right]$ is independent of r and K_i.

By assumption (a) the auditor cannot commit to an auditing schedule p and so the equilibrium is characterized by the following conditions.

$$\{p_k^*(\cdot)\}_k \in \arg\max_{\{p_k(\cdot)\}_k} \sum_k G(k) \int E\left[\pi(\Xi_k, r_k(\mathbf{x}), p_k(r_k(\mathbf{x})))|\mathbf{x}, r_k(\mathbf{x})\right] \ f_k(\mathbf{x})\, d\mathbf{x}$$

$$\text{st: } \sum_k G(k) \int_a^b p_k(r_k(\mathbf{x}))\, f_k(\mathbf{x})\, d\mathbf{x} \leq B$$

$$r_k(\mathbf{x}) \in \arg\max_r E\left[\kappa(\Xi_k, \mathbf{x}, r, p_k^*(r))|\mathbf{x}, r\right] \text{ for each } k, \mathbf{x}. \tag{4}$$

Let $r_k^*(\mathbf{x}; p_k^*(r))$ denote the reporting strategy that solves (4). Since condition (4) involves $p^*(r)$, not $p(r)$, the behavior of firms is a function of the auditor's expected equilibrium strategy, not of the actual strategy employed by the auditor. We shall therefore write, for ease of notation, $r_k^*(\mathbf{x}; p_k^*(r)) = r_k^*(\mathbf{x})$. Form the Lagrangean for the auditor's problem:

$$\begin{aligned}\mathcal{L}(\{p_k(\cdot)\}_k; \lambda) = {} & \sum_k G(k) \int E\left[\pi(\Xi_k, r_k^*(\mathbf{x}), p_k(r_k^*(\mathbf{x})))|\mathbf{x}, r_k^*(x)\right] \ f_k(\mathbf{x})\, d\mathbf{x} \\ & - \lambda\left[\sum_k G(k) \int p_k(r_k^*(\mathbf{x}))\, f_k(\mathbf{x})\, d\mathbf{x} - B\right].\end{aligned}$$

Use assumption 3 to substitute into the Lagrangean, which upon rearrangement reads

$$\begin{aligned}& \sum_k G(k) \int \{E\left[C(\Xi_k, r_k^*(\mathbf{x}))|\mathbf{x}, r_k^*(\mathbf{x})\right] - \lambda\}\, p_k(r_k^*(\mathbf{x})) \ f_k(\mathbf{x})\, d\mathbf{x} \\ & + \sum_k G(k) \int E\left[A(\Xi_k, r_k^*(\mathbf{x}))|\mathbf{x}, r_k^*(\mathbf{x})\right] f_k(\mathbf{x})\, d\mathbf{x} + \lambda B.\end{aligned}$$

The first term of the Lagrangean can be written as

$$\begin{aligned}& \sum_k G(k) \sum_r \int_{X_k^*(r)} \{E\left[C(\Xi_k, r)|\mathbf{x}, r\right] - \lambda\}\, p_k(r) \ f_k(\mathbf{x})\, d\mathbf{x} \\ & \sum_k G(k) \sum_r p_k(r) \left[\int_{X_k^*(r)} \{E\left[C(\Xi_k, r)|\mathbf{x}, r\right] - \lambda\} \ f_k(\mathbf{x})\, d\mathbf{x}\right]\end{aligned}$$

As the Lagrangean is linear in each $p_k(\cdot)$, the necessary conditions for optimality of the auditor's strategy are that, if $p_k^*(r) > 0$ then $E\left[C(\Xi_k, r)|\mathbf{x} \in X_k^*(r), r\right] \geq \lambda$.

Now, suppose by contradiction that the strict inequality $E\left[C(\Xi_k, r)|\mathbf{x} \in X_k^*(r), r\right] > \lambda$ holds for some r. Then at the optimum it must be $p_k^*(r) = 1$. Because $p_k^*(r) = 1$ Assumptions 1 and 2 together imply that

$$E\left[A(\Xi_k, r)|\mathbf{x} \in X_k^*(r), r\right] + E\left[C(\Xi_k, r)|\mathbf{x} \in X_k^*(r), r\right] \leq 0 \tag{5}$$

for that r. But, since $E\left[C(\Xi_k, r)|\mathbf{x} \in X_k^*(r), r\right] > \lambda \geq 0$, and $E\left[A(\Xi_k, r)|\mathbf{x} \in X_k^*(r), r\right] \geq 0$ by Assumption 3, inequality (5) cannot hold. This contradiction proves that at the optimum it must be $E\left[C(\Xi_k, r)|\mathbf{x} \in X_k^*(r), r\right] = \lambda$ for all r such that $p_k^*(r) > 0$. We may rewrite this condition as

$$E\left[C(\Xi_k, r)|\mathbf{x} \in X_k^*(r), r\right] = \lambda \text{ for all } r \text{ such that } p_k^*(r) > 0. \tag{6}$$

Now, remember that from (3) we had

$$\begin{aligned} E\left[\overline{h}(r, K_i)\right] &= \frac{\sum_{k \in K_i} G(k)\, p_k^*(r) \sum_{\mathbf{x} \in X_k^*(r)} E\left[h(\Xi_k, r)|\mathbf{x}, r\right] \; f_k(\mathbf{x})}{\sum_{k \in K_i} G(k)\, p_k^*(r) \sum_{\mathbf{x} \in X_k^*(r)} f_k(\mathbf{x})} \\ &= \frac{\sum_{k \in K_i} G(k)\, p_k^*(r) \left(\sum_{\mathbf{x} \in X_k^*(r)} f_k(\mathbf{x})\right) \; E\left[h(\Xi_k, r)|\mathbf{x} \in X_k^*(r), r\right]}{\sum_{k \in K_i} G(k)\, p_k^*(r) \sum_{\mathbf{x} \in X_k^*(r)} f_k(\mathbf{x})} \end{aligned}$$

From assumption (b) we know that $h(\xi, r) = C(\xi, r)$, and substituting into $E\left[\overline{h}(r, K_i)\right]$ we get

$$E\left[\overline{h}(r, K_i)\right] = \frac{\sum_{k \in K_i} G(k)\, p_k^*(r) \left(\sum_{\mathbf{x} \in X_k^*(r)} f_k(\mathbf{x})\right) \; E\left[C(\Xi_k, r)|\mathbf{x} \in X_k^*(r), r\right]}{\sum_{k \in K_i} G(k)\, p_k^*(r) \sum_{\mathbf{x} \in X_k^*(r)} f_k(\mathbf{x})} = \lambda$$

where the last equality makes use of (6). We have shown that, if hypotheses (a) and (b) hold then $E\left[\overline{h}(r, K_i)\right]$ is equal to a constant independent of r and K_i. ■

This proposition provides a straightforward identification strategy: if we suspect that we are in the no-commitment case, we can try out various "economically reasonable" functions $h(\xi, r)$ and check which, if any, has the property that it is equalized across all reports that are audited. If such a function is found, then this is identified as $C(\xi, r)$, and we cannot reject the hypothesis of lack of commitment.

This identification strategy is robust to details, in the sense that it is robust to the many frictions we have built into our model, and it is informationally parsimonious—it does not require us to know $G(k)$, $f_k(\cdot)$, or $p_k^*(\cdot)$, or even solve for the equilibrium behavior of firms.

The identification strategy can be extended to the case in which Assumption 3 need not hold—and so the payoff function is allowed to be nonlinear in the probability of auditing—but only at a heavy price in terms of additional assumptions. First, it is necessary to form $p(r)$, and therefore data on the universe of non-audited firms are needed. Second, it is no longer possible to accommodate the presence of latent auditing classes; we need to assume that we can observe auditing classes just as well as the auditor. Finally, and perhaps most onerous, we will require the existence of exogenous variation in the auditor's budget, not observed by the firms but observed by us. (Of course, it is perfectly all right if the firms are aware of the existence of this variation, provided they cannot observe the realization of it). Under these stringent conditions is is possible to test whether "economically reasonable" functions $h(\xi, r, p)$ can generate the data we observe. This result is presented in Appendix 7.

3.2. Identification with commitment

When the auditor can/does publicly commit to an auditing schedule it is more difficult to give a "detail-free," characterization, even a partial one, of the equilibrium of the game between auditor and firms.

An important special case, which is well-studied case in the literature, is the case in which auditor and audited play a constant-sum game, so that $\pi(x, r, p(r)) = -\kappa(x, r, p(r))$, and in addition $\pi(x, r, p(r))$ is as in Example 1. In this case the solution of the auditing game has been characterized, see e.g. Sanchez and Sobel (1993). Generically, the optimal auditing policy is to set $p_k(r_k(x)) = \frac{t}{t+\theta}$ for those types x such that $\frac{1-F_k(x)}{f_k(x)}$ exceeds a threshold, and zero for all other types. Under this auditing policy, the probability of auditing any report r is so high (when it is positive) that no-one wants to cheat by reporting r. Instead, the cheaters in group k will choose to underreport at those levels r' which are not audited at all under p_k. In terms of testable identification, this model has some very sharp predictions: the testable implications are that all audits must be unsuccessful. Clearly, this prediction is preserved even in the presence of latent audit classes. So one might be inclined to think that, at least within this specific set of primitives, we have "robust" identification. However, Macho-Stadler and Perez-Castrillo (1997) show that this result is not robust to introducing class-specific inaccuracies in audits, of the kind that we have introduced through Ξ_k. In addition, it is clear from the proofs in Sanchez and Sobel (1993) that the assumption that $\pi(x, r, p(r)) = -\kappa(x, r, p(r))$ is essential to get the "zero success" result to hold in equilibrium. We read this body of literature as suggesting that a robust and informationally parsimonious identification strategy does not exist.

3.3. Summary of identification

The message from this section is that a robust, informationally parsimonious identification strategy is available which jointly tests for the assumption of no commitment and any specification of the auditor's objective function. It is based on checking what it is that the auditor is equalizing across audit classes. If we find an economically resonable objective h which is being equalized across audit classes then we cannot reject the hypothesis that the auditor has no commitment and that the return from an audit is given by h. It is noteworthy that this identification strategy is basically agnostic about the objective function of firms. This is convenient in that it is not necessary to make specific assumptions about the nature of the firms' decision problem, in order to get identification. It is a drawback, however, in that the identification analysis per se does not give us any information about what the firms might be maximizing.

If we are unsuccessful and do not find a reasonable h that is equalized across audit classes, then we need to consider the hypothesis that the auditor has commitment power. In this case we believe that a robust and informationally parsimonious identification strategy does not exist. This is because the whole body of theoretical literature on strategic auditing highlights the sensitivity of the equilibrium to different assumptions about the objective function of the firms, about the accuracy of audits, etc.

4. Empirical Analysis of INPS Data

In this section we apply this strategy to the INPS data. We check whether there is any objective h which is being equalized across audit classes. We will argue that the data do not allow us to reject the joint assumptions of no commitment *and* maximization of detection of underreporters. In a later section we will return to the question of whether these joint assumptions about INPS's motives and behavior are reasonable.

4.1. The environment

Our data comes from labor-tax auditing of Italian firms. In Italy it is the employers' responsibility to pay labor taxes on its employees. These taxes are analogous to Social Security contributions in the US, but they are higher (they hover around 40% of the worker's gross compensation).[8] Every year the Italian Social Security Institute (INPS) inspects a number of firms in order to verify that they paid their labor taxes. This is done by auditors who visit the firms' locations and check for violations. The auditor can interview the workers he finds and check administrative and accounting records. An employer found underreporting is assessed a fine equal to the money underreported plus 33% of it.[9]

Our dataset is composed of the universe of INPS audits in 2000-2005, except for two sectors: agriculture and self-employed workers.[10] This unique dataset was created in order to get some insight into labor tax evasion and undocumented work.[11] Each observation is an audit. For each audit, the data consists in some firm characteristics (number of declared workers, production sector, regional location) and some characteristic of the audit and its outcome (lenght of the time window that is the object of audit,[12] the amount of underreported taxes, the number of undeclared workers detected). In all, we have 474,645 inspections developed on 396,065 different firms, an average of around 80,000 per year.[13] Most of these firms (90%, or about 430,000) report 10 or fewer workers, reflecting the well-known prevalence of small firms in Italy.

The variable *dipendenti* is the number of workers declared by the firm in the month in which the audit is performed. The variable *risultato* is a dummy which is 1 if the audit resulted in a fine in any amount. The variable *evasioni* is the amount of money that INPS assesses is owed, as we highlighted before this is a reliable proxy of the fines that a firms should pay. The variable *settori* codes the

[8] For most workers these taxes amount to 40-42% of gross wages, but they are 38% for workers classidied as "artisans," and only 23% for specific types of workers who are not permanently employed. Our data does not distinguish among these various types of workers.

[9] To be more precise is 33% on annual base. WHAT DOES THAT MEAN?

[10] These two sectors are subjected to a separate auditing process on which we have no data.

[11] Describe what is in Di Porto, 2009 and add to references.

[12] Every audit examines only a specific time window, say, the two most recent years of activity. If a firm is audited twice, the window of the second audit cannot by law overlap with the first audit's.

[13] Since there are around 1,660,000 Italian firms, this means that INPS audits almost 5% of them every year.

ATECO industry sector codes to which the audited firm belongs.[14]

4.2. The data

In order to be consistent with the theoretical framework of optimal auditing, our sample should only contain audits which are discretionarily intiated by INPS with the goal of uncovering underreports. However, the administrative process that generates our data is multi-faceted, and thus we need to decide what to do with "anomalous" audits that are not discretionarily intiated by INPS with the goal of uncovering underreporters. Our strategy will be to exclude them from the sample. It is important to note that we are not only erring on the side of caution; this strategy also has a theoretical justification, because eliminating these "anomalous" audits does not invalidate the analysis we intend to carry out. As mentioned on page 5 when we discussed the interpretation of κ, these excluded audits may influence the behavior of the firms, but that will not matter for our analysis: the impact of the extraneous audits folds into the definition of κ, and Proposition 3 holds regardless of their presence.

Our sample is determined as follows. First, we drop the roughly 171,000 observations in which firms are audited in a month in which they declare zero workers. These are not audits of self-employed workers, which as we mentioned do not appear in our data. Rather, these are firms which closed down (or went bankrupt) before the month in which they are audited, and who therefore report zero workers in the month in which they are audited. Unfortunately we do not know what number of workers they did report before they closed down, so even if we wanted to correlate the audit with their true report we could not do that. But, in fact, in many cases a post-bankruptcy audit is not an audit aimed at uncovering underreports of taxes, but rather part of a procedure aimed at recovering unpaid taxes (about which there is no uncertainy in INPS's records) out of the bankruptcy process. For both these reasons we eliminate observations where *dipendenti* equals zero.

Next we use the variable *origine* to screen out several types of interactions between INPS and the public which are not audits in the sense of our models. We keep in the sample only the roughly 175,000 audits which are coded as *controlli incrociati* and *mirate*. These are the audits that are discretionarily intiated by

[14] There are nine such sectors, with the numbers from 1 to 9 corresponding to, respectively: Energy, Water, Gas; Mining and Chemical; Manufacturing and Mechanical; Food and Textiles; Constructions; Commerce; Transportation; Credit and Insurance; and, finally, Services.

INPS with the goal of uncovering underreporters.

What is left out is, first, about 5,000 audits coded *fallimenti* which are initiated in connection with bankruptcy and which we eliminate for the same reason mentioned above—these are part of bankruptcy process and not true audits. Next, we have 27,000 interactions coded *scoperture* which are triggered when INPS detects a mismatch between the number of workers declared by the firm and the amount of taxes paid. This mismatch is not cheating in the sense that our models intend it: a firm who wanted to cheat would underreport both the number of workers and the taxes paid. Moreover, these audits are triggered automatically and they are not discretionary. So we eliminate them from the sample. A third type of anomalous audit is the almost 79,000 *segnalazioni*, "whistleblower audits" initiated following a complaint, typically by an alleged employee who claims that they were not declared to the tax authority—in other words, that the firm underreported its employee count. These audits are (a) not discretionary, because INPS is required by law to follow up; and (b) they are based on a piece of information (the whistleblower) which is not contemplated in auditing models, including ours. Therefore, we eliminate whistleblower audits for our sample.

After eliminating the audits mentioned above we are left with those discretionary audits which are initiated by INPS based only on documentary information about the firm. These audits are allocated following a strategy devised by the top management at INPS. The strategic guidelines, which are updated throughout the year, direct auditors in a given region to focus on specific types of businesses, such as truckers, or ice-cream parlors, etc. These discretionary audits correspond to the auditing activity contemplated in the auditing models. Therefore, we will restrict attention to these audits. To the extent that the auditing strategy is centrally designed, our model with a single auditor fits well the institutional environment.[15] There is no explicit statement, however, about INPS's objective function. Therefore, it is left to us to infer it empirically from the data.

Finally, we divide the sample into audits of small firms, which we define as firms which declare 10 or fewer employees, and audits of large firms. In Italy small

[15] While on paper the auditing strategy is decided centrally, the reader may wonder about the incentives of individual auditors to potentially subvert the centrally-decided strategy. Individual auditors are compensated on a fixed wage plus a "productivity premium" based on the amount of unpaid taxes recovered in their region. We view these incentives as rather low-powered for two reasons. First, an individual auditor has a negligible effect on how much is recovered in his entire region. Second, as a practical matter the auditor's union has always resisted the notion that the productivity premium might be withheld. Perhaps as a result, the productivity premium has historically never been denied to any region.

firms represent a very large fraction of all the firms (and roughly 90% of our entire sample), and accordingly we will focus on the small firms sample, but nonetheless we will also analyze the large firms sample. As we will see, our results change somewhat depending on the sample.

A notable featureof the summary statistics is that audits of small and large firms differ in several dimensions. First, the composition by industry changes, as one would expect, with small firms being a larger fraction of the audited population in certain sectors. Second, the average evasion detected is larger in the large firms sample. Third, and more interesting, in the probability of a successful audit is smaller for small firms (40% versus 54%). We will return to this observation later.

4.3. Results for small firms

The dependent variable in the regression of Table 2, *risultato,* is a dummy which is 1 if the audit resulted in a fine of any amount. According to our test, if the firm maximizes the probability of detecting evasion, then the probability of detection should be the same for any reported number of *dipendenti* and for any sector (the variables *settori#*). In our empirical specification we allow for additional flexibility by controlling for the interaction between dipendenti and industry codes (the variables *sett#dip*). This allows for the probability of detection to vary with dipendenti in a sector-specific way. Despite this flexibility, the coefficients on almost all the variables are not significantly different from zero. We interpret this widespread lack of explanatory power as evidence that none of the independent variables in our regression help improve the probability of detection. One must not overemphasize this interpretation, because the F statistic indicates joint significance of the independent variables. Nevertheless, Table 2 does point to the difficulty for an auditor of predicting the probability of the success of an audit based on the variables we have, so that any audit is perceived as "equally likely to succeed" by INPS. This is consistent with the assumption that INPS maximizes the probability of a successful audit, and that INPS does not have, or does not make strategic use of, the power to commit.

The next regression, Table 3, is identical to the first except that the dependent variable is *evasioni*, the amount of money that INPS assesses it is owed. According to our theory, if F/f is increasing, then there is a positive correlation between number of reported employees and amount of the misreport, i.e., firms who report more workers also misreport by more. The coefficient on dipendenti is positive

	Small Firms	**Large Firms**
EnGasAcq	453 (0.3%)	83 (0.3%)
IndEstrChim	1,600 (1%)	656 (2,7%)
ManMetmecc	8,563 (5.7%)	3,303 (13.7%)
IndAliTess	16,807 (11.1%)	4,419 (18.3%)
Const	35,427 (23.3%)	6,910 (28.6%)
ComPubbEs	69,385 (45.7%)	5,087 (21%)
Trasp	1,587 (1%)	775 (3.2%)
CredAss	5,927 (3.9%)	1,684 (6.7%)
Serv	12,057 (7.9%)	1,268 (5.2%)
Total	151,806 (100%)	24,185 (100%)
Risultato (std. dev.)	0.40 (0.49)	0.54 (0.50)
Evasioni (std. dev.)	7,287 (36,987)	85,230 (3,611,768)
Evasioni conditional on evasioni > 0 (std. dev.)	18,421 (57,038)	158,590 (4,925,675)
Dipendenti (std. dev.)	3.09 (2.35)	51.13 (346.32)

Table 1: Summary Statistics. Large firms are those with more than 10 reported employees.

Linear regression

Number of obs =	151806
F(17,151788) =	125.86
Prob > F =	0.0000
R-squared =	0.0141
Root MSE =	.48565

risultato	Coef.	Robust Std. Err.	t	P>|t|	[95% Conf.	Interval]
dipendenti	.0113146	.0107671	1.05	0.293	-.0097886	.0324178
settori2	.0323488	.0405487	0.80	0.425	-.0471259	.1118235
settori3	.0297063	.0357099	0.83	0.405	-.0402844	.0996969
settori4	.0706489	.0351828	2.01	0.045	.0016913	.1396065
settori5	.0412871	.03489	1.18	0.237	-.0270966	.1096708
settori6	.0248906	.0347275	0.72	0.474	-.0431745	.0929557
settori7	.263766	.040259	6.55	0.000	.1848593	.3426728
settori8	.0852163	.0360221	2.37	0.018	.0146138	.1558189
settori9	-.0192634	.0352476	-0.55	0.585	-.0883479	.0498211
sett2dip	.0106818	.0117732	0.91	0.364	-.0123935	.033757
sett3dip	.0076794	.0109501	0.70	0.483	-.0137825	.0291413
sett4dip	.0085667	.0108717	0.79	0.431	-.0127415	.0298749
sett5dip	.0059091	.0108192	0.55	0.585	-.0152962	.0271145
sett6dip	.0125863	.0108006	1.17	0.244	-.0085826	.0337551
sett7dip	-.0026538	.011755	-0.23	0.821	-.0256934	.0203857
sett8dip	.0085434	.0111466	0.77	0.443	-.0133036	.0303905
sett9dip	.0258065	.0110239	2.34	0.019	.0041999	.0474132
_cons	.2935722	.0345996	8.48	0.000	.2257578	.3613866

Table 2: Predicting successful audits in small firms (robust standard errors).

```
Linear regression                                      Number of obs =  151806
                                                       F( 17,151788) =   91.54
                                                       Prob > F      =  0.0000
                                                       R-squared     =  0.0181
                                                       Root MSE      =   36675
```

evasioni	Coef.	Robust Std. Err.	t	P>|t|	[95% Conf.	Interval]
dipendenti	778.7195	507.3539	1.53	0.125	-215.6839	1773.123
settori2	-1515.371	2378.336	-0.64	0.524	-6176.862	3146.119
settori3	-587.936	1853	-0.32	0.751	-4219.779	3043.907
settori4	-1319.229	1659.549	-0.79	0.427	-4571.911	1933.454
settori5	-1534.837	1615.869	-0.95	0.342	-4701.908	1632.233
settori6	-2166.489	1591.495	-1.36	0.173	-5285.787	952.81
settori7	4441.855	2517.581	1.76	0.078	-492.5533	9376.263
settori8	-3520.811	2552.691	-1.38	0.168	-8524.034	1482.412
settori9	-4425.674	1632.514	-2.71	0.007	-7625.368	-1225.98
sett2dip	1652.25	789.3061	2.09	0.036	105.2258	3199.273
sett3dip	1606.68	600.034	2.68	0.007	430.6256	2782.734
sett4dip	1291.939	538.1668	2.40	0.016	237.1431	2346.735
sett5dip	780.8399	517.8516	1.51	0.132	-234.1388	1795.819
sett6dip	416.3283	512.5052	0.81	0.417	-588.1714	1420.828
sett7dip	2096.388	802.8324	2.61	0.009	522.8531	3669.923
sett8dip	3478.786	1093.871	3.18	0.001	1334.82	5622.751
sett9dip	1183.175	557.8244	2.12	0.034	89.85087	2276.5
_cons	4123.077	1581.367	2.61	0.009	1023.629	7222.524

Table 3: Predicting returns from audits in small firms (robust standard errors).

and close to significant at the 10 percent level, and moreover coefficients on the interaction terms are all positive and most are robustly significant. This evidence supports the finding that firms who report more employees also underreport by more. This, as we will see in the next section, is a prediction of our theoreitcal model under the assumption that F/f is increasing.

4.4. Results for large firms

For large firms, our identification strategy delivers mixed results. From the summary statistics we know that, on average, the probability of a successful audit is larger for large than for small firms (54% versus 40%). In other words, an inspector could substitute a search of a small firms with a search of a large firm and increase his probability of success.

One might attribute this difference to the (unobserved) cost of carrying out an audit in a larger firm. This interpretation not consistent with the evidence presented in Table 4, however. The regression in Table 4 is identical to that of Table 2, except that it is performed on the sample of firms which report more than

Linear regression

Number of obs =	24185
F(17, 24167) =	27.93
Prob > F =	0.0000
R-squared =	0.0129
Root MSE =	.49553

risultato	Coef.	Robust Std. Err.	t	P>|t|	[95% Conf.	Interval]
dipendenti	.0000339	.0000113	2.99	0.003	.0000117	.0000561
settori2	.0677106	.058369	1.16	0.246	-.0466963	.1821175
settori3	.1732998	.0556894	3.11	0.002	.0641451	.2824544
settori4	.1043166	.0556147	1.88	0.061	-.0046916	.2133247
settori5	.09056	.0553061	1.64	0.102	-.0178434	.1989634
settori6	.1064535	.0554327	1.92	0.055	-.0021981	.215105
settori7	.1833129	.0578605	3.17	0.002	.0699028	.2967231
settori8	.2866815	.056091	5.11	0.000	.1767397	.3966234
settori9	.165802	.0567756	2.92	0.004	.0545183	.2770857
sett2dip	.0000168	.0000122	1.38	0.169	-7.11e-06	.0000407
sett3dip	-.0001349	.0000581	-2.32	0.020	-.0002489	-.000021
sett4dip	-.0002117	.000096	-2.21	0.027	-.0003999	-.0000235
sett5dip	-.0000141	.000056	-0.25	0.801	-.0001239	.0000956
sett6dip	-.000141	.0000736	-1.92	0.055	-.0002853	3.26e-06
sett7dip	-.0000187	.0000412	-0.45	0.650	-.0000994	.000062
sett8dip	-.0000656	.0000177	-3.70	0.000	-.0001003	-.0000309
sett9dip	.0000632	.0000315	2.00	0.045	1.33e-06	.000125
_cons	.4125532	.0549474	7.51	0.000	.3048528	.5202536

Table 4: Predicting successful audits for large firms (robust standard errors).

10 employees. First, we notice that the coefficients of the variable *dipendenti*, as well as those of the variables interacted with *dipendenti*, are very small. This means that size per se does not appreciably raise the probability of a successful audit above the 41% level (the constant in the regression which, incidentally, is about equal to the average success rate among small firms). Rather, the "excess return" from audits comes through some of the sector dummies. This observation casts doubt on the interpretation that the difference in success rates of audits is due to the larger cost of performing audits on bigger firms. Adding to the puzzle, the sector with the biggest "excess return" to an audit is *settori8*, corresponding to credit and insurance.

It is possible that the cross-sector differences in returns to an audit that are present in the large firm sample reflect an unobserved "complexity of audit" cost that varies across large firms in different sectors. It is difficult to rule out the existence of such unobservable differences; however, we saw no evidence of them in the small firms sample. In our own reading, the evidence in the large firm sample provides weak support for the no-commitment, success-maximizing model.

4.5. Discussion of findings

ADD DISCUSSION OF WHY RESULT DOES NOT WORK IN LARGE FIRMS

For the "small firms" sample, which comprises about 90% of the firms in Italy (IS THIS TRUE?), our results suggest that INPS behaves, in the aggregate, as an agency which maximizes the probability of finding tax cheaters and, moreover, has no ability to publicly commit to an auditing strategy. Is such a conclusion plausible? We think it is, and that the observed auditing behavior reflects an implicit incentive scheme whereby individual inspectors are rewarded (with promotions, for example, or by other means) as a function of the number of times they find a firm which is not reporting correctly. Furthermore, we believe that this implicit compensation scheme deals with an agency problem between INPS and its inspectors. We will now outline the elements of the agency problem

The first element of the agency relationship is that effort exerted by the inspector is not observable. This means that it is difficult to induce an inspector to monitor very intensely a group of firms which in equilibrium do not to evade taxes: the inspector will simply shirk and report finding no evasion. Notice that this is precisely the behavior required by the optimal strategy with commitment! The general point is that commitment bites exactly when it is necessary to audit well-behaved groups. If, as seems natural, the problem of unobservable effort is solved by rewarding the productivity if audits, then it will not be possible to generate "commitment behavior."

This argument does not explain why the implicit incentive scheme should reward detection and disregard the *amount* of evasion detected. We believe that a possible reason might lie in the peculiar technology of assessing tax evasion. Suppose that the inspector can manipulate the amount of evasion ascertained, but cannot as easily manipulate whether any evasion is ascertained. For example, ADD EXAMPLE. In this case it would be optimal not to reward the size of the tax evasion ascertained, because this is more easily manipulable; rather, it would be optimal to tie compensation to the less manipulable correlate of effort, which is whether any evasion was ascertained. In support of our argument, we note that there is a very large amount of overascertaining

CAN WE PRODUCE EVIDENCE THAT AMOUNTS ASCERTAINED ARE SHAVED BY JUDGES, BUT THAT THE JUDGE RARELY TURNS A POSITIVE ASCERTAINMENT INTO A ZERO?

5. A New Theoretical Model: Auditing to Maximize Detections

We develop a novel model of tax enforcement/evasion that is consistent with the empirical findings of the previous section.

In particular, we identified the objective of the auditing agency to be consistent with the absence of commitment in its auditing policy and maximization of audit success. This is the main novelty of our theoretical model. The data is also consistent with the presence of honest firms that we introduce in the model in the same way as Erard and Feinstein (1994). We restrict attention to a single auditing class.

We assume a continuum of firms with different labor taxes to be reported to the government. True tax base x lies in the interval $[a, b]$, and is distributed on this interval according to the density $f(x)$. A firm reporting a type r pays taxes $t \cdot r$. There are two types of frims, strategi frims and honest firms. We assume that there is a proportion λ of honest firms and a proportion $(1 - \lambda)$ of strategic firms. A strategic firm chooses which tax base r to report to the tax authority in order to minimize its expected taxes. In doing so, the firm recognizes that it faces an probability schedule $p(r)$ that relates the report r to the probability of being audited by INPS. Firms are aware that in case of an audit, the true characteristic x will be discovered. The taxes are then determined on the true level x and a penalty is added that is proportional to the amount of evasion. In case of an audit, a firm x that reported r, pays a total of $t \cdot x + \theta(x - r)$. Honest firms always report the true value x and pay taxes $t \cdot x$.

INPS observes the report of the firms and chooses an audit schedule $p(r)$. INPS' objective is assumed to be to maximize the probability of successful audits. This is consistent with every successful audit bringing a utility of S to INPS. INPS has a finite amount of resources to audit firms. The total number of firms that can be audited is N. There is thus an opportunity cost of auditing one firm rather than another. INPS only audits firms that have the highest probability of being caught cheating.

5.1. The firm's problem

A firm with type x chooses which $r \leq x$ to report so as to maximize

$$\pi(r; x, p(\cdot)) = p(r)(x - tx - \theta(x - r)) + (1 - p(r))(x - tr). \tag{7}$$

The gain from reporting $r \leq x$ is

$$\pi(r; x, p(\cdot)) - \pi(x; x, p(\cdot)) = [t - p(r)(t+\theta)](x - r)$$

This is positive for any $r < x$ unless $p(r) \geq \frac{t}{t+\theta}$ Thus, to deter any strategic firm x from misreporting at any level r, the audit probability has to be at least $p(r) = \frac{t}{t+\theta}$. Conversely, if $p(r) > \frac{t}{t+\theta}$ then no firm will ever misreport at level r. But then in equilibrium it must be $p(r) = 0$, which is a contradiction. This proves that in equilibrium $p(r) \leq \frac{t}{t+\theta}$.

We will construct an equilibrium in which all strategic firms will misreport. In that case the constraint $r \leq x$ is never binding and the first-order conditions associated with (7) are necessary conditions for a maximum. They are

$$p'(r)(r - x)(t+\theta) + p(r)(t+\theta) - t = 0, \tag{8}$$

that can be rewritten as

$$x(r) = r + \frac{p(r) - \frac{t}{\theta+t}}{p'(r)}, \tag{9}$$

where we denoted by $x(r)$ the true type of a strategic firm which reports $r < x(r)$.

Concavity of the objective function with respect to r is a sufficient condition for the first order conditions to identify a global maximum.. Concavity means that, for all x and $r < x$, the second derivative of the objective function with respect to r must be negative:

$$(t+\theta)[p''(r)(r - x) + 2p'(r)] \leq 0. \tag{10}$$

5.2. The auditor's problem

We are looking for a separating equilibrium in which strategic firms uses a report function $\rho(x)$ that is strictly increasing in x. Observing a report r, INPS realizes that it can come from an honest firm with true type $x = r$, or from a strategic firm that underreported taxes with true type $x(r) > r$. INPS has to assess the probability that it faces a strategic firm that underreported its taxes.

Reports audited by INPS with probability $p(r)$, $0 < p(r) < 1$, need to lead to the same probability of success.

The proportion of honest types making a report r can be calculated using Bayes' Rule. Among the honest types, the mass who report in the interval Δr are approximately $f(r) \cdot \Delta r$. Among the dishonest types, the mass who report in the interval Δr are approximately $f(x(r)) \cdot \Delta x(r)$. Therefore, the probability of an honest types conditional on reporting in Δr is

$$\frac{\lambda f(r)\,\Delta r}{\lambda f(r)\,\Delta r + (1-\lambda) f(x(r))\,\Delta x(r)}.$$

Dividing by Δr and letting $\Delta r \to 0$ yields

$$\frac{\lambda f(r)}{\lambda f(r) + (1-\lambda) f(x(r)) \frac{dx(r)}{dr}}. \tag{11}$$

A constant success of audits means that on the range of reports audited, the probability of honest types must be constant and equal to some $q \in (0,1)$. (Therefore $1-q$ represents the success rate of audits). The precise value of q is determined by the shadow value of the resources allocated to INPS's auditing function. Let, therefore

$$\frac{\lambda f(r)}{\lambda f(r) + (1-\lambda) f(x(r)) \frac{dx(r)}{dr}} = q \tag{12}$$

Lemma 4. *In any equilibrium of the auditing game it must be $\lambda \geq q$.*

Proof. See appendix. ■

We can rewrite (12) as

$$\frac{\lambda(1-q) f(r)}{(1-\lambda) q} = f(x(r)) \frac{dx}{dr}.$$

Before solving the differential equation, we can say a few more things on the equilibrium strategies. The interval of reports must be of the form $[a, r(b)]$. This comes from the fact that any report not reported would lead to a probability of auditing of zero since the audits at that report would be only of honest firms. But a zero probability of audits would lead all firms that report higher to want to deviate to that report. This means that if tax report r is made in equilibrium, then all reports below r are also used by some firm. This is a first-order differential equation in $x(\cdot)$. Denoting $\frac{\lambda(1-q)}{(1-\lambda)q}$ as γ, and integrating both sides yields:

$$\gamma F(r) = F(x(r)) + k \tag{13}$$

We get $k = 0$ since the lowest type must correspond to the lowest possible tax base.[16] Hence:

$$\begin{aligned} x(r) &= F^{-1}(\gamma F(r)), or \\ r(x) &= F^{-1}(F(x)/\gamma). \end{aligned}$$

We then get the reports of the extreme types:

$$\begin{aligned} r(b) &= F^{-1}(1/\gamma) \\ r(a) &= a \end{aligned}$$

We now characterize the equilibrium auditing schedule. Using

$$x(r) = r + \frac{p^*(r) - \frac{t}{\theta+t}}{p^{*\prime}(r)},$$

we get

$$\frac{p^{*\prime}(r)}{p^*(r) - \frac{t}{\theta+t}} = 1/\left(F^{-1}(\gamma F(r)) - r\right).$$

Integrating both sides yields:

$$\ln\left(p^*(r) - \frac{t}{\theta+t}\right) = -\int_r^{r(b)} 1/\left(F^{-1}(\gamma F(y)) - y\right) dy + k$$

$$p^*(r) = \frac{t}{\theta+t} + K \exp\left(-\int_r^{r(b)} 1/\left(F^{-1}(\gamma F(y)) - y\right) dy\right)$$

K is computed such that $p(r(b)) = 0$.[17]

$$p^*(r) = \frac{t}{\theta+t}(1 - \exp\left(-\int_r^{r(b)} 1/\left(F^{-1}(\gamma F(y)) - y\right) dy\right).$$

We have that $lim_{r\to a}(F^{-1}(\gamma F(r)) - r) = 0$ and $lim_{r\to a}\frac{\partial}{\partial r}(F^{-1}(\gamma F(r)) - r) = lim_{r\to a}\left(\frac{\gamma f(r)}{f(F^{-1}(\gamma F(r)))} - 1\right) = \gamma - 1$. This proves that $1/(F^{-1}(\gamma F(y)) - y)$ goes to infinity as the same rate as $1/x$ and that the integral diverges.

[16] This comes from the fact that the interval of reports at a and that $r(x)$ is increasing.

[17] This boundary condition comes from the following observation. If $p(r(b)) > 0$ and $r(b) < b$, then we would have an immediate contradiction since a firm with type b would rather report a bit higher than $r(b)$ avoiding all audits.

Note that if the integral diverges as r goes to a and $p^*(r)$ goes to $\frac{t}{\theta+t}$ as r goes to a. The audit policy thus displays full deterrence at the bottom of the distibution $p(a) = \frac{t}{\theta+t}$.

We are now able to verify the second-order conditions of the firm's problem and conlude that the startegies proposed are indeed an equilibrium of the game.

Lemma 5. *Given the audit schedule, the first-order conditions of the firm's problem are necessary and sufficient to characterize the firm's optimal behavior.*

Proof. See appendix. ■

The probability that a strategic type x is audited is defined as $p^*(r(x))$.

We have

$$p^*(r) = \frac{t}{\theta+t}(1 - \exp\left(-\int_r^{r(b)} 1/\left(x(y) - y\right) dy\right).$$

With the change of variable $x = x(r)$, we get :

$$p(x) = p^*(r(x)) = \frac{t}{\theta+t}(1 - \exp\left(-\int_x^b \frac{r'(z)}{(z - r(z))} dz\right).$$

$$p^*(r(x)) = \frac{t}{\theta+t}(1 - \exp\left(-\int_x^b \frac{f(z)}{\gamma f(r(z))(z - r(z))} dz\right)$$

since $r'(z) = \frac{f(z)}{\gamma f(r(z))}$.

We still need to find the value of γ (or q) as a function of the primitives of the model in terms of the agency budget. We will show that the number of audits made given a value of q is increasing in q. (More audits leads to a lower success of audits). For that, we will make the following assumption on the distribution $F(\cdot)$.

Assumption 4. *$F(\cdot)$ has a decreasing hazard rate : $f(x)/F(x)$ is decreasing in x.*

This allows us to prove that the amount of underreporting is increasing in the true income, that is $x - r(x)$ is increasing in x. We can also show that when q increases, the probability of audit corresponding to a report x is increasing. That means that honest firms are audited more often when q increases. We can also show that this is also the case for the probability that a strategic firm with income.x. When q increases, he is increasing his report (which goes toward a lower audit probability) but the audit schedule shifts up. The total effect is positive, which shows that honest firms are audited more.

This means that an increase in q leads to more audits. This proves that for a given budget for the auditing agency, there exists a unique equlibrium of the auditing game.

Lemma 6. ***(increasing cheating)*** *$x-r(x)$ is increasing in x, that is, firms with higher true tax base cheat more.*

Lemma 7. *The number of strategic and honest firms audited are both increasing in q.*

The proofs are in the appendix.

The total number of audits for a given q is

$$N(q)=\lambda\int_a^{r(b)}p^*\left(r\right)f\left(r\right)dr+(1-\lambda)\int_a^b p^*\left(r\left(x\right)\right)f\left(x\right)dx.$$

Given the previous calculations, we have $\frac{\partial}{\partial q}N(q)\geq 0$. There is thus a unique q for which the number of audits implied by the equlibrium audit schedule adds to the budget of the agency.

We can then summarize our findings in the following proposition.

Proposition 8. *There is a solution to the auditing game. It is characterized by:*

$$r\left(x\right)=F^{-1}\left(F\left(x\right)/\gamma\right)$$

$$p^*\left(r\right)=\frac{t}{\theta+t}(1-\exp\left(-\int_r^{F^{-1}(1/\gamma)}1/\left(x(y)-y\right)\cdot dy\right)$$

$$\gamma=\frac{\lambda\left(1-q\right)}{\left(1-\lambda\right)q}$$

$$B/c=\int_a^{r(b)}p^*\left(r\right)\lambda f\left(r\right)+\int_a^{r(b)}p^*\left(r\right)\left(1-\lambda\right)f\left(x(r)\right)x^{'}\left(r\right)dr.$$

5.3. The case of the general uniform

6. Conclusion

If our model is correct then get upper bound on fraction of tax evasors in Italy.

7. Appendix: Identification with nonlinear π

$$D(r,k;B) = \{d : r_d = r, K(d) = k | B\}.$$

$$\widehat{p}_k(r;B) = \frac{\sum_{d\in D(r,k;B)} 1}{\#\text{ of firms in the population which are of class } k \text{ and report } r}$$

$$\overline{h}(r,k;B) = \frac{\sum_{d\in D(r,k;B)} h(x_d, r_d, \widehat{p}_k(r;B))}{\sum_{d\in D(r,k;B)} 1}$$

$$p\lim \overline{h}(r,k;B) = \pi(x_k^*(r), r, p_k^*(r;B))$$

Proposition 9. *Suppose there are no latent auditing classes, and suppose we observe the universe of firms subjected to auditing. Suppose further that we observe exogenous variation in the auditing budget which the firms cannot observe. If, as the number of observations grows large and* $B_2 \to B_1$, *the ratio* $\frac{\overline{h}(r,k;B_2)-\overline{h}(r,k;B_1)}{\widehat{p}_k(r;B_2)-\widehat{p}_k(r;B_1)}$ *is not independent of* r *and* k, *then one can reject the hypotheses that (a) auditor can/does not publicly commit to an auditing schedule, and (b) the auditor maximizes* $h(x,r,p)$. *Conversely, if a function* $h(x,r,p)$ *can be found such that one cannot reject the hypothesis that as the number of observations grows large and* $B_2 \to B_1$, *the ratio* $\frac{\overline{h}(r,k;B_2)-\overline{h}(r,k;B_1)}{\widehat{p}_k(r;B_2)-\widehat{p}_k(r;B_1)}$ *is not independent of* r *and* k, *then one cannot reject the hypotheses that (a) auditor can/does not publicly commit to an auditing schedule, and (b) the auditor maximizes* $h(x,r,p)$.

Proof. Denote by $\widetilde{B}$ the budget constraint, which in this case is a random variable from the point of view of the firms. Let $r_k^*(x)$ represent the firms' optimal reporting strategy in this incomplete-information game. The auditor, observing the realization B of $\widetilde{B}$, solves

$$\mathcal{L}(\{p_k(\cdot)\}_k;\lambda) = \sum_k G(k)\int_a^b \pi(x, r_k^*(x), p_k(r_k^*(x)))\ f_k(x)\,dx$$
$$-\lambda\left[\sum_k G(k)\int_a^b p_k(r_k^*(x))\,f_k(x)\,dx - B\right].$$

Denote by $\{p_k^*(r_k^*(x);B)\}_k$ the solution to the above problem, that is, the probability of auditing a report equal to $r_k^*(x)$ when the budget constraint equals B. The FOCs are given by

$$\pi_p(x, r_k^*(x), p_k^*(r_k^*(x);B)) - \lambda = 0 \text{ for each } x,k \text{ such that } p_k^*(r_k^*(x);B) > 0. \tag{14}$$

Now, take two different budget constraints B_1, B_2, and form the ratio

$$\frac{\overline{h}\,(r,k;B_2)-\overline{h}\,(r,k;B_1)}{\widehat{p}_k\,(r;B_2)-\widehat{p}_k\,(r;B_1)}. \tag{15}$$

As the number of observations grows large, if (b) is true and the auditor maximizes $h(x,r,p)$, then this ratio converges to

$$\frac{\pi(x^*\,(r)\,,r,p_k^*\,(r;B_2))-\pi(x^*\,(r)\,,r,p_k^*\,(r;B_1))}{p_k^*\,(r;B_2)-p_k^*\,(r;B_1)}.$$

Taking the limit as $B_2 \rightarrow B_1$, the ratio converges to $\pi_p(x^*\,(r)\,,r,p_k^*\,(r;B_1))$, which by (14) is equal to λ for all r,k such that $p_k^*\,(r) > 0$. Thus we have shown that if the auditor maximizes $h(x,r,p)$ then as the number of observations grows and $B_2 \rightarrow B_1$ the ratio (15) converges to a constant λ independent of r,k. ■

7.1. Extra Material on Identification

7.2. No Commitment

Proposition 10. *Suppose Assumption 3 holds. Then the expected returns from each audit are equalized. Moreover, it is possible to identify* $\pi(x, r^*(x), p(r^*(x)))$ *up to an additive term involving* $(x, r(x))$.

Proof. If the auditor cannot commit to an auditing schedule p the equilibrium is given by the following conditions.

$$\begin{aligned} p^*(\cdot) &\in \arg\max_{p(\cdot)} \int_a^b \pi(x, r(x), p(r(x)))\ f(x)\,dx \\ \text{st: } &\int_a^b p(r(x))\, f(x)\, dx \leq B \\ r(x) &\in \arg\max_r \kappa(x, r, p^*(r)). \end{aligned} \tag{16}$$

Let $r^*(x; p^*(r))$ denote the reporting strategy of firms that solves (16). Note a key property of the no-commitment case: since condition (16) involves $p^*(r)$, not $p(r)$, the behavior of firms is a function of the auditor's expected equilibrium strategy, not of the actual strategy employed by the auditor. We shall write, for ease of notation, $r^*(x; p^*(r)) = r^*(x)$. Form the Lagrangean for the auditor's problem:

$$\mathcal{L}(p(\cdot); \lambda) = \int_a^b \pi(x, r^*(x), p(r^*(x)))\ f(x)\,dx - \lambda\left[\int_a^b p(r^*(x))\, f(x)\,dx - B\right].$$

Due to Assumption 3 we can write

$$\pi(x, r^*(x), p(r^*(x))) = A(x, r^*(x)) + p(r^*(x))\, C(x, r^*(x));$$

substituting into the Lagrangean and rearranging yields

$$\mathcal{L}(p(\cdot); \lambda) = \int_a^b [C(x, r^*(x)) - \lambda]\, p(r^*(x))\ f(x)\,dx + \int_a^b A(x, r^*(x)) f(x)\,dx + \lambda B.$$

As the Lagrangean is linear in $p(\cdot)$, the necessary conditions for optimality of the auditor's strategy are that, for any r such that $p(r) > 0$, we must have

$$C(x, r^*(x)) = \lambda + \text{const.} \tag{17}$$

Empirically, $C(x, r^*(x))$ represents the return from an audit, and equation (17) says that in equilibrium this return is equalized for all audits of all r. ■

This equilibrium condition suggests a straightforward identification strategy: if we know that we are in the commitment case, we can try out various "economically reasonable" functions $C(x, r^*(x))$ and check which, if any, has the property that it is equalized across all reports that are audited. If such a function is found, then this is identified as π.

Proposition 11. *Suppose the budget constraint varies in a way that is observable to the econometrician but not to the firms. Then, as the constraint varies, the marginal returns from audits are equalized. Moreover, it is possible to identify* $\pi(x, r^*(x), p(r^*(x)))$ *locally up to an additive term involving* $(x, r(x))$.

Proof. Denote by $\widetilde{B}$ the budget constraint, which in this case is a random variable from the point of view of the firms. The auditor observes the realization B of $\widetilde{B}$, and solves

$$\mathcal{L}(p(\cdot);\lambda) = \int_a^b \pi(x, r^*(x), p(r^*(x)))\ f(x)\,dx - \lambda\left[\int_a^b p(r^*(x)) f(x)\,dx - B\right].$$

The FOCs are given by

$$\pi_3(x, r^*(x), p(r^*(x))) - \lambda = 0 \text{ for each } x$$

Denote by $p(r^*(x); B_i)$ the probability of auditing a report $r^*(x)$ when $\widetilde{B} = B_i$. Then

$$\pi_3(x, r^*(x), p(r^*(x))) = \lim_{B_2 \to B_1} \frac{\pi(x, r^*(x), p(r^*(x); B_2)) - \pi(x, r^*(x), p(r^*(x); B_1))}{p(r^*(x); B_2) - p(r^*(x); B_1)}$$

■

7.3. Commitment

8. Appendix (proofs)

Proof. (Lemma 2):

Let R denote the set of all reports used by strategic firms. Then from (12) we have

$$\int_R \lambda f(r)\,dr = q\left(\int_R \lambda f(r)\,dr + \int_R (1-\lambda) f(x(r)) \frac{dx(r)}{dr} dr\right),$$

or equivalently

$$(1-q)\lambda\int_R f(r)\,dr = q(1-\lambda)\int_R f(x(r))\frac{dx(r)}{dr}dr = q(1-\lambda),$$

where the last equality follows from the definition of R. Now, in any equilibrium $R\subseteq[a,b]$, and so $\int_R f(r)\,dr\leq 1$ Then the previous equality implies

$$(1-q)\lambda\geq q(1-\lambda),$$

whence $\lambda\geq q$.

Proof. (Lemma 4) ■

From, 10, we need to show that $(t+\theta)[p''(r)(r-x)+2p'(r)]\leq 0$.

We have

$$p^{*\prime}(r) = -\frac{t}{\theta+t}\exp\left(-\int_r^{r(b)}\frac{1}{(x(y)-y)}dy\right)\frac{1}{(x(r)-r)},$$

$$p^{*\prime\prime}(r) = -\frac{t}{\theta+t}\exp\left(-\int_r^{r(b)}\frac{1}{(x(y)-y)}dy\right)\left[\frac{1}{(x(r)-r)^2}+\frac{\partial}{\partial r}\frac{1}{((x(r)-r))}\right].$$

We can then compute

$$\begin{aligned}&p''(r)(r-x(r))+2p'(r)\\&=\frac{t}{\theta+t}\exp\left(-\int_r^{r(b)}\frac{1}{(x(y)-y)}dy\right)(r-x(r))\left[\frac{1}{(x(r)-r)^2}-\frac{\partial}{\partial r}\frac{1}{(x(r)-r)}\right].\end{aligned}$$

The sign of $(p''(r)(r-x)+2p'(r))$ is the opposite as the sign of $\left(\frac{1}{(x(r)-r)^2}-\frac{\partial}{\partial r}\frac{1}{(x(r)-r)}\right)$.

Now, we have

$$\begin{aligned}\frac{1}{(x(r)-r)^2}-\frac{\partial}{\partial r}\frac{1}{(x(r)-r)} &= \frac{1}{(x(r)-r)^2}+\frac{x'(r)-1}{(x(r)-r)^2}\\&=\frac{x'(r)}{(x(r)-r)^2}>0.\end{aligned}$$

Thus reporting $r^*(x)$ is indeed a local maximum for a firm with true tax base x. If any other local maxima exist which do not satisfy the first order conditions, they must be at the corners of the feasible report sets, either $r = 0$ or $r = x$. But neither can be a local maximum, for otherwise there would have to be at least one other solution to the first order conditions between $r^*(x)$ and 0, or $r^*(x)$ and x, whereas we know that $r^*(x)$ is the unique solution to the first order conditions. Therefore, reporting $r^*(x)$ is also a *global* maximum for a firm with true tax base x. ■

Proof. (Lemma 5)

Proving that that $x - r(x)$ is increasing is equivalent to show that $r'(x) \leq 1$. We have :

$$r'(x) = \frac{f(x)}{\gamma f(r(x))}$$

From decreasing hazard rate property we have

$$\frac{f(r(x))}{F(r(x))} \geq \frac{f(x)}{F(x)},$$

using the fact that $F(x) = \gamma F(r(x))$, we get:

$$\gamma f(r(x)) \geq f(x).$$

whence $r'(x) \leq 1$.

Proof. (Lemma 6) ■

Note first that $\partial\gamma/\partial q \leq 0$. So we will do the calculation with respect to γ. We just need to take the opposite sign to get the result for q. Let's now show that honest firms are audited more when q increases, that is, $\partial p(r)/\partial q \geq 0$ or $\partial p(r)/\partial\gamma \leq 0$. We have:

$$\frac{\partial p(r)}{\partial\gamma} = \frac{t}{\theta + t} \exp\left(-\int_r^{r(b)} (1/(x(y) - y))\, dy\right) \frac{\partial}{\partial\gamma} \int_r^{r(b)} (1/(x(y) - y))\, dy$$

which is the same sign as

$$\frac{\partial}{\partial\gamma} \int_r^{r(b)} (1/(x(y) - y))\, dy = \frac{\partial r(b)}{\partial\gamma} \frac{1}{b - r(b)} + \int_r^{r(b)} \frac{\partial}{\partial\gamma} (1/(x(y) - y))\, dy$$

which is negative since

$$\frac{\partial}{\partial\gamma}r\left(b\right)=\frac{\partial}{\partial\gamma}F^{-1}(1/\gamma)=\frac{-1}{\gamma^{2}f\left(1/\gamma\right)}<0,$$

$$\text{and } \frac{\partial}{\partial\gamma}\left(1/\left(F^{-1}\left(\gamma F\left(y\right)\right)-y\right)\right)=-\frac{F(y)/f\left(\gamma F\left(y\right)\right)}{\left(F^{-1}\left(\gamma F\left(y\right)\right)-y\right)^{2}}<0.$$

It remains to show that strategic firms are audited more, that is $\partial p(x)/\partial q\geq 0$ or $\partial p(x)/\partial\gamma\leq 0$.

We have that

$$p(x)=p^{*}\left(r(x)\right)=\frac{t}{\theta+t}(1-\exp\left(-\int_{x}^{b}\frac{r^{'}\left(z\right)}{\left(z-r\left(z\right)\right)}dz\right).$$

The sign of $\partial p(x)/\partial\gamma$ is the same as the sign as $\partial\left(\int_{x}^{b}\frac{r^{'}(z)}{(z-r(z))}dz\right)/\partial\gamma$.

We have :

$$\frac{\partial}{\partial\gamma}\left(\int_{x}^{b}\frac{r^{'}\left(z\right)}{\left(z-r\left(z\right)\right)}dz\right)=\int_{x}^{b}\frac{\partial}{\partial\gamma}\left(\frac{r^{'}\left(z\right)}{\left(z-r\left(z\right)\right)}\right)dz.$$

Now we get:

$$\begin{aligned}\frac{\partial}{\partial\gamma}\left(\frac{r^{'}\left(z\right)}{\left(z-r\left(z\right)\right)}\right)&=\frac{\left(\frac{\partial}{\partial\gamma}r^{'}\left(z\right)\right)\left(z-r\left(z\right)\right)-r^{'}\left(z\right)\left(\frac{\partial}{\partial\gamma}\left(z-r\left(z\right)\right)\right)}{\left(z-r\left(z\right)\right)^{2}}\\&=\frac{\left(\frac{\partial}{\partial\gamma}r^{'}\left(z\right)\right)\left(z-r\left(z\right)\right)+r^{'}\left(z\right)\frac{\partial}{\partial\gamma}r\left(z\right)}{\left(z-r\left(z\right)\right)^{2}}.\end{aligned}$$

Since $\frac{\partial}{\partial\gamma}r\left(z\right)=\frac{-F(r(z))}{\gamma f(r(z))}\leq 0$, we just need to show that $\frac{\partial}{\partial\gamma}r^{'}\left(z\right)\leq 0$ to complete the proof.

$$\frac{\partial}{\partial\gamma}r^{'}\left(z\right)=\frac{\partial}{\partial\gamma}\left(\frac{f\left(z\right)}{\gamma f\left(r\left(z\right)\right)}\right)=-\frac{f\left(z\right)}{\left(\gamma f\left(r\left(z\right)\right)\right)^{2}}\left(f\left(r\left(z\right)\right)+f^{'}\left(r\left(z\right)\right)\frac{\partial}{\partial\gamma}r\left(z\right)\right)$$

Now,

$$f\left(r\left(z\right)\right)+f^{'}\left(r\left(z\right)\right)\frac{\partial r\left(z\right)}{\partial\gamma}=f\left(r\left(z\right)\right)-f^{'}\left(r\left(z\right)\right)\frac{F(r\left(z\right))}{\gamma f\left(r\left(z\right)\right)}$$

The decreasing hazard rate property can be rewritten as:

$$\frac{\partial}{\partial x}\left(f(x)/F(x)\right) = \frac{f^{'}(x)\,F(x) - f^{2}(x)}{F^{2}(x)} \leq 0$$
$$or \;\; f^{'}(x)\,F(x) \leq f^{2}(x)\,.$$

$f\left(r\left(z\right)\right) - f^{'}\left(r\left(z\right)\right)\frac{F(r(z))}{\gamma f(r(z))} \geq 0$ is equivalent to showing that $f^{'}\left(r\left(z\right)\right)F\left(\left(r\left(z\right)\right)\right) \leq \gamma f^{2}\left(\left(r\left(z\right)\right)\right)$ which is true from the previous inequality and the fact that $\gamma \geq 1$. ■

9. References

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