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Multidimensional poverty indices: A critical assessment

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Abstract:
This paper reviews and assesses issues involved in the measurement of multidimensional poverty, in particular the soundness of the various "axioms" and properties often imposed on poverty indices. It argues that some of these properties (such as those relating poverty and inequality) may be sound in a unidimensional setting but not so in a multidimensional one. Second, it addresses critically some of the features of recently proposed multidimensional poverty indices, in particular the Multidimensional Poverty Index (MPI) recently put forward by the United Nations Development Program (UNDP). The MPI suffers from several unattractive features that need to be better understood (given the prominence of the index). The MPI fails in particular to meet all of three properties that one would expect multidimensional poverty indices to obey: continuity, monotonicity, and sensitivity to multiple deprivation. Robustness techniques to address some of the shortcomings of the use of such indices are briefly advocated.

Keywords: Multidimensional poverty; United Nations Development Program; Poverty; Inequality.

JEL Classification: D31; D63; I32; O15;

1 Introduction

1.1 Beware of the use of popular indices

The last four decades have witnessed considerable progress in welfare economics. Seminal contributions, such as those of Watts (1968), Kolm (1969), Atkinson (1970), and Sen (1976), have led to a better understanding of the procedures and indices for measuring and comparing social welfare, inequality, and poverty. They have also made analysts better aware of the pitfalls associated with the use of specific indices.

In particular, the flaws of the use of the popular headcount (ratio) index for measuring poverty are now well understood (see, for instance, Cowell (2016)). The headcount — namely, the proportion of a population living in poverty — is discontinuous at the poverty line: because of the headcount's 0/1 dichotomization between the poor and the non-poor, small changes in the living standards of those close to the poverty line can bring about substantial changes to the headcount. Because of this discontinuity, the headcount can also be disproportionately sensitive to small changes in the poverty line, it can increase following an equalization of living standards, and it can also support policies that benefit the not-so-poor at the expense of the poorest of the poor. Furthermore, the headcount does not necessarily fall when the living standards of poor individuals increase; technically speaking, the headcount is not strictly decreasing in the living standards of the poor. Because of this, the use of the headcount can unfortunately lead to misleading rankings of poverty across distributions and to unfortunate policy guidance.

In part because of the above-mentioned difficulties associated with the use of specific poverty indices, techniques to assess the normative soundness and to check the robustness of unidimensional poverty comparisons have also been designed and developed in the last decades, both theoretically and statistically. The use of these techniques (derived from the stochastic dominance literature) can help identify those situations in which poverty comparisons are sensitive to the choice of indices. They can also help researchers understand why such sensitivity exists and further serve to quantify the degree of support for stating, for instance, that one distribution has more poverty than another (see, for example, Atkinson 1987, Foster and Shorrocks 1988a, Foster and Shorrocks 1988b and Davidson and Duclos 2000).

Overall, because of the greater awareness of the pitfalls associated with specific and potentially flawed indices, it can reasonably be stated that greater care is nowadays usually applied to the analysis of unidimensional poverty.

1.2 From unidimensional to multidimensional welfare

Recent developments in welfare economics have also stressed the need to incorporate multidimensional aspects of wellbeing in assessing poverty — in large part because of the influence of Sen (1983) and Sen (1985); see also, for instance, Tsui (2002), Atkinson (2003), Bourguignon and Chakravarty (2003) and Chakravarty and Lugo (2016). In this context, it would seem natural to check whether apparently natural multidimensional poverty indices may lead to inappropriate poverty rankings and undesirable policy guidance.

The conceptual issues for multidimensional poverty measurement are both similar to and different from those found in a unidimensional setting. First, there is the role of discontinuity. As mentioned above, discontinuity can introduce unfortunate features into poverty measurement and poverty alleviation policy. This paper discusses in details below why these features are particularly worrying in a multidimensional setting. Second, there is the view that multidimensional poverty measures should be strictly decreasing in the living standards of the poor. Third, there is the fact that inequality does not necessarily have the same interpretation in a unidimensional and in a multidimensional setting. For instance, multidimensional welfare can become *more* unequal following *decreases* in the inequality of one of the component dimensions. Therefore, it may well be fine to expect that decreases in unidimensional inequality (*i.e.*, in one of the component dimensions) should lower unidimensional poverty but that such decreases should not necessarily lower (or should even increase) multidimensional poverty. Finally, decreases in multidimensional inequality can sometimes be reasonably argued to increase multidimensional poverty.

Properties that may make sense in a unidimensional setting may therefore not be sensible in a multidimensional one. It is thus important first to investigate which properties are truly desirable (in the sense of not being simply copied from the unidimensional literature) in a multidimensional setting. It may then be found inappropriate to impose certain unidimensional poverty measurement properties on multidimensional poverty indices.

There also exist properties that are specific to multidimensional poverty measurement. One such important and reasonable property requires poverty indices to increase with the extent of *multiple* deprivation. For fixed distributions of deprivation in all of the component dimensions, this increase in multiple deprivation follows from an increase in component correlation. This property would seem to lie at the core of multidimensional poverty measurement: it says that if poverty stays constant in all of the component dimensions, but that instances of multiple deprivation fall, then multidimensional poverty should also fall. Unfortunately, as we will see below, several of the popular multidimensional poverty indices fail to obey that property. This is particularly the case of UNDP's recent multidimensional poverty index (MPI).

1.3 Structure of the paper

The rest of this paper proceeds as follows. Section 2 reviews some of the indices and some of the important features of the framework usually considered for measuring and comparing multidimensional poverty — see, for instance, Tsui (2002), Atkinson (2003) and Bourguignon and Chakravarty (2003) for early examples. In addition to introducing a number of multidimensional indices introduced in the literature, the section lists a number of properties (in the form of axioms) and assesses which of these would appear desirable within a multidimensional poverty setting.

Some of the desirable properties (such as monotonicity) discussed in Section 2 may be more relevant for some types of data (such as cardinal component dimensions), but others (such as sensitivity to multiple deprivation) are important *even* for binary, categorical, or other discrete data indicators of component welfare. At a more general level, the objective of the Section is to delimit a set of useful properties for multidimensional poverty measurement and see which of the existing indices fulfill those properties. The best data procedures can then be sought for estimating these indices.

This leads to Section 3, which describes some of the possible difficulties associated with the use of indices such as the MPI. Section 4 briefly advocates ways of addressing important issues of arbitrariness and possible lack of robustness in making multidimensional poverty comparisons.

2 Concepts and Measurement

2.1 Measuring multidimensional poverty in practice

Considering the multiple dimensions of welfare and poverty has become increasingly important for understanding development over the recent years (see, for instance, Alkire (2016)). Several prominent contributions have been made in this regard. One of the most influential has been UNDP's human development index (HDI) (see for instance the annual issues of UNDP's Human Development Reports since 1990, UNDP 1990-2014), which has popularized the use of three dimensions — life expectancy, educational attainment, and living standards — in constructing multidimensional indices of development.

Before proceeding to a more formal definition of multidimensional poverty indices, note that their construction involves several steps. First, the number, the nature, and the weight of the dimensions of interest must be chosen, an exercise that is fraught with difficulties and that can generate considerable disagreement. Second, one must decide whether aggregation within each dimensional attribute must first be performed (as in the case of the HDI/HPI, see equations (1) and (2) below), or whether aggregation across attributes and for each individual must first be carried out (as for the MPI).[1]

If, in the measurement exercise, one wishes to discriminate between multiple deprivation for a single individual and single deprivation for multiple individuals, then one must necessarily aggregate first across attributes for each individual and then across individuals — this is, for instance, an implication of the multiple-deprivation-sensitivity Axiom 10 described on page 15 below. Another

[1]Other decisions include unit of identification and choice of dimensions.

implication of this is that such multidimensional indices cannot be “attribute-additive” (Axiom 8 on page 13); multidimensional poverty indices must take into account the joint distribution of deprivations.

The alternative, known as the “dashboard” approach (see *inter alia* Ravallion 2011), is to compute dimension-specific poverty indices. These indices can then be aggregated to form a composite index of poverty. One example of this is given by the HDI_j, as originally proposed in UNDP’s 1990 Human Development Report for a country j:

$$HDI_j = (1 - \frac{1}{3}\sum_{i=1}^{3} \delta_{i,j}), \tag{1}$$

where $\delta_{i,j} = \frac{(\max x_{i,.} - x_{i,j})}{(\max x_{i,.} - \min x_{i,.})}$, with $\delta_{i,j}$ being a measure of the deprivation of country j in attribute x_i (*i.e.*, life expectancy, literacy/average years of schooling and the log of per capita income/GDP) and where the min and the max operators are taken over the values $x_{i,.}$ of the entire sample of countries.

As should be clear from (1), the HDI is a weighted average of dimension-specific indicators of development — or one minus a composite indicator of deprivation. Because it does not take into account the *distribution* of individual welfare within each of the dimensions, the HDI does not capture *inequalities* in the distributions of life expectancy, educational attainment and income.

An alternative index, the Human Poverty Index (HPI), is a composite indicator of poverty proposed in the UNDP’s 1997 Human Development Report (see also Anand and Sen 1997). Let HPI_1, HPI_2, and HPI_3 represent respectively the probability at birth of not surviving to age 40 in a particular country; the adult illiteracy rate; and the mean of the percentage of the population without access to safe water, of the percentage of children under five that are underweight and of the percentage of the population without access to healthcare. HPI is then defined as

$$HPI = (\lambda_1 HPI_1^{\varepsilon} + \lambda_2 HPI_2^{\varepsilon} + \lambda_3 HPI_3^{\varepsilon})^{\frac{1}{\varepsilon}}, \tag{2}$$

with λ_i (typically 1/3) being the weight associated to dimension i, $\lambda_1 + \lambda_2 + \lambda_3 = 1$, and $\varepsilon \geq 1$ (for the iso-poverty curves to be concave to the origin in the HPI_i; points on the same iso-poverty curve yield the same poverty level). $1/(\varepsilon - 1)$ is the elasticity of substitution between the dimensions. With $\varepsilon = 1$, the HPI is the arithmetic average of dimensional HPI; the three attributes are then perfect substitutes. With ε tending to infinity, the index approaches the maximum value of its three components; the iso-poverty curves are then rectangular and the attributes are completely imperfect substitutes.

Seen in a Pigou-Dalton-transfer perspective (see Axiom 9 on page 14 for a statement), this composite indicator of poverty can *decrease* when a transfer in some attribute is made from a poorer to a richer individual. This can happen when such a transfer in one dimension of welfare (such as weight or life duration) pulls a poorer individual even further away from the poverty line while pushing a less poor individual out of poverty. As with the HDI, the contribution of one individual's deprivation in each of the dimensions is assessed independently of the contribution of that same individual in other dimensions, since individual deprivations are first summed across individuals to form a dimension-specific index and then aggregated. Hence, *ceteris paribus*, the HDI and the HPI cannot tell whether in some societies the poor suffer more from multiple deprivation than in other societies. Because of this, both of these indices are better understood as composite indicators of unidimensional indices than as truly multidimensional poverty indices.

The distinction between composite indicators of dimensional deprivation and multidimensional poverty is important since there are good reasons to believe that multidimensional poverty is more than just the aggregation of dimensional poverty. The interaction across dimensions is important both for normative and for positive reasons. From a normative perspective, a society with a greater extent of multiple deprivation would be judged by most analysts as worse, everything else being the same. From a positive perspective, a conjunction of deprivations in multiple dimensions would be expected to cause greater individual harm and have a greater impact on individual and social behavior (such as participation in credit markets and the emergence of social uprisings), both in the short term and in the longer term, than if these deprivations were more evenly spread across individuals.

The UNDP has recently implemented a measure that tries to remedy those shortcomings of the HDI and HPI (see UNDP 2010). The remedy takes the form of a multidimensional poverty index (MPI). The UNDP has since then continued to use the MPI in its annual reports on human development. A considerable number of studies applying the MPI to developing countries have also been produced (see, for example, Alkire and Santos 2014, which discusses results obtained for over 100 developing countries).

The MPI aggregates individual deprivations in areas such as education, health outcomes, and assets and services *only* of those whose (weighted) number of deprived dimensions exceeds a "dual" cutoff, namely, a cutoff in the number of dimensions in which individuals may be poor — see equation (14) for a precise definition. By focusing only on those individuals, the MPI gives more importance to deprivation in multiple dimensions than the HDI and the HPI, which are neutral to the incidence of multiple deprivation.

As will be discussed further below in section 3, the MPI does suffer, however, from several unattractive features that have not yet been sufficiently noted in the literature (though mentioned in Alkire and Foster 2011). These are mostly caused by measurement discontinuities. These discontinuities lead to unfortunate properties: the index may, for instance, increase following 1) a transfer of one attribute from a richer to a poorer person; 2) an equalization of an attribute across the poor; 3) an equalization of all attributes across the population; 4) a fall in the incidence of multiple deprivation. This last point is particularly important: greater multiple deprivation, for fixed levels of unidimensional poverty, may lead to lower multidimensional poverty as measured by the MPI.

2.2 Formal setting

To proceed to the formal definition of multidimensional poverty indices, it is useful to let $\mathbf{x}_i$, $i = 1, 2, ..., N$, be a $(1 \times K)$ vector of K dimensions of welfare ("attribute" or "component" k) for the ith individual, $x_{i,k}$ be attribute k for individual i, $\mathbb{X}$ be a $(N \times K)$-matrix (whose ith row is $\mathbf{x}_i$) showing the distribution of the K dimensions across the N persons, and $\mathbf{z} = (z_1, ..., z_k, ..., z_K)$

be a $(1 \times K)$-vector of poverty lines (in units of $\mathbf{x}_i$). Multidimensional poverty indices P can be generally defined as $P(\mathbb{X}; \mathbf{z})$. Let the cumulative joint distribution function (normalized to lying between 0 and 1) of the distribution of attributes in $\mathbb{X}$ be given by $F_{\mathbb{X}}(\mathbf{x})$.

Population-additive multidimensional poverty indices can then be defined as

$$P(\mathbb{X}; \mathbf{z}) = \int \pi(\mathbf{x}; \mathbf{z}) dF_{\mathbb{X}}(\mathbf{x}), \tag{3}$$

where $\pi(\mathbf{x}; \mathbf{z})$ is an individual-level aggregation of deprivation. Equation (3) integrates ("adds up") the deprivation $\pi(\mathbf{x}; \mathbf{z})$ of all individuals, as distributed according to $F_{\mathbb{X}}(\mathbf{x})$. $\int \ldots dF_{\mathbb{X}}(\mathbf{x})$ can effectively be replaced by $N^{-1} \sum$ when the distribution is discrete, which gives

$$P(\mathbb{X}; \mathbf{z}) = N^{-1} \sum_{i=1}^{N} \pi(\mathbf{x}_i; \mathbf{z}). \tag{4}$$

Additive multidimensional poverty indices are thus an average of individual poverty.

This general definition of multidimensional poverty indices can serve to distinguish between $union$ and $intersection$ indices. Intersection indices take the particular form of

$$P(\mathbb{X}; \mathbf{z}) = \int_{0}^{\mathbf{z}} \pi(\mathbf{x}; \mathbf{z}) dF_{\mathbb{X}}(\mathbf{x}); \tag{5}$$

only those individuals i that are poor in *all* dimensions ($x_{i,k} < z_k$, for all k) — that is, those whose attributes lie between $(0, ..., 0)$ and $(z_1, ..., z_K)$, which is the domain of the integral — contribute to total poverty. A particular form for union indices is given by

$$P(\mathbb{X}; \mathbf{z}) = \int \pi\left(\pi_1(\mathbf{x}; \mathbf{z}), \ldots, \pi_K(\mathbf{x}; \mathbf{z})\right) dF_{\mathbb{X}}(\mathbf{x}), \tag{6}$$

with the property that the contribution of dimension k, $\pi_k(\mathbf{x}; \mathbf{z})$, is larger than zero if and only if $x_k < z_k$. Equation (6) says that someone's poverty contribution π cannot be reduced through increases in x_k whenever $x_k \geq z_k$ since $\pi_k(\mathbf{x}; \mathbf{z})$ is then fixed to 0 for those values of x_k.

A specific formulation of π leads to the multidimensional intersection headcount index, namely, the proportion of individuals whose attributes all lie below the attributes' respective poverty lines. Let $\pi(\mathbf{x};\mathbf{z}) = 1$ in (5); this leads to

$$P(\mathbb{X};\mathbf{z}) = F_{\mathbb{X}}(\mathbf{z}), \tag{7}$$

which is the multidimensional intersection headcount poverty index.

Now let $\pi(\pi_1(\mathbf{x};\mathbf{z}), \dots, \pi_K(\mathbf{x};\mathbf{z})) = \max(I(x_1 < z_1), \dots, I(x_K < z_K))$ in (6), with $I(\cdot)$ being the indicator function, that is, equal to 1 if its argument is true and to 0 otherwise. (6) then becomes:

$$P(\mathbb{X};\mathbf{z}) = 1 - \int_{\mathbf{z}}^{\infty} dF_{\mathbb{X}}(\mathbf{x}). \tag{8}$$

(8) is the multidimensional union headcount index: this is the proportion of individuals with at least one attribute below that attribute's poverty line.

We can distinguish the different notions of intersection and union poverty in a different manner. With two dimensions, let

$$\lambda(x_1, x_2) \tag{9}$$

be an aggregate (or 'inclusive') indicator of individual welfare (as opposed to π, which aggregates deprivation). It can be safely assumed that both dimensions contribute positively to the individual's aggregate welfare. Then suppose that an unknown poverty frontier separates the poor from the rich; as in Figure 1, let that frontier be defined by $\lambda(\mathbf{x}) = 0$.

< insert Figure 1 about here >

The set of the poor is then obtained as:

$$\Lambda(\lambda) = \{(x_1, x_2)\,|(\lambda(\mathbf{x}) \leq 0\}. \tag{10}$$

Figure 1 illustrates this two-dimensional case by showing two dimensions of welfare, x_1 and x_2, and those combinations of x_1 and x_2 that produce the same frontier level of overall individual welfare (those combinations appear on the iso-welfare lines denoted by $\lambda(\mathbf{x}) = 0$). The three lines in Figure 1 also serve to illustrate three different general ways of thinking about multidimensional poverty. The first ($\lambda_1(\mathbf{x}) = 0$) is the intersection view. Under that first view, someone is poor if and only if he is poor in both dimensions. As soon as the value of welfare in one dimension exceeds the dimensional poverty line, the person is judged overall to be above poverty. The second line ($\lambda_2(\mathbf{x}) = 0$) illustrates the union view. However large the value of one dimension may be, the person will be considered to be in poverty if he is poor in no less than one dimension. The third, intermediate, view (with $\lambda_3(\mathbf{x}) = 0$), is more in line with the way in which economists traditionally think about welfare; it requires a joint consideration of the component dimensions before identifying who is poor.

2.3 Axioms

Before introducing additional indices, it is convenient to present some of the axioms that have been traditionally used in the welfare economics literature to assess the soundness of such indices. Many of these axioms are immediate generalizations of axioms used to characterize unidimensional poverty measures (see, for instance, Cowell (2016)); others are specific to a multidimensional poverty setting. The axioms are stated below in a "strong form" (*e.g.*, following a 'change', an index should 'increase'). For more formal and complete statements of these axioms, also see, for instance, Chakravarty and Lugo (2016).

Axiom 1

Symmetry: permutations of the rows of $\mathbb{X}$ *should not affect* $P(\mathbb{X}; \mathbf{z})$.

The position of individuals in the matrix $\mathbb{X}$ does not matter; the poverty measure is "anonymous". (This is implicitly imposed in (3) by the fact that $F_{\mathbb{X}}(\mathbf{x})$ is anonymous.)

Axiom 2

Population size invariance: if the N rows of $\mathbb{X}$ are replicated and added to $\mathbb{X}$, poverty should remain unchanged.

Poverty $P(\mathbb{X};\mathbf{z})$ is independent of N for given $F_{\mathbb{X}}$. (This is implicitly imposed in (3) through the normalization of $F_{\mathbb{X}}(\mathbf{x})$.)

Axiom 3

Scale invariance: poverty should be homogeneous of degree zero with respect to $\mathbb{X}$ and $\mathbf{z}$.

Poverty $P(\mathbb{X};\mathbf{z})$ should remain unchanged following changes in those measurement units used for $\mathbb{X}$ and $\mathbf{z}$; attributes can be normalized by their respective poverty line, and poverty is unaffected if each dimension is separately rescaled.

Axioms 1, 2, and 3 are obeyed by all commonly used unidimensional and multidimensional poverty indices. This does not mean that they are uncontroversial. The symmetry axiom, for instance, can be problematic if issues of horizontal equity and/or temporal poverty matter; see, for instance, Bibi and Duclos (2007) and Grimm (2007). Population size invariance may not be appropriate if, for ethical reasons, we are concerned with the absolute number of poor people, as discussed recently by Cockburn, Duclos, and Zabsonré (2014). Finally, we may not wish scale invariance to hold if we want poverty indices to capture the extent of absolute distances between living standards and the poverty line. This being said, however, most of poverty analysis would typically fit comfortably under Axioms 1, 2, and 3, and we will therefore take them for granted.

Axiom 4

Continuity (Con): poverty should not be overly sensitive to a small variation in the quantity of an attribute.

The poverty measure $P(\mathbb{X};\mathbf{z})$ should be continuous with respect to $\mathbb{X}$. Small measurement errors or small changes in the values of attributes should not cause a jump in the value of the poverty measure. This is not the case of the poverty headcount index: a small change that pushes a poor individual above the poverty line can generate a large variation in the index, which is not the case of the average poverty gap, for instance. This is a crucial axiom for poverty analysis; failure to obey that axiom has important consequences for poverty indices. One such consequence is to make it difficult for poverty indices to react properly to changes in inequality and/or changes in correlations across attributes, as will be discussed later in the context of multidimensional poverty.

The axiom of continuity is not relevant when component dimensions are measured by binary data, since it is not convenient to think of "small changes" with those data, but indices that obey the continuity property can still be used with binary data just as with any other types of welfare data. With discrete/non-binary data, the Continuity axiom can be relaxed so as to (intuitively) require that the size of the discontinuity "jumps" (if they exist) falls as dimensional welfare increases.

Axiom 5

Focus (Foc): poverty should be unchanged if the attribute of a person that is not poor in that attribute changes.

This axiom is called the *strong focus axiom* by Bourguignon and Chakravarty (2003). A less strong version (*weak focus axiom*) requires that the *person be non-poor* in the statement of Axiom 5. The *strong* and *weak* focus axioms are equivalent for intersection poverty indices since for such indices a non-poor person must be non-poor in all attributes.

It is not obvious that this focus axiom is desirable. It is, in particular, difficult to imagine why an increase in a non-poor attribute of a poor person, whatever that attribute may be, should not affect the poverty level of that person. This seems particularly doubtful in a context in which welfare is judged as a whole, that is, as a joint function of the levels of welfare in each and in all dimensions. In the context of Figure 1, for instance, it is clear that an increase in x_2 (for someone below $\lambda_3(\mathbf{x})$) will affect both the level of overall welfare and the proximity of that person's welfare to the poverty frontier, even though his x_2 may be above z_2.

Axiom 6

Monotonicity (Mon): poverty should strictly decrease if any poor attribute of a poor individual increases.

This axiom is important if we want poverty alleviation strategies to promote increases in the welfare of poor individuals and if we want those strategies potentially to discriminate in favor of the poorest among the poor individuals. The axiom of strict monotonicity is not necessary when component dimensions are measured by binary data, but such data can be used with indices that obey that property.

The following two axioms address decompositions of total poverty across population subgroups and attributes.

Axiom 7

Subgroup additivity (SA): total poverty should be a population-weighted average of population subgroup poverty.

This axiom implies that poverty reduction for some population subgroup should translate into an overall poverty reduction that is exactly proportional to the population share of that subgroup. The additive formulation of equation (3) implies Axiom 7.

Subgroup additivity (or separability, more generally speaking) is essentially a technical property that has been used in the literature to facilitate the estimation of aggregate (national) poverty and the provision of subnational poverty decompositions. There seems to be no obvious ethical reason why poverty indices must be separable across individuals and/or groups. It may well be, for instance, that poverty measures should take into account the extent of relative deprivation, that is, the extent of differences across individuals. In such a perspective, non-additive poverty index formulations such as those of Gini-type indices (see for instance Sen 1976, Thon 1979 and Duclos and Grégoire 2002) may be normatively better, although they may then not obey Axiom 7.

Axiom 8

Attribute additivity (AA): total poverty should be a weighted average of attribute specific poverty.

This implies that poverty should be measured by *union* indices, as shown by the formulation in (6), with the additional constraint that $\pi\left(\pi_1(\mathbf{x};\mathbf{z}),\ldots,\pi_K(\mathbf{x};\mathbf{z})\right)=\sum_{k=1}^{K}\pi_k(\mathbf{x};\mathbf{z})$. Attribute additivity is therefore a strong property. It is, in particular, incapable of capturing the extent of multiple deprivation: attribute additivity forces multidimensional poverty to be a composite indicator of disjoint dimensional poverty. Because of this and despite its possible attractiveness for facilitating poverty decompositions, the axiom therefore seems inappropriate for multidimensional poverty measurement.

Axiom 9

Pigou-Dalton transfer: poverty should decrease following a progressive transfer in some deprived attribute k from a richer to a poorer person.

This axiom has both a single-attribute formulation (UPDT) and a multiple-attribute one (MPDT). Sen (1976) argues, in a unidimensional setting, that poverty should be sensitive to inequalities: an increase in inequality (at least among the poor) should lead to an increase in poverty. This is an application of the well-known Pigou-Dalton transfer principle (originally used by Pigou and Dalton for thinking about sources of inequality changes) to the measurement of poverty (see also Cowell (2016)).[2]

Properties such as UPDT and MPDT may seem uncontroversial at first sight. Consider, for instance, Figure 2, in which two individuals a and b are brought closer together because of some unidimensional equalization process shown by the arrows. UPDT requires poverty to fall following this change, which would seem intuitive: the greater level of b's poverty is offset by the fall in a's poverty.

< insert Figure 2 about here >

This is not, however, generally true: single-attribute equalization does not necessarily reduce poverty, at least when considered in a multidimensional perspective. To see why, consider Figure 3. Again, two individuals a and b are brought closer together unidimensionally by the arrows. In a multidimensional perspective, however, the two individuals can be reasonably deemed to be farther from each other than initially: a is now unambiguously better off than b, which was not the case initially. If we consider multidimensional welfare, it is therefore reasonable to believe that

[2] Note that, in a unidimensional setting, a vector $\mathbf{x}$ can be said to be more equal than a vector $\mathbf{y}$ if $\mathbf{x}$ can be obtained as the product of a bistochastic matrix and $\mathbf{y}$. A bistochastic matrix is a matrix made of non-negative proportions, whose rows and columns sum to one. The product of such a matrix with $\mathbf{x}$ equalizes welfare across individuals. The generalization to multidimensional setting is analogous: the product of a bistochastic matrix and a matrix $\mathbb{X}$ is more equal than $\mathbb{X}$. The bistochastic transformation equalizes simultaneously the cross-individual distribution of each and every attribute.

poverty has *increased* after this unidimensional equalization process. This is an important point that suggests that we should *avoid* insisting that multidimensional poverty indices obey UPDT. A fall in unidimensional poverty can increase multidimensional poverty.

< insert Figure 3 about here >

Whether multiple-attribute equalization reduces poverty is also uncertain. To see why, consider Figure 4. The equalization movements of a and b in the two-dimensional space illustrate how and why a similar equalization of attributes can reasonably be argued to *increase* overall poverty, at least in some circumstances: a and b are initially close to the poverty frontier; the equalization moves both of them into greater poverty.

< insert Figure 4 about here >

The general message is therefore that the seemingly sensible Pigou-Dalton axiom linking inequality and poverty in a unidimensional setting does not extend into a multidimensional setting. Because of that, it would seem prudent *not* to impose inequality-type axioms for the measurement of multidimensional poverty.

Axiom 10

Sensitivity to multiple deprivation (SMD): let transfers of attributes across individuals leave unchanged the marginal distributions of attributes but increase their correlation among the poor and thus also increase the incidence of multiple deprivation; then poverty should increase.

Axiom 10 says that increasing the incidence of multiple deprivation, without changing the incidence of dimensional deprivation, should increase the measure of multidimensional poverty.

To understand that property better, consider Figure 5. Figure 5 shows the effect of a transfer that moves individual a to position a' and individual b to position b'. This is called in the literature a "correlation-increasing switch". Note that unidimensional poverty is not affected in either dimension: the distribution of both x_1 and x_2 is unchanged. The effect of this switch is generally supposed to increase total poverty.

< insert Figure 5 about here >

Suppose we have two individuals a and b lacking relatively in one of the attributes — see again Figure 5. Both individuals are able to compensate their lower achievement in some attribute with a higher performance in the other. After the transfer, individual b will be better off and individual a will be worse off. Because individual a is less able to "protect himself", his increase in poverty will be greater than b's fall in poverty. Overall poverty should then increase.

This interrelation issue is the main aspect that distinguishes unidimensional and multidimensional poverty analysis. Axiom 10 is indeed designed specifically for *multidimensional* poverty measurement; it is arguably the most important property for such measurement purposes. It would indeed be sound to argue in most applications that an increase in attribute correlation in Figure 5 increases a's poverty more than it decreases b's. Policy advocates often urge us to care first for those who suffer from multiple deprivation, suggesting that these individuals matter more for poverty reduction and measurement than those that are not multiply deprived.

Early attempts at motivating and applying Axiom 10 can be found in work by Atkinson and Bourguignon (1982) (for social welfare and inequality) and Tsui (2002) (for poverty). Both intersection and union poverty indices can obey this axiom. This axiom is important *even* when component dimensions are measured by binary data: the example of Figure 5 is relevant with binary data just as with any other types of discrete/continuous data.

Axiom 10 can also be understood as introducing a cross-individual "substitutability" property: loosely speaking, the more someone has of x_1, the less is overall poverty deemed to be reduced if his value of x_2 is increased. One way to identify whether the poverty measure in (3) exhibits substitutability across individuals is to consider the second-order cross derivative of the individual poverty measure π with respect to any pair of attributes. If the attributes are substitutes, then that derivative is positive:

$$\frac{\partial^2 \pi(\mathbf{x};\mathbf{z})}{\partial x_j \partial x_k} \geq 0. \tag{11}$$

Poverty will then increase following a correlation-increasing switch. When the attributes are "complements", we have instead

$$\frac{\partial^2 \pi(\mathbf{x}_i;\ \mathbf{z})}{\partial x_{ij} \partial x_{i,k}} \leq 0, \tag{12}$$

that is, poverty should decrease following a correlation-increasing switch. If attributes are complements, a rise in one attribute increases the "marginal utility" of the other. After the correlation-increasing transfer, those facing better conditions will see a decrease in their level of individual poverty that is greater than the increase in the poverty of the lesser-off persons.

It should be clear from the above that attribute-additive indices (Axiom 8) cannot strictly obey Axiom 10 or, alternatively, the complementarity property of equation (12).

2.4 Examples of indices

We can now revert to some of the indices already introduced above, present additional ones and interpret all of them in the light of the properties and axioms discussed above. The indices considered are listed in Table 1; whether they obey the main axioms presented above is also indicated in that Table. The axioms that we judge to be the most relevant multidimensionally speaking (**Con**, **Mon**, and **SMD**) are in bold font. The indices (*MFGT*, *Tsui*, *Datt* and *BC*) that obey these three axioms are italicized.

< insert Table 1 >

The HDI does not obey the focus axiom since it depends on the welfare of the non-poor. The HDI is also insensitive to inequality (UPDT and MPDT) and to the incidence of multiple deprivation (SMD).

The HPI is discontinuous at the poverty frontier; it is not monotonically decreasing in the poor's welfare; and it does not consistently penalize inequality and the incidence of multiple deprivation.

We introduced above (see equations (7) and (8)) the two most popular and simplest measures of multidimensional poverty, called the union and intersection headcount indices. These simple measures can be generalized to "intermediate" headcount indices.

Let ζ be the minimal (possibly weighted) number of dimensions in which someone needs to be deprived to be considered as multidimensionally poor. Let $H(\zeta)$ be the resulting multidimensional headcount: the proportion of individuals that are poor in at least ζ dimensions. $H(1)$ is the union headcount: on the two-dimensional Figure 1, this is given by the proportion of those that are to the left of or below $\lambda_2(\mathbf{x})$. $H(K)$ is the intersection headcount: with two dimensions, this is the proportion of people that are poor in the two dimensions. On Figure 1, this is given by the proportion of those that are to the left of or below $\lambda_1(\mathbf{x})$.

With more than two dimensions ($K > 2$), $H(\zeta)$ becomes an intermediate headcount index when $1 < \zeta < K$; it gives the proportion of the population that is poor in at least ζ dimensions. $H(\zeta)$ indices are discontinuous whatever the value of ζ; they are also not strictly monotonically decreasing with the poor's welfare and do not consistently penalize inequality. $H(1)$ is attribute additive, and it therefore does not penalize consistently the correlation across attributes. In fact, for *all values* of $\zeta < K$, $H(\zeta)$ indices do not account consistently for the incidence of multiple deprivation and therefore do not obey SMD.

Apart from $H(\zeta)$, there are other simple ways to extend intersection and union indices. One such method has been proposed by Alkire and Foster (2011) (AF for short). The AF index is given by

$$\pi(\mathbf{x}_i;\mathbf{z}) = \sum_{k=1}^{K} w_k g_{i,k}^{\alpha} I(d_i \geq \zeta), \tag{13}$$

where $\sum_{k=1}^{K} w_k = 1$, with w_k being the weight attached to dimension k (often taken to be K^{-1}), $g_{i,k} = \max(0, 1 - z_k^{-1} x_{i,k})$ being the normalized poverty gap in dimension k, $d_i = \sum_k g_{i,k}^0$ denoting the (possibly weighted) number of dimensions in which individual i is deprived, and ζ denoting a cross-dimensional 'dual' cut-off. $I(d_i \geq \zeta)$ is the identification condition: individual i is considered multidimensionally poor only if he is deprived in at least ζ (possibly weighted) dimensions.

Table 1 indicates that the AF index is discontinuous: a small change in the value of an individual's attribute can lead to a large change in the poverty level of that individual because of the identification condition. It will also be seen in the context of the discussion of the MPI in section 3 that (and why) the AF index does not obey the SMD property.

Attribute additive (conditional on identification) formulations such as the one of (13) raise important concerns. The AF index (and other attribute additive indices such as the special form of AF given by the MPI, see equation (14)) does not consistently penalize inequality in total deprivation across individuals; an individual's poverty contribution to total poverty is (conditional on identification $I(d_i \geq \zeta)$)) linear in that individual's total deprivation across dimensions. Conditional on two individuals being multidimensionally deprived, AF is therefore insensitive to the distribution of deprivation across those two individuals. This is true for any value of α.

As in (13), UNDP's MPI index also uses ζ first to identify those that are multidimensionally poor. Unlike the traditional headcount indices, which measure the total number of multidimensionally poor people as a proportion of the total number of people, the MPI measures the total number of *dimensions* in which the multidimensionally poor are poor, as a proportion of the total number of dimensions for which welfare is measured (this is the number of people times the number of dimensions, $N \cdot K$). Let the MPI be denoted by $M(\zeta)$; it is given by (13) when $\alpha = 0$ (and assuming $w_k = 0$):

$$\pi(\mathbf{x}_i; \mathbf{z}) = K^{-1} d_i I(d_i \geq \zeta). \tag{14}$$

The MPI, unlike the simple multidimensional headcount ratio, is thus sensitive to the number of dimensions in which the poor are deprived. In the case of two dimensions, it equals

$$M(1) = 0.5 \left(F_{\mathbb{X}}(z_1, \infty) + F_{\mathbb{X}}(\infty, z_2) \right) \tag{15}$$

and

$$M(2) = F(z_1, z_2). \tag{16}$$

Note that the intersection headcount $H(2)$ and $M(2)$ are thus the same.

Figures 6 (a), 6 (b) and 6 (c) are useful in understanding the distinction between the traditional multidimensional headcount indices and the MPI. The numbers in larger font in the figures show the contribution to total poverty of individuals with different values of x_1 and x_2. Figure 6 (a) indicates that, for the intersection headcount index, only those that are in the lower rectangle count for total poverty. Figures 6 (b) and 6 (c) show why the union headcount is different from the union MPI. The union headcount counts people in poverty; the union MPI counts the proportion of dimensions in which the poor are poor. Hence, those poor individuals that are poor in fewer dimensions contribute less to MPI poverty than to the union headcount. In Figure 6 (c), this is shown by the contribution of 0.5 for those individuals that are poor in only one dimension out of two.

< insert Figure 6 about here >

As shown in Table 1, the MPI is discontinuous, it does not obey the monotonicity axiom, it obeys consistently neither the unidimensional nor the multidimensional transfer principles, and it does not obey SMD. We discuss these features in more details later in Section 3.

Based upon the original Foster, Greer, and Thorbecke (1984) (FGT) class of unidimensional indices, Chakravarty, Mukherjee, and Ranade (1998) propose a multidimensional extension of the FGT poverty measures. It is a union index defined by:

$$\pi(\mathbf{x}_i;\ \mathbf{z}) = \sum_{k=1}^{K} w_k g_{i,k}^{\alpha}, \tag{17}$$

where w_k is the weight attached to dimension *k*. That index obeys all of the axioms listed in Table 1 except SMD (because of its attribute additivity). A natural multiplicative extension (MFGT) is given by:

$$\pi(\mathbf{x};\mathbf{z}) = \prod_{k=1}^{K} g_k^{\alpha_k}. \tag{18}$$

This does not respect attribute additivity since it corresponds to *intersection* poverty. MFGT obeys all of the relevant multidimensional axioms listed in Table 1. (18) is also a useful generalization of the dominance curves that will be defined in (24).

Chakravarty, Deutsch, and Silber (2008) introduce a multidimensional extension of the Watts index (Watts 1968):

$$\pi\left(\mathbf{x};\mathbf{z}\right)=\sum_{i\in S_k}\sum_{k=1}^{K}w_k\ln\left[\frac{z_k}{\min\left(x_k,\ z_k\right)}\right], \tag{19}$$

where w_k is the weight on attribute k. Because of its logarithmic form, (19) gives a greater weight to poorer individuals. (19) is also a union index. It obeys all of the axioms listed in Table 1 except for SMD.

Tsui (2002)'s index allows for sensitivity to multiple deprivation. The index is defined as

$$\pi\left(\mathbf{x};\mathbf{z}\right)=\sum_{i=1}^{n}w_k\left[\prod_{k=1}^{K}\left(\frac{z_k}{\min\left(x_k,\ z_k\right)}\right)^{\delta_k}-1\right], \tag{20}$$

with $\delta_k \geq 0$ and w_k again a weight. This index satisfies all of the useful axioms highlighted in Table 1 (including SMD).

Datt (2013) introduces the index

$$\pi(\mathbf{x};\mathbf{z})=\left(K^{-1}\sum_{k=1}^{K}g_k^{\alpha}\right)^{\beta}, \tag{21}$$

with $\beta \geqslant 1$ measuring the degree of "cross-dimensional convexity". For $\beta = 1$, the index is equivalent to the index in (17). Unlike the AF index, the contribution of a poor person's attribute to total deprivation in (21) depends on the distribution of the other deprivations. When $\alpha, \beta \geq 1$, (21) obeys the multidimensional transfer principle MPDT. (21) is generally not attribute additive and does not obey the unidimensional transfer principle UPDT; for $\alpha > 0$, it is, however, continuous, monotonically decreasing with the welfare of the poor, and it does respect SMD for $\beta > 1$.

Bourguignon and Chakravarty (2003) propose an extension of the standard unidimensional FGT index that allows for either substitutability or complementarity between attributes. Their multidimensional index (here presented with 2 dimensions) takes the form

$$\pi(\mathbf{x}_i;\mathbf{z})=\left[\beta_1 g_{i,1}^{\varepsilon}+(1-\beta_1)g_{i,2}^{\varepsilon}\right]^{\frac{\alpha}{\varepsilon}}, \tag{22}$$

where β_1 and β_2 are greater than zero and are the weights given to each dimension. $\alpha = 0$ gives the multidimensional union headcount, a discontinuous measure of poverty. The elasticity of substitution between the two poverty gaps is $\frac{1}{\varepsilon - 1}$. With $\varepsilon = 1$, isopoverty curves are straight lines in the two dimensions, and the elasticity of substitution is infinite. As ε tends towards *infinity*, the measure tends to

$$\pi(\mathbf{x}; \mathbf{z}) = \max\left(g_1, g_2\right)^{\alpha}, \tag{23}$$

in which case the two attributes are completely imperfect substitutes and the iso-poverty curves are rectangular. An individual's poverty level is then given by his worst attainment in any single attribute.

The parameter α captures the degree of aversion to *multidimensional* poverty and not, as in the AF index, to *unidimensional* poverty. Because of this, the index in (22) is able to take into account coherently the incidence of multiple deprivation; that is, it is able to obey the SMD principle. The larger the α parameter, the higher the weight given to multidimensionally poorer individuals. Variations in α do not affect the shape of the isopoverty curves across the two attributes; ε alone affects the shape of those curves.

For $\varepsilon, \alpha > 0$, the index in (23) satisfies the monotonicity axiom. When $\alpha > \varepsilon$, the two attributes are *substitutes* and (22) is sensitive to multiple deprivation (SMD). When $\varepsilon = \alpha$, changes in the association between attributes do not affect the poverty measure; indeed, only when $\varepsilon = \alpha$ does the index obey the attribute additivity property, but it then fails to obey SMD.

3 Difficulties with UNDP's MPI

We have seen above that only a few indices — MFGT, (18); Tsui, (20); Datt, (21); Bourguignon-Chakaravarty, (22) — obey what we consider to be three fundamental axioms for multidimensional poverty measurement (continuity, monotonicity, and sensitivity to multiple deprivation). These indices, however, have not been commonly used. Most multidimensional poverty measurement exercises have instead focused on indices that fail to respect those fundamental axioms. The currently most popular index among such indices is the MPI. For this reason, it seems useful to analyze it in greater details.

The problems with the MPI are of two major sorts. The first comes from the discretization that the MPI makes of deprivation in the various dimensions: individuals are either poor or non-poor. This creates discontinuities in the measurement of poverty that may penalize welfare-equalizing policies and development processes. These discontinuity problems are well understood in the unidimensional literature. It is, for instance, well-known that the popular unidimensional (discontinuous) headcount index can *increase* following a policy that redistributes resources from richer to poorer individuals. This is necessarily exacerbated when multidimensional discontinuities are introduced across several dimensions.

The second sort of problem comes from the MPI's use of an additional "poverty line" ζ. This "dual" poverty line serves to identify those individuals that are deemed to be multiply deprived; it unfortunately introduces a second type of discontinuities. Whenever a slight change in someone's welfare changes the number of dimensions in which that person is deprived, there is a risk that the person moves suddenly into or out of the set of individuals that are considered to be multiply deprived. This movement thus introduces measurement discontinuities that may again penalize development processes that have the feature of equalizing welfare. Such features also have the effect that a society that seeks to alleviate multiple deprivation may, because of this, see its MPI *increase* over time.

The first sort of problems can be alleviated through the use of less dichotomous (and perhaps more precise) measures of welfare in the various dimensions of interest, as is implicit for instance with the AF index in (13) when $\alpha > 0$. Unfortunately, the second sort of problems cannot generally be avoided with poverty indices of the AF or MPI type, unless ζ is set to 1. Setting ζ to 1, however, makes the MPI an exclusively "union" index — a feature that would not be desirable since that would prevent the MPI from focusing on those that are multiply deprived.

We can now explore more precisely how the MPI reacts to changes in levels of welfare. This can be done using changes in welfare involving either a single individual or several.

3.1 Continuity

It is clear from Figures 6 (a) and 6 (b) that headcount-type multidimensional indices cannot obey the continuity axiom. This is because a small change in dimensional welfare or in the dimensional poverty lines can change from 1 to 0, or from 0 to 1, the contribution of any individual to total poverty. When that happens depends on the value set for ζ.

Figure 6 (c) also shows how the union MPI introduces further instances of such discontinuities. This is because this form of MPI can jump whenever the number of poor dimensions that an individual experiences changes, even though the person may still be considered to be a multidimensionally poor person. Although the size of the jumps is quantitatively less important than for the traditional headcount indices (0.5 as opposed to 1 in our Figures), they occur more often with the union MPI than with the traditional union headcount.

To avoid such sensitivity for the (union) $M(1)$, it would be necessary to use a poverty valuation function that is continuous in dimensional welfare. To avoid such sensitivity for the $H(\zeta)$ and $M(\zeta)$ types of indices is impossible. All multidimensional poverty indices that are of those types will indeed necessarily jump whenever the number of poverty dimensions of a particular individual moves up or down the ζ parameter.

3.2 Inequality

Figures 7 and 8 show why the MPI can increase following policies or distributional changes that decrease inequality, either within the poor or between the poor and the non-poor. Consider first Figure 7, which shows a transfer from (non-poor) person b to (poor) person a. The usual union headcount index increases whenever the richer person falls into poverty following such a transfer. The union MPI ($M(1)$) also displays this property, as shown in Figure 8.

< insert Figure 7 about here >

< insert Figure 8 about here >

Figure 8 shows that the union MPI ($M(1)$) has an additional shortcoming, which arises when a transfer is made from a less poor individual to a poorer individual. This happens on Figure 8 when individual d becomes poor in dimension 2 when a transfer is made from him to individual c. This is a shortcoming that does not arise with the traditional headcount indices. All in all, Figure 8 shows how the discontinuity of the MPI creates two difficulties in reconciling inequality reduction with poverty alleviation: when inequality reduction affects both the non-poor and the poor and when its effects are limited to the poor.

Figure 9 demonstrates another shortcoming of the MPI. In that Figure, individuals c and d are brought closer together through an equalizing transfer. Both individuals, however, see their individual levels of poverty increase, and total poverty therefore also increases.

< insert Figure 9 about here >

3.3 Multiple deprivation

As discussed above, the sensitivity to multiple deprivation (SDM) property would seem essential to multidimensional poverty measurement.

Figure 10 shows how and why a fall in multiple deprivation can increase the MPI, using a case of two individuals, three dimensions and $\zeta = 2$. It is assumed that both individuals are poor in the third dimension, with $x_3 < z_3$. The first individual moves from position a to position c, and the second individual goes from position d to position b. Although the correlation of dimensional welfare has fallen across individuals, the MPI has increased. This shortcoming also arises with other discontinuous union indices, such as the union headcount index (see Figure 7).

< insert Figure 10 about here >

Figure 11 illustrates that the problem can be worse. It now assumes that the first individual (a) is also poor in the third dimension, whereas the second one (d) is not. Hence, the correlation-decreasing switch shown by the arrows benefits clearly the poorest of the two individuals, in addition to decreasing the inequality that exists between them. Here again, however, the MPI increases.

< insert Figure 11 about here >

It may be hoped that the problems with MPI indices would be alleviated if one posited that attributes should be complementary — that is, that poverty should decrease with the incidence of multiple deprivation. It is not difficult, however, to find examples in which the MPI increases following an increase in correlation. This is shown in Figure 12, for instance, where a greater incidence of multiple deprivation does indeed raise $M(2)$.

< insert Figure 12 about here >

4 What can be done?

Given the above, a natural question that can be asked is whether we can think of other procedures, such as the use of alternative indices, that would perform better than UNDP's MPI. As we saw, the problem with the MPI comes from the dual sources of discontinuity that it introduces, first with respect to each dimension, second with respect to the cut-off number (ζ) of dimensions that serves to identify the multidimensional poor. The first source can be corrected by using the more general MPI found in (13) (the AF indices). The second source is, unfortunately, a general feature of *all* indices that make use of a cut-off of the ζ type.

Other forms of multidimensional poverty indices will be more robust to such shortcomings (*e.g.*, those highlighted in Table 1). The choice of any particular multidimensional poverty index is bound, however, to be arbitrary, because it will impose a particular choice of poverty line(s), a particular choice of attribute-aggregating procedures, and a particular choice of individual-aggregating procedures. All such choices can be contested, a feature that can then undermine the reliability of the findings (including the policy guidance) obtained through the use of such particular poverty indices. The significance of those problems further grows with the number of dimensions considered.

4.1 Dominance

Given this, it would seem important to verify that poverty assessments, poverty profiles, and poverty alleviation policies are not inadvertently distorted by the possible shortcomings of the MPI or of other multidimensional indices. One effective and simple manner to guard against such distortions is through dominance testing. Dominance testing techniques have been around for some time in the unidimensional poverty literature; see for instance Foster and Shorrocks (1988b) as well as Cowell (2016). They are also now available for multidimensional poverty (Duclos, Sahn, and Younger 2006). Such dominance tests are easily and rigorously applied using readily available software (such as DASP [Araar and Duclos 2007]). They are also easily understood since they consist in comparisons of intersection headcounts and other simple poverty indices — though not at specific poverty lines and frontiers, but over ranges and areas of them.

To understand how to carry out these tests, let again $\lambda(x_1, x_2)$ be an individual aggregation of two indicators of individual welfare. For dominance tests, it is useful to think of an uppermost poverty frontier, given by $\lambda^*(x_1, x_2) = 0$, such that $\Lambda(\lambda^*)$ includes the maximal set of possibly poor persons (recall (9) and (10)). To check whether multidimensional poverty comparisons are robust over this maximal set of possibly poor persons and over classes of poverty indices, we need to make use of the multidimensional intersection indices given by

$$P^{\alpha_1,\alpha_2}(\mathbb{X}; \mathbf{z}) = \int_0^{z_1} \int_0^{z_2} (z_1 - x_1)^{\alpha_1} (z_2 - x_2)^{\alpha_2} \, dF(x_1, x_2) \tag{24}$$

for integers $\alpha_1 \geq 0$ and $\alpha_2 \geq 0$. The $\alpha_1 + 1$ and $\alpha_2 + 1$ parameters correspond to orders of dominance in each respective dimension. The order $\alpha_1 + 1$ of dominance in the first dimension imposes a set of conditions on how the π functions in equation (3) incorporate deprivation in that first dimension, and analogously for the order $\alpha_2 + 1$ in the second dimension. The dominance surfaces are generated by varying the poverty lines z_1 and z_2 in (24) over a domain (z_1, z_2) that belongs to $\Lambda(\lambda^*)$. These surfaces can be interpreted as two-dimensional generalizations of the FGT index. The most commonly used such surface is for first-order dominance ($\alpha_1 = 0$ and

$\alpha_2 = 0$), and for expositional simplicity we will focus on that case here. $P^{0,0}(\mathbb{X}; \mathbf{z})$ is then the intersection headcount index. Multidimensional dominance of distribution A over distribution B is then obtained whenever

$$F_{\mathbb{X}_B}(z_1, z_2) - F_{\mathbb{X}_A}(z_1, z_2) > 0, \ \forall (z_1, z_2) \in \Lambda(\pi^*). \tag{25}$$

Consider Figure 13 (see Duclos, Sahn, and Younger 2006), which graphs a typical first-order dominance surface: the multidimensional intersection headcount with household *per capita* expenditures and height-for-age ("z-scores") measures as the welfare dimensions of 1989 Ghanaian children. A larger "hump" in the middle of the surface corresponds to a larger positive correlation between the two welfare dimensions. To test for dominance, the difference between two surfaces of the type shown in Figure 13 is considered; Figure 14 depicts such a difference.

< insert Figure 13 about here >

< insert Figure 14 about here >

Clearly, condition (25) says that the multidimensional intersection headcount will be lower in A regardless of the choice of poverty frontiers in $\Lambda(\lambda^*)$. But it can be shown that multidimensional dominance implies more than just that. If condition (25) holds, for *all* multidimensional poverty indices 1) whose poverty frontiers lie within $\Lambda(\lambda^*)$, 2) that are continuous and decreasing in x_1 and x_2 and 3) that obey Axiom 10 (SMD), poverty in A will be lower than in B.

It is important to stress that when A dominates B, A has less poverty than B over *classes of indices* of possibly different functional forms and shapes. This is a stronger result than simply checking differences in particular poverty indices (such as the MPI) over different choices of weights (such as w_k) or poverty cutoffs (*e.g.* Alkire and Santos 2014).

The above dominance conditions require sensitivity to multiple deprivation, which (as we noted above) is important even with binary data. They do not demand cardinality of the component dimensions; not requiring cardinality seems appropriate in most poverty applications. They require monotonicity, which is important for all non-binary data. They also suppose that poverty indices should be continuous; this, however, and as discussed above in the context of the MPI, is a property that can be regarded as *necessary* in order to make appropriate poverty judgments, especially when data are not binary. They do not require having to choose between union, intersection, or intermediate views of multidimensional poverty: all such views are allowed, the only restriction being that the frontier lies within $\Lambda(\lambda^*)$. They do not force the choice of a particular functional form for the poverty indices: the comparisons are valid for classes of poverty indices. They do not force the choice of particular poverty frontiers: the comparisons are valid for areas of such frontiers.

Such dominance procedures are therefore useful for complementing the information provided by peculiar multidimensional poverty indices, such as UNDP's MPI, especially given the concerns that the use of these indices may raise. Note, however, that the above dominance procedures do not include those poverty indices that fail the continuity, monotonicity, and sensitivity to multiple deprivation conditions that we judge important; for such other indices, other types of robustness tests would need to be designed.

4.2 Are dominance tests an attractive procedure?

Despite its attractiveness, dominance testing can nevertheless be, and has been, criticized. The arguments against dominance testing are, however, not as persuasive as may appear at first sight. They can be grouped as follows.

Argument 1: *Dominance tests do not precisely measure poverty differences across distributions*.

This is true. Precise poverty differences, however, necessarily come at the expense of making highly specific measurement assumptions. Such assumptions are problematic for two main reasons.

First, ordinal comparisons of poverty indices (comparisons across time, regions, socio-demographic groups, or policy regimes, for instance) may be disturbingly sensitive to the choice of indices and poverty lines. Second, cardinal differences are even more sensitive to the choices of indices and lines. In general, any cardinal valuation of differences across distributions will be specific to those choices; any other choice will lead to different valuations.

If cardinal valuations of differences are absolutely needed, then numerical differences in dominance curves or surfaces can be used to do this (with care) since these curves and surfaces do correspond to simple poverty measures.

Argument 2: *Dominance tests are not always conclusive*.

This is true, but this is in fact a valuable feature of the dominance procedures. It is important to know if comparisons of distributions are robust: when dominance tests are not conclusive, any ranking inferred from the use of cardinal poverty indices will be due to the choice of specific assumptions, such as the choice of specific poverty frontiers and poverty indices. If the dominance tests are inconclusive, then it may justify investing energy and time on estimating specific poverty frontiers and investigating numerical sensitivity to the choice of poverty indices.

When dominance tests are inconclusive, it is also possible to know why and where the non-robustness arises. Inspection of the dominance surfaces indeed reveals why a distribution may be better than another for some poverty frontiers/poverty indices and not for others. Such inspection of dominance surfaces can also provide critical poverty lines/frontiers, as in Davidson and Duclos (2000) and Duclos, Sahn, and Younger (2006).

When dominance tests are conclusive, however, such time and energy can be saved off conducting difficult debates on the choice of appropriate theoretical and econometric models for estimating poverty lines and on discussions of the relative merits and properties of the many poverty indices that have been proposed in the literature.

Note again that when multidimensional dominance comparisons are conclusive, the rankings are valid for a wide area of poverty frontiers – all those such that $\Lambda(\lambda) \subset \Lambda(\lambda^*)$. Hence, even though (25) is an intersection poverty index, the class of indices that is ordered by it includes intersection, union, and intermediate poverty frontiers.

Argument 3: *Dominance tests suffer from the curse of dimensionality*.

This is also true: the curse of dimensionality says that, as the number of dimensions increases, it becomes increasingly difficult to compare (graphically, numerically and statistically) distributions. But this curse pertains in fact to all multidimensional poverty comparison procedures. However, dominance tests make the consequences of the curse explicit. The computation of cardinal multidimensional indices usually hides this curse by not highlighting the importance of the choice of functional forms, poverty lines, and parameters in making multidimensional poverty comparisons: the greater the number of dimensions, the greater the number of these choices and the greater the difficulty of checking the cardinal and ordinal sensitivity of multidimensional indices.

Argument 4: *Dominance tests are difficult to understand and to apply*.

This is incorrect. It may be difficult to understand the analytical equivalence between multidimensional poverty orderings and orderings of poverty surfaces. But the poverty surfaces are themselves straightforward to understand since they correspond to simple intersection indices. Applying dominance tests is also simple and transparent.

Argument 5: *Dominance tests impose arbitrary normative assumptions*.

This is misleading. The multidimensional poverty dominance tests do impose conditions on the properties of multidimensional poverty measurement, but these properties are made explicit and are designed to be as widely acceptable as possible. First-order dominance tests (which are the most robust ones) suppose, for instance, that multidimensional poverty indices should be continuous, should decrease with dimensional welfare, and should increase with the incidence of multiple deprivation — these are important properties in most poverty measurement exercises.

One can, however, imagine that some dimensions of welfare may be complementary in producing overall welfare. For instance, the dimensions of education and nutritional status for children might be complementary because better-nourished children may learn better. Overall child poverty would decline by more if education were transferred from the poorly nourished to the better nourished children. If this complementarity were strong enough, it might overcome the usually convincing judgement that would favor assisting the multiply-deprived children;

Bourguignon and Chakravarty (2002) derive the dominance criteria for the case of complementary attributes. The dominance surfaces are then of the union type; in such cases, group A

dominates group B (for some fixed poverty frontier) if and only if the headcount union indices are always lower in A than in B. Robustness over classes of such complementary attributes is, however, difficult to obtain when "poverty-frontier robustness" is also sought; the literature has not yet provided reasonably powerful procedures that can rank multidimensional poverty both over classes of attribute-complementary indices and over areas of poverty frontiers.

5 Conclusion

The exercise of measuring and comparing multidimensional poverty over time and across countries has become increasingly important and frequent in welfare economics. This paper argues that the multidimensional poverty indices used in that exercise should obey the properties of continuity, monotonicity, and sensitivity to multiple deprivation. Not all commonly used indices do that, however. One important example of an index that fails all of these three properties is the recently proposed UNDP's Multidimensional Poverty Index (MPI). The paper discusses in particular how the discontinuities introduced by the MPI (*i.e.*, discretization of deprivation and use of a dual poverty line) may cause it to increase following 1) a transfer from a richer to a poorer individual in one dimension; 2) a decrease in the inequality in one component dimension among the poor; 3) a simultaneous decrease in inequality across all dimensions; or 4) a fall in the incidence of multiple deprivation. This last failure is particularly important since it can arise with all types of data — continuous, binary, or other sorts of discrete data.

This paper notes that only a few of the multidimensional indices found in the literature obey the above properties: the Multiplicative FGT index and the classes of indices proposed by Tsui, Datt, and Bourguignon/Chakravarty. Since any choice of multidimensional poverty index introduces some degree of arbitrariness — a particular choice of poverty frontiers, a specific choice of attribute-aggregating procedures, and a particular choice of individual-aggregating procedures — this paper also discusses and encourages the use of poverty dominance testing as an effective and simple tool to guard against the risk of non-robustness when measuring and comparing poverty.

References

ALKIRE, S. (2016): "The Capability Approach," in *The Oxford Handbook of Well-Being and Public Policy*, ed. by M. Adler and M. Fleurbaey, Oxford: Oxford University Press, chap. 21.

ALKIRE, S. AND J. FOSTER (2011): "Counting and multidimensional poverty measurement," *Journal of Public Economics*, 95, 476–487.

ALKIRE, S. AND M. E. SANTOS (2014): "Measuring Acute Poverty in the Developing World: Robustness and Scope of the Multidimensional Poverty Index," *World Development*, 59, 251 – 274.

ANAND, S. AND A. K. SEN (1997): "Concepts of Human Development and Poverty: A Multidimensional Perspective," .

ARAAR, A. AND J.-Y. DUCLOS (2007): "*DASP*: Distributive Analysis Stata Package," PEP, CIRPÉE and World Bank, Université Laval.

ATKINSON, A. (1970): "On the Measurement of Inequality," *Journal of Economic Theory*, 2, 244–63.

——— (1987): "On the Measurement of Poverty," *Econometrica*, 55, 749 –764.

——— (2003): "Multidimensional Deprivation: Contrasting Social Welfare And Counting Approaches," *Journal of Economic Inequality*, 1, 51–65.

ATKINSON, A. B. AND F. BOURGUIGNON (1982): "The Comparison of Multidimensional Distributions of Economic Status," *Review of Economic Studies*, 49, 183–201.

BIBI, S. AND J.-Y. DUCLOS (2007): "Equity and Policy Effectiveness with Imperfect Targeting," *Journal of Development Economics*, 83, 109–140.

BOURGUIGNON, F. AND S. CHAKRAVARTY (2002): "Multi-dimensional poverty orderings," Tech. Rep. 2002-22, DELTA.

BOURGUIGNON, F. AND S. R. CHAKRAVARTY (2003): "The Measurement of Multidimensional Poverty," *Journal of Economic Inequality*, 1, 25–49.

CHAKRAVARTY, S., D. MUKHERJEE, AND R. RANADE (1998): "On the Family of Subgroup and Factor Decomposable Measures of Multidimensional Poverty," *Research on Economic Inequality*, 8, 175–194.

CHAKRAVARTY, S. R., J. DEUTSCH, AND J. SILBER (2008): "On the Watts Multidimensional Poverty Index and its Decomposition," *World Development*, 36, 1067–1077.

CHAKRAVARTY, S. R. AND M. A. LUGO (2016): "Multidimensional Indicators of Inequality and Poverty," in *The Oxford Handbook of Well-Being and Public Policy*, ed. by M. Adler and M. Fleurbaey, Oxford: Oxford University Press, chap. 9.

COCKBURN, J., J.-Y. DUCLOS, AND A. ZABSONRÉ (2014): "Is global social welfare increasing? A critical-level enquiry," *Journal of Public Economics*, 118, 151 – 162.

COWELL, F. A. (2016): "Inequality and Poverty Measures," in *The Oxford Handbook of Well-Being and Public Policy*, ed. by M. Adler and M. Fleurbaey, Oxford: Oxford University Press, chap. 4.

DATT, G. (2013): "Making every dimension count: multidimensional poverty without the "dual cut-off"," Monash Economics Working Papers 32-13, Monash University, Department of Economics.

DAVIDSON, R. AND J.-Y. DUCLOS (2000): "Statistical Inference for Stochastic Dominance and for the Measurement of Poverty and Inequality," *Econometrica*, 68, 1435–64.

DUCLOS, J.-Y. (2011): "UNDP's Multidimensional poverty index," Working document I-11, FERDI, http://www.ferdi.fr/uploads/sfCmsContent/html/112/I-11.pdf.

DUCLOS, J.-Y. AND P. GRÉGOIRE (2002): "Absolute and Relative Deprivation and the Measurement of Poverty," *Review of Income and Wealth*, 48, 471–92.

DUCLOS, J.-Y., D. E. SAHN, AND S. D. YOUNGER (2006): "Robust Multimensional

Poverty Comparison," *Economic Journal*, 113, 943–968.

FOSTER, J., J. GREER, AND E. THORBECKE (1984): "A Class of Decomposable Poverty Measures," *Econometrica*, 52, 761–776.

FOSTER, J. AND A. SHORROCKS (1988a): "Inequality and Poverty Orderings," *European Economic Review*, 32, 654–661.

——— (1988b): "Poverty Orderings," *Econometrica*, 56, 173–177.

GRIMM, M. (2007): "Removing the anonymity axiom in assessing pro-poor growth," *Journal of Economic Inequality*, 5, 179–197.

KOLM, S.-C. (1969): "The Optimal Production of Justice," in *Public Economics*, ed. by J. Margolis and S. Guitton, London: MacMillan.

RAVALLION, M. (2011): "On multidimensional indices of poverty," *Journal of Economic Inequality*, 9, 235–248.

SEN, A. K. (1976): "Poverty: An Ordinal Approach to Measurement," *Econometrica*, 44, 219– 231.

——— (1983): "Poor, Relatively Speaking," *Oxford Economic Papers*, 35, 153–169.

——— (1985): *Commodities and Capabilities*, Amsterdam: North-Holland.

THON, D. (1979): "On Measuring Poverty," *Review of Income and Wealth*, 25, 429 –439.

TSUI, K. Y. (2002): "Multidimensional Poverty Indices," *Social Choice and Welfare*, 19, 69–93.

UNDP (1990-2014): *Human Development Report*, Oxford University Press, New York.

——— (2010): *Human Development Report*, Oxford University Press, New York.

WATTS, H. W. (1968): "An Economic Definition of Poverty," in *Understanding Poverty*, ed. by D. Moynihan, New York: Basic Books.

Table 1: Indices and axioms

	Con	Foc	**Mon**	SA	AA	UPDT	MPDT	**SMD**
HDI	√	no	√	√	√	no	no	no
HPI	no	√	no	√ (if $\varepsilon = 1$)	√ (if $\varepsilon = 1$)	no	no	no
UH	no	√	no	√	√	no	no	no
IH	no	√	no	√	no	no	no	√
AF	no	√	√ (if $\alpha > 0$)	√	no	no	no	no
MPI	no	√	no	√	no	no	no	no
CMR	√	√	√	√	√	√	√	no
MFGT	√	√	√	√	no	no	no	√
CDS	√	√	√	√	√	√	√	no
Tsui	√	√	√	√	no	no	no	√
Datt	√	√	√ (if $\alpha, \beta > 0$)	√	√ (if $\beta = 1$)	no	√[1]	√ (if $\alpha > 0$,$\beta > 1$)
BC	√	√	√	√	√ (if $\alpha = \varepsilon$)	no	√[2]	√[2]

Axioms:
Con: continuity
Foc: focus
Mon: monotonicity
SA: subgroup additivity
AA: attribute additivity
UPDT: unidimensionsal Pigou-Dalton transfer
MPDT: multidimensionsal Pigou-Dalton transfer
SMD: sensitivity to multiple deprivation
[1] depending on the values of α and β — see in the text
[2] depending on the values of α and ε — see in the text

Indices: HDI=Human Development Index eq. (1); HPI=Human Poverty Index eq. (2); UH=Union Headcount eq. (8); IH=Intersection Headcount eq. (7); AF=Alkire-Foster (2011) eq. (13); MPI=Multidimensional Poverty Index eq. (14); CMR=Chakravarty-Mukherjee-Ranade (1998) eq. (17); MFGT= multiplicative FGT eq. (18); CDS=Chakravarty-Deutsch-Silber (2008) eq. (19); Tsui=Tsui (2002) eq. (20); Datt=Datt (2013) eq. (21); BC=Bourguignon-Chakravarty (2003) eq. (22)

Figure 1: Union, intersection, and intermediate poverty measurement with two dimensions; λ are the poverty frontiers

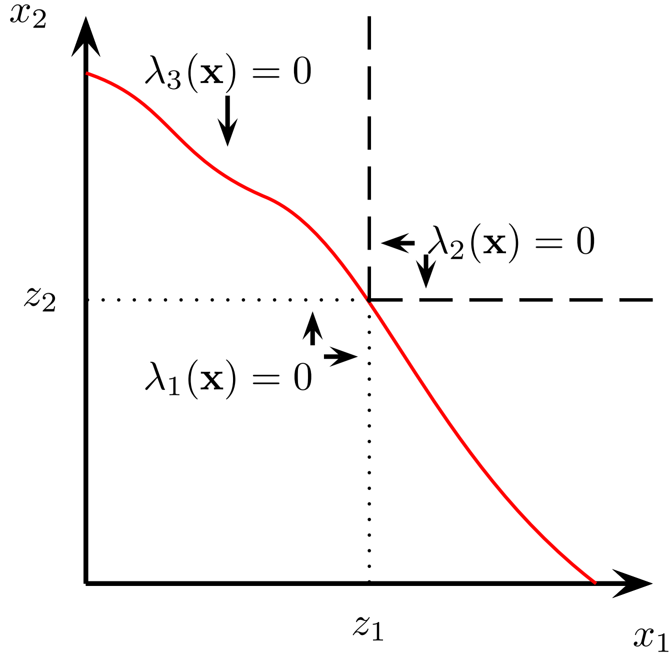


Figure 2: Unidimensional equalization decreases poverty

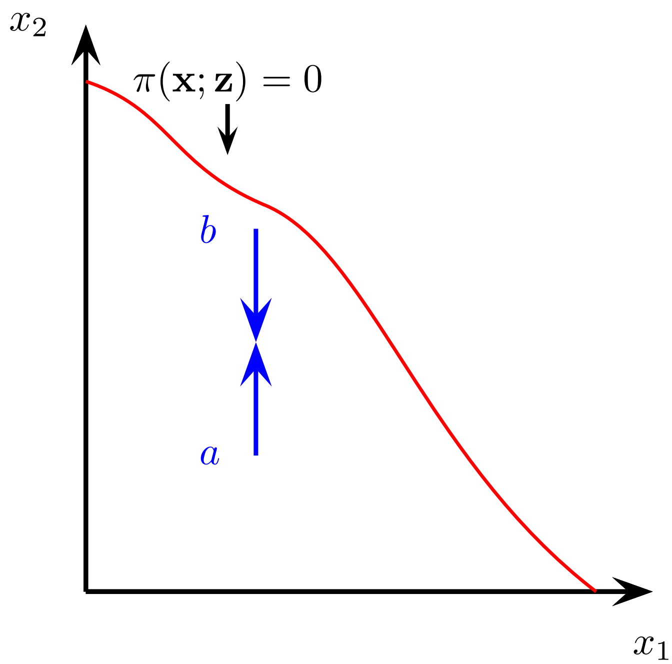


Figure 3: Unidimensional equalization of attributes can increase poverty

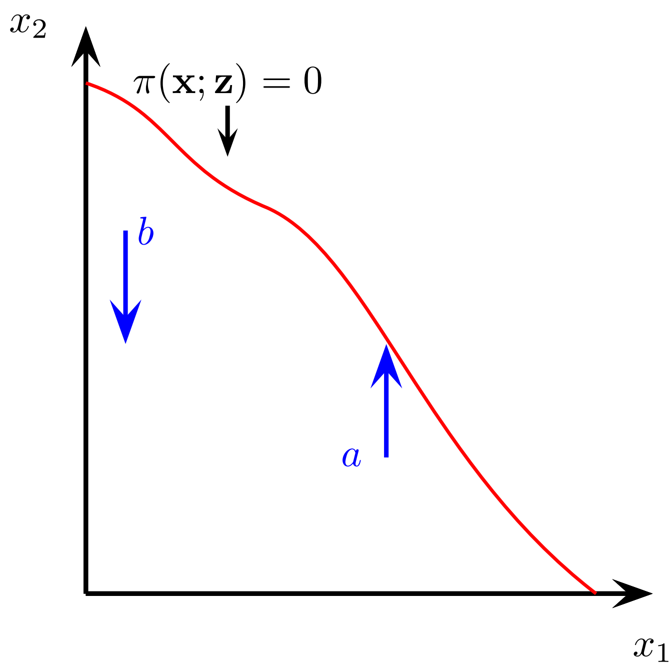


Figure 4: Multidimensional equalization of attributes can increase poverty

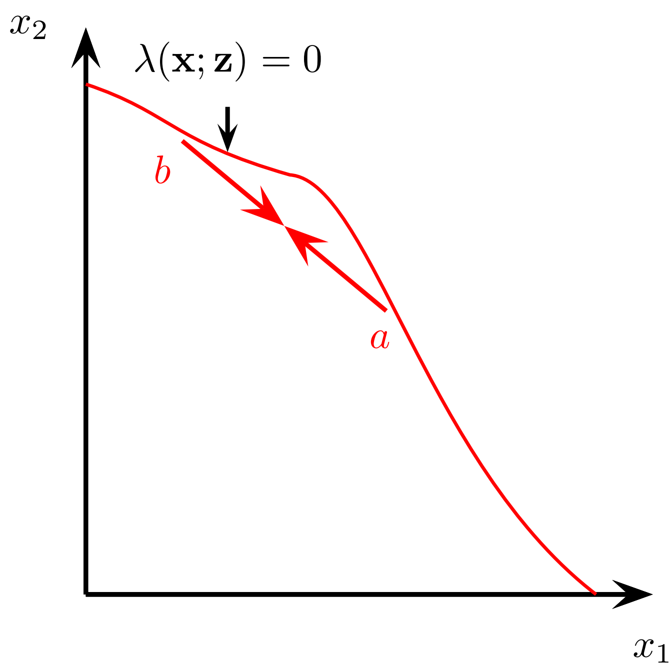


Figure 5: An increase in correlation and in multiple deprivation increases multidimensional poverty

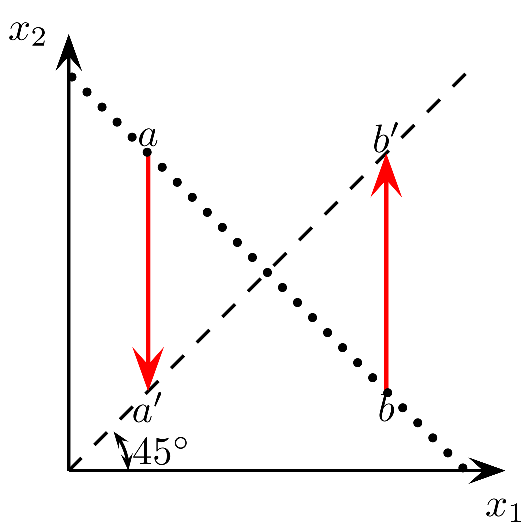
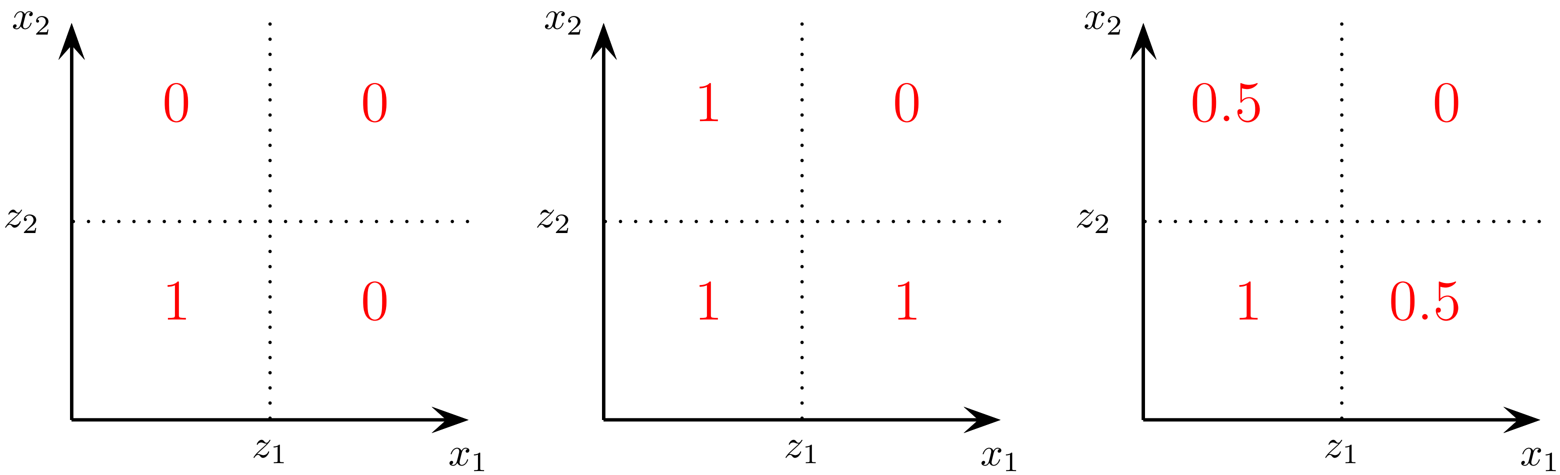


Figure 6: Poverty weights with the intersection headcount, the traditional union headcount and the union MPI



(a) Intersection headcount: $H(2) = M(2)$ (b) Traditional union headcount: $H(1)$ (c) Union MPI: $M(1)$

Figure 7: A rich-to-poor transfer increases the union headcount ($H(1)$)

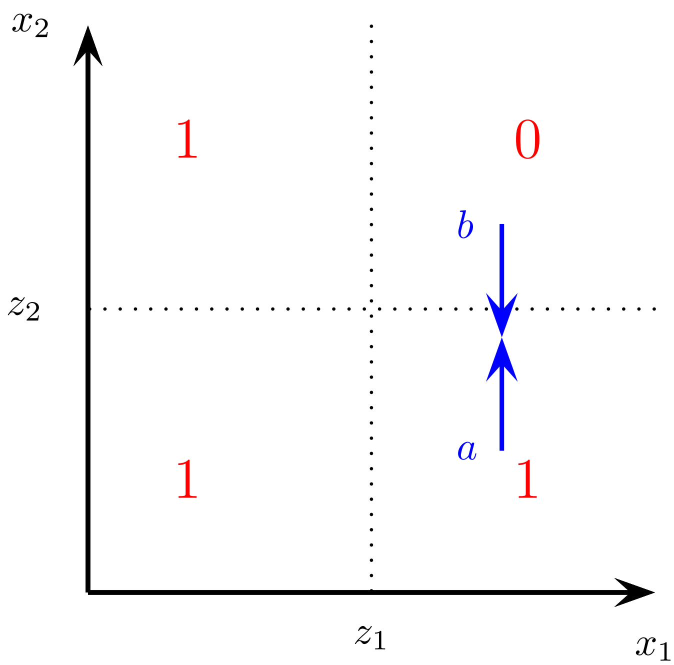


Figure 8: Each of these two rich-to-poor transfers increases the MPI ($M(1)$)

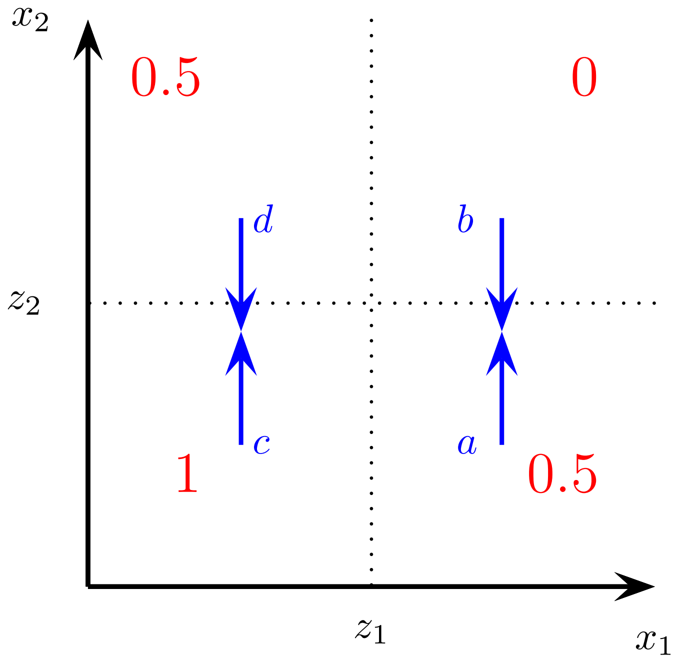


Figure 9: Greater multi-attribute equality increases the MPI ($M(1)$)

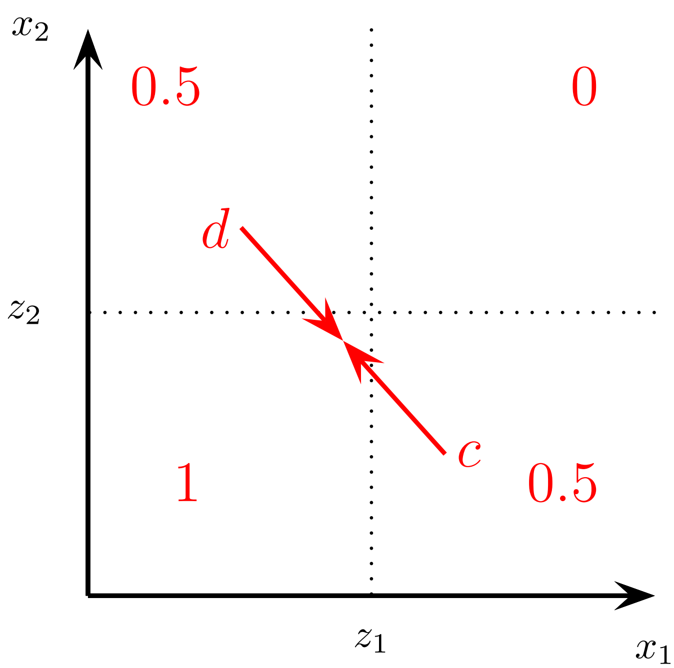


Figure 10: A fall in multiple deprivation increases the MPI $M(2)$, with three dimensions and $x_{a,3} < z_3$, $x_{b,3} < z_3$

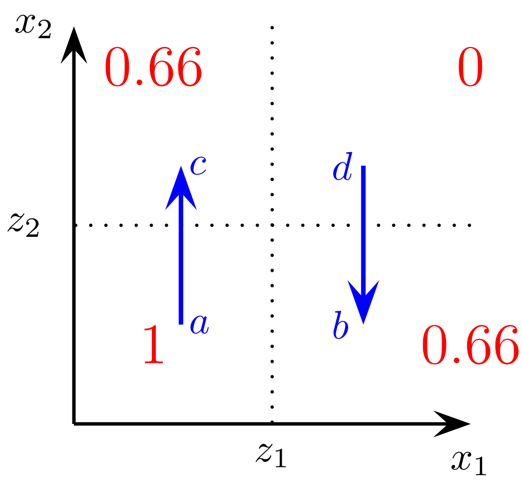


Figure 11: Decreasing multiple deprivation and helping the poorest ($x_{a,3} < z_3$, $x_{d,3} > z_3$) increases the MPI ($M(2)$) with three dimensions

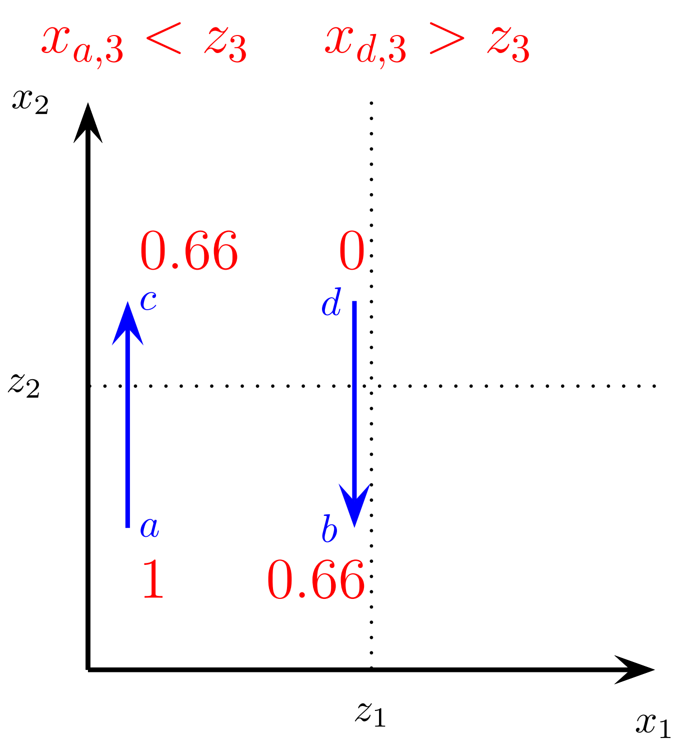


Figure 12: But an increase in multiple deprivation can also increase the MPI ($M(2)$)

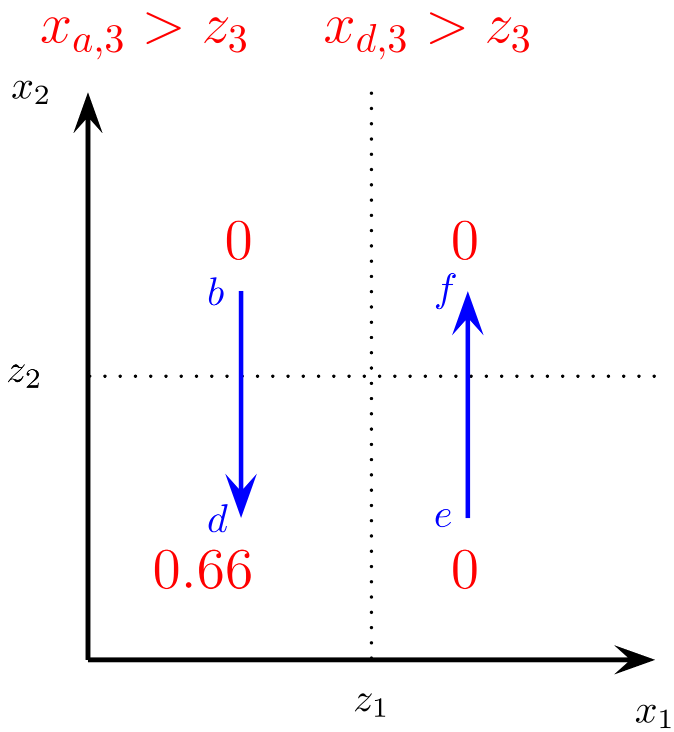
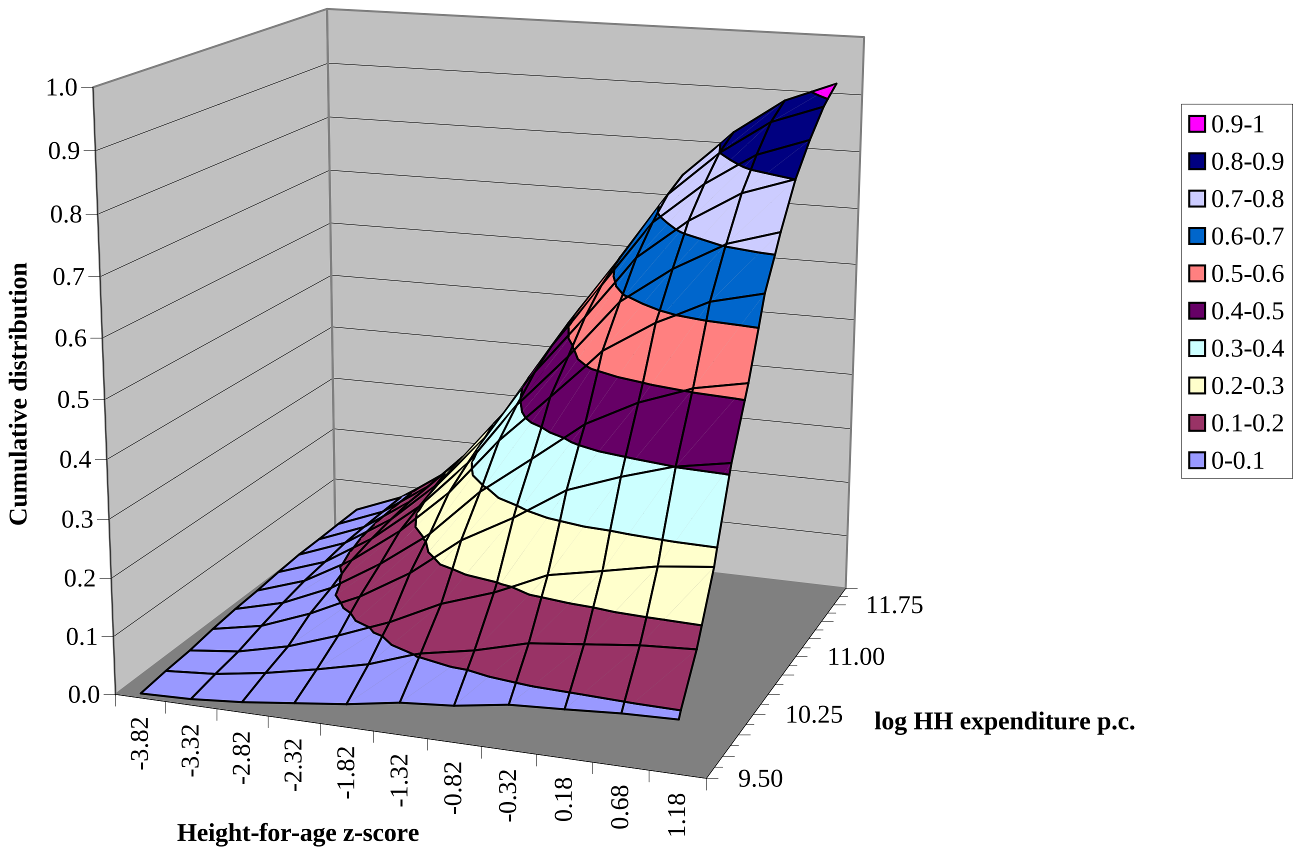
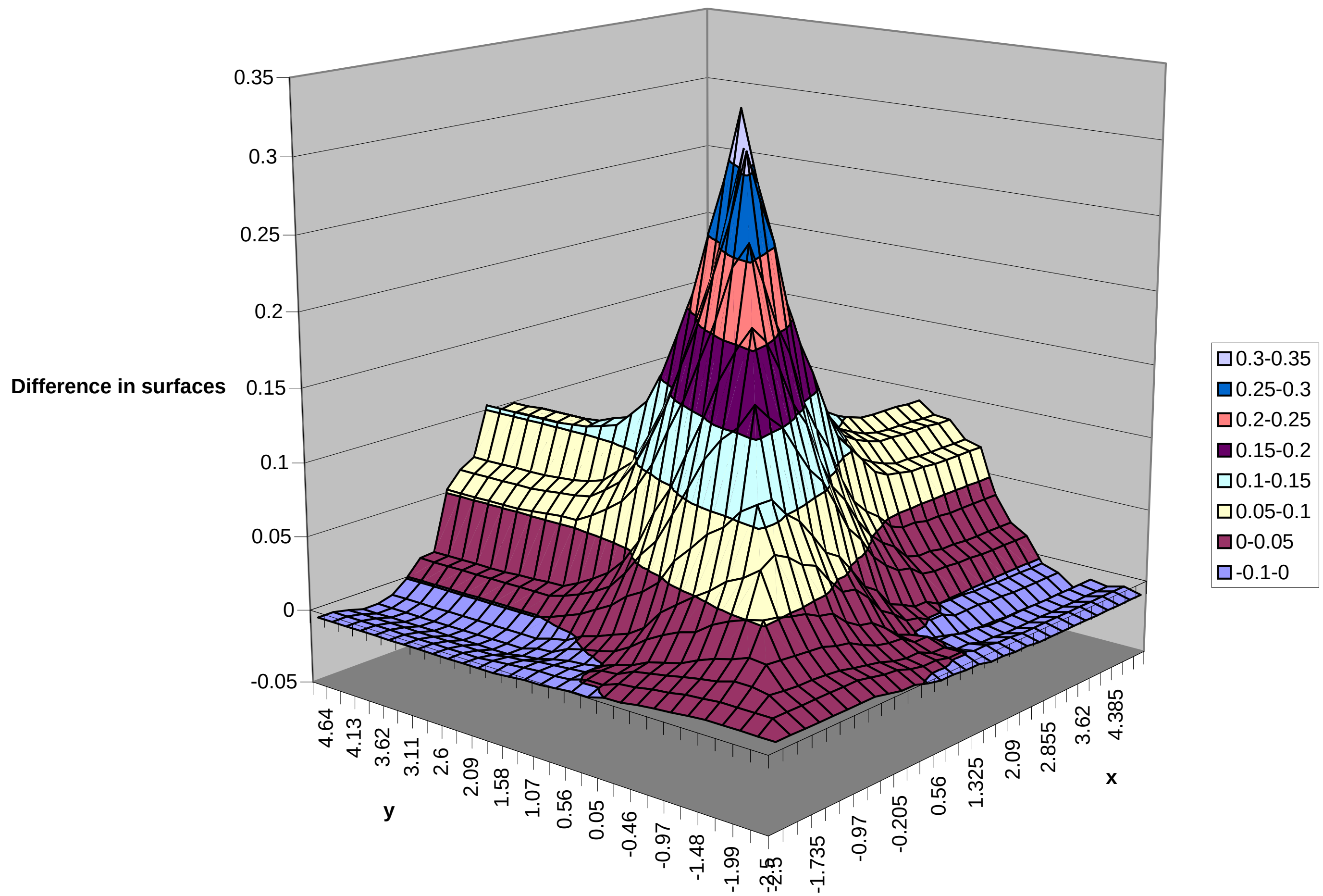


Figure 13: Dominance surface for Ghanaian children, 1989



Note: From Duclos, Sahn, and Younger 2006 (Figure 3, page 254).

Figure 14: Example of difference in first-order dominance surfaces



Note: From Duclos, Sahn, and Younger 2006 (Figure 4, page 255).