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Resource Extraction under Heterogeneous Growth in Demand

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Abstract:
We study the effect of heterogeneous growth in demand on resource extraction. Using the Great Fish War framework of Levhari and Mirman (1980), we show that heterogeneity in demand growth has a profound effect on both non-cooperative (Cournot-Nash and Stackelberg) and cooperative solutions.

Keywords: Common-pool resource, Dynamic games, Heterogeneous growth, Strategic extraction, Tragedy of the commons

JEL Classification: Q20, C72, C71, C73

1 Introduction

Since the beginning of the 20th century, the use of global materials has increased 8-fold (Krausmann et al., 2009). This increase in world demand ranges from natural resources such as fish to energy-related resources. See Figures 1 and 2 in Appendix A. Moreover, the growth of demand for natural resources varies considerably among countries. For instance, the annual fish consumption growth rate for the years 1999-2013 is only 1.06% for the US, but 3.43% for China. Similarly, for total primary energy consumption, the annual growth rate for the years 2006-2013 is negative for the US (-0.44%), but positive for both India (5.14%) and China (7.16%). Figures 3 and 4 in Appendix A further illustrate this heterogeneity of demand growth among countries for both fish and primary energy consumption. In view of such heterogeneity with the particular case of China's exploding demand for resources, it is important to understand how the anticipation of growing demand affects extraction and thus welfare.

In this paper, we study the effect of heterogeneous growth in demand on extraction. To that end, we extend the Great Fish War framework (Levhari and Mirman, 1980) to a situation in which demand for the resource grows exogenously and heterogeneously.[1] Specifically, we consider the case of two countries with heterogeneous growth in demand. The growth in demand is assumed exogenous in order to identify clearly the effect of growing demand on behavior, thereby abstracting from the effect of natural resource utilization on demand growth.

In order to provide a general view of the effect of demand growth on extraction, we solve for non-cooperative and cooperative solutions. Under non-cooperation, both Cournot-Nash and Stackelberg frameworks are considered. Under Cournot-Nash non-cooperation, a higher demand for one country leads both countries to extract more. In contrast, in a Stackelberg environment, a higher demand for the leader induces him to increase extraction while the follower reduces his. On the other hand, an increase in

[1] See Long (2011) for an exhaustive survey of models of dynamic games in the exploitation of renewable and exhaustible resources. None considers exogenous growth in demand with the possibility of heterogeneity in the growth rates, as in our paper.

the demand of the follower yields an increase in the follower's extraction without any effect on the leader's. Next, under cooperation, each country increases extraction along with an increase in his own demand, but decreases extraction when demand of the other country decreases.

The rest of the paper is organized as follows. In Section 2, we present the model. Section 3 provides Cournot-Nash non-cooperative and cooperative solutions, which are then analyzed in Section 4. We then extend the analysis to the Stackelberg environment in Section 5. Finally, Section 6 provides concluding remarks.

2 The Model

Consider the Great Fish War (Levhari and Mirman, 1980) dynamic game in which two countries derive utility from the utilization of a common-pool resource. Let y_t be the stock of the resource at time t. In the absence of extraction, the stock evolves according to the following rule,

$$y_{t+1} = y_t^{\alpha} \tag{1}$$

where $\alpha \in (0,1]$. From (1), the evolution of the stock applies to both renewable resources (i.e., $\alpha \in (0,1)$) and depletable resources (i.e., $\alpha = 1$).

At time t, for $i = 1,2$, country i utilizes $q_{i,t}$ units of the stock. Using (1), the evolution of the stock under exploitation is given by

$$y_{t+1} = (y_t - q_{1,t} - q_{2,t})^{\alpha} \tag{2}$$

where a total of $q_{1,t} + q_{2,t}$ is utilized at time t. For country i at time t, utilizing $q_{i,t}$ yields utility $u_i(q_{i,t}) = g_{i,t} \ln q_{i,t}$ where $g_{i,t} > 0$ reflects country i's present level of demand.[2]

In order to study the effect of exogenous and heterogeneous growth in demand on behavior, we assume that the demand parameter evolves over

[2]In Levhari and Mirman (1980), $g_{i,t} = 1$ for all i and t.

time. For $i = 1, 2$ and $t = 1, 2, \ldots$,

$$g_{i,t+1} = \lambda_i g_{i,t} + \theta_i \tag{3}$$

where $\lambda_i \in [0, 1)$ and $\theta_i > 0$ are country-specific parameters.[3] Given the initial value $g_{i,0} > 0$, the complete solution to (3) is

$$g_{i,t} = \lambda_i^t \left(g_{i,0} - \frac{\theta_i}{1 - \lambda_i} \right) + \frac{\theta_i}{1 - \lambda_i} \tag{4}$$

and the system converges asymptotically to the steady state

$$\overline{g}_i = \frac{\theta_i}{1 - \lambda_i} > 0. \tag{5}$$

Hence, from (5), the difference in demand between the two countries converges asymptotically to $\left| \frac{\theta_1}{1-\lambda_1} - \frac{\theta_2}{1-\lambda_2} \right|$.

3 Non-cooperation vs. Cooperation

In this section, we first characterize the Cournot-Nash non-cooperative solution. We then provide the solution when the two countries cooperate.

Definition 3.1 states the feedback-Nash equilibrium in the infinite-horizon case. To simplify notation, we drop the subscript t and we use a hat sign to mark the evolution over time. Specifically, g_i and $\hat{g}_i$ represent the level of demand today and tomorrow, respectively. Analogously, y and $\hat{y} = (y - q_1 - q_2)^\alpha$ are stock today and tomorrow, respectively. Let $\delta \in (0, 1)$ be the discount factor. The superscript N stands for *Non-cooperation*.

[3] The restrictions on $\lambda_i \in [0, 1)$ and $\theta_i > 0$ ensure that, for any $g_{i,0} > 0$, the system converges asymptotically to a positive steady state. If $\lambda_i = 1$, then the system explodes since $g_{i,t} = g_{i,0} + t\theta_i$ and $\theta_i > 0$.

Definition 3.1. *The tuple* $\{Q_1^N(y, g_1, g_2), Q_2^N(y, g_2, g_1)\}$ *is a feedback-Nash equilibrium if, for* $i, j = 1, 2$, $i \neq j$, *given* $Q_j^N(y, g_j, g_i)$,[4]

$$\begin{aligned} Q_i^N(y, g_i, g_j) = & \arg\max_{q_i \in (0,y)} \{g_i \ln q_i \\ & + \delta V_i^N \left((y - q_i - Q_j^N(y, g_j, g_i))^\alpha, \lambda_i g_i + \theta_i, \lambda_j g_j + \theta_j\right)\} \end{aligned} \quad (6)$$

such that $q_i \in (0, y - Q_j^N(y, g_j, g_i))$ *and where, for any* $\{y', g_i', g_j'\}$,

$$\begin{aligned} V_i^N(y', g_i', g_j') = & \, g_i \ln Q_i^N(y', g_i', g_j') \\ & + \delta V_i^N((y' - Q_1(y', g_1', g_2') - Q_2(y', g_2', g_1'))^\alpha, \lambda_i g_i' + \theta_i, \lambda_j g_j' + \theta_j). \end{aligned} \quad (7)$$

Proposition 3.2 presents the Cournot-Nash non-cooperative solution.

Proposition 3.2. *In the feedback Cournot-Nash equilibrium, for* $i, j = 1, 2, i \neq j$,

$$Q_i^N(y, g_i, g_j) = \frac{g_i/A_i^N}{g_i/A_i^N + g_j/A_j^N + \alpha\delta} y \quad (8)$$

where

$$A_i^N \equiv \frac{\lambda_i g_i + \frac{\theta_i}{1-\alpha\delta}}{1 - \alpha\delta\lambda_i}. \quad (9)$$

Proof. See Appendix B. □

Having characterized the non-cooperative solution, we now turn to the case of cooperation. When countries cooperate, individual extractions are chosen so as to maximize the sum of present and future discounted utilities, i.e., $\{Q_1^C(y, g_1, g_2), Q_2^C(y, g_2, g_1)\}$ are the optimal solutions consistent with the Bellman equation

$$\begin{aligned} V^C(y, g_1, g_2) = & \max_{q_1, q_2 \in (0,y)} \{g_1 \ln q_1 + g_2 \ln q_2 \\ & + \delta V^C((y - q_1 - q_2)^\alpha, \lambda_1 g_1 + \theta_1, \lambda_2 g_2 + \theta_2)\} \end{aligned} \quad (10)$$

[4]In principle, countries could choose $q_i = 0$ or $q_i = y$. However, these cases are excluded because the log utility and an infinite horizon ensure interior solutions at every period, i.e., $q_i \in (0, y)$.

subject to $q_1 + q_2 < y$. Here, the superscript C stands for *Cooperation*.

Proposition 3.3 characterizes the cooperative solution. As in the non-cooperative case, extraction policies depend on both countries' demand parameters. However, the effect of demand growth on extraction is different between non-cooperation and cooperation.

Proposition 3.3. *From* (10)*, for* $i, j = 1, 2, i \neq j$*,*

$$Q_i^C(y, g_i, g_j) = \frac{g_i/A^C}{\frac{g_1+g_2}{A^C} + \alpha\delta} y \tag{11}$$

where

$$A^C \equiv \frac{\lambda_1 g_1 + \frac{\theta_1}{1-\alpha\delta}}{1 - \alpha\delta\lambda_1} + \frac{\lambda_2 g_2 + \frac{\theta_2}{1-\alpha\delta}}{1 - \alpha\delta\lambda_2}. \tag{12}$$

Proof. See Appendix B. □

From Propositions 3.2 and 3.3, it follows that, compared to cooperation, non-cooperation induces both countries to extract more, which yields a lower level of the resource stock in the steady state. Remark 3.4 restates the tragedy of the commons in the context of heterogeneous growth in demand.

Remark 3.4. *From* (8) *and* (11)*,*

$$Q_1^C(y, g_1, g_2) + Q_2^C(y, g_2, g_1) \leq Q_1^N(y, g_1, g_2) + Q_2^N(y, g_2, g_1). \tag{13}$$

In the subsequent sections, we discuss the effect of demand growth on the equilibrium. We proceed as follows. We first consider policy functions under Cournot-Nash non-cooperation and cooperation. We then extend the analysis to the Stackelberg environment as done in Levhari and Mirman (1980). The discussion on the steady state with a particular focus on welfare is relegated to Appendix C.

4 Extraction Policies

In this section, we study how the presence of heterogeneous growth in demand affects extraction policies. We begin by considering the intermediate case in which the level of demand is different across the two countries, i.e., $g_1 \neq g_2$, but without growth in demand. Proposition 4.1 states that in the presence of differences in demand, non-cooperation distorts the allocation of the resource between the two countries in favor of the smaller country. Specifically, the cooperative solution allocates more resource toward the largest country whereas in the non-cooperative solution, each country extracts the same amount each period, regardless of demand size.

Proposition 4.1. *Suppose that $\lambda_i = 1$ and $\theta_i = 0$. Then,*

1. *Under non-cooperation, for $i = 1, 2$,*

$$Q_i^N(y, g_i, g_j) = \frac{1 - \alpha\delta}{2 - \alpha\delta} y. \tag{14}$$

2. *Under cooperation, for $i = 1, 2$,*

$$Q_i^C(y, g_1, g_2) = \frac{g_i(1 - \alpha\delta)}{g_i + g_j} y. \tag{15}$$

Proof. Evaluating (8) and (11) at $\lambda_i = 1$ and $\theta_i = 0$ yields (14) and (15), respectively. □

Having considered the intermediate case of heterogeneity in demand with no growth, we now study how the non-cooperative solution compares to the cooperative solution when there is heterogeneous growth in demand. Proposition 4.2 states that in the presence of heterogeneous growth in demand, similar to the case of no growth, the cooperative solution yields higher extraction for the country with the largest present size of demand. However, unlike the no-growth case, under non-cooperation, countries with heterogeneous growth in demand extract unequally. Nevertheless, the allocation of the resource between the two countries is distorted in a different way. Indeed, under non-cooperation, countries take account of growing demand (i.e., the terms A_i^N and A_j^N) for their current consumption decisions.

Proposition 4.2. *Suppose that* $\lambda_i \in [0,1)$ *and* $\theta_i > 0$*. Then,*

1. *Under non-cooperation, from* (8) *and* (9), $Q_i^N(y, g_1, g_2) > Q_j^N(y, g_1, g_2)$ *if and only if* [5]
$$\frac{g_i}{A_i^N} > \frac{g_j}{A_j^N}. \tag{18}$$

2. *Under cooperation, from* (11) *and* (12), $Q_i^C(y, g_1, g_2) > Q_j^C(y, g_1, g_2)$ *if and only if* $g_i > g_j$.

Proposition 4.3 provides the effect of an increase in the size of demand on the cooperative and the non-cooperative solutions. Consider first an increase in the current demand of one country (part (a)). Under cooperation, such a change increases extraction for the growing country, but reduces extraction of the other country. However, under non-cooperation, both countries increase their resource extraction. Consider next an increase in the future demand of one country (part (b)). Under cooperation, each country reduces present extraction to preserve the stock of resources for the future enlarged demand. Under non-cooperation, the country whose future demand size has increased, cuts current resource extraction. However, the other country increases extraction in anticipation to lower availability of the resource in future. Hence, heterogeneity in demand growth has an effect in over-exploitation of the resources and the tragedy of the commons. The reason is that, under non-cooperation, countries' competition to extract resources is exacerbated when they anticipate higher future demand from their competitor.

[5]We can also define these conditions in terms of the parameters of the model. Specifically, $g_i > g_j$ is equivalent to

$$\lambda_i^t \left(g_{i,0} - \frac{\theta_i}{1-\lambda_i}\right) + \frac{\theta_i}{1-\lambda_i} > \lambda_j^t \left(g_{j,0} - \frac{\theta_j}{1-\lambda_j}\right) + \frac{\theta_j}{1-\lambda_j}, \tag{16}$$

and $g_i/A_i^N > g_j/A_j^N$ is equivalent to

$$\frac{(1-\alpha\delta\lambda_i)\left(\lambda_i^t\left(g_{i,0} - \frac{\theta_i}{1-\lambda_i}\right) + \frac{\theta_i}{1-\lambda_i}\right)}{\lambda_i^{t+1}\left(g_{i,0} - \frac{\theta_i}{1-\lambda_i}\right) + \frac{\theta_i}{1-\alpha\delta}} > \frac{(1-\alpha\delta\lambda_j)\left(\lambda_j^t\left(g_{j,0} - \frac{\theta_j}{1-\lambda_j}\right) + \frac{\theta_j}{1-\lambda_j}\right)}{\lambda_j^{t+1}\left(g_{j,0} - \frac{\theta_j}{1-\lambda_j}\right) + \frac{\theta_j}{1-\alpha\delta}}. \tag{17}$$

Proposition 4.3. *For $i, j = 1, 2,\ i \neq j$,*

1. *Under non-cooperation, from* (8) *and* (9),
   (a) $\frac{\partial Q_i^N(y,g_i,g_j)}{\partial g_i} > 0,\ \frac{\partial Q_j^N(y,g_i,g_j)}{\partial g_i} > 0,$
   (b) $\frac{\partial Q_i^N(y,g_i,g_j)}{\partial \hat{g}_i} < 0,\ \frac{\partial Q_j^N(y,g_i,g_j)}{\partial \hat{g}_i} > 0.$
2. *Under cooperation, from* (11) *and* (12),
   (a) $\frac{\partial Q_i^C(y,g_i,g_j)}{\partial g_i} > 0,\ \frac{\partial Q_j^C(y,g_i,g_j)}{\partial g_i} < 0,$
   (b) $\frac{\partial Q_i^C(y,g_i,g_j)}{\partial \hat{g}_i} < 0,\ \frac{\partial Q_j^C(y,g_i,g_j)}{\partial \hat{g}_i} < 0.$

In order to understand better the distortion resulting from non-cooperation pointed out in Proposition 4.3, we consider each country's share of extraction. Let r_i^C and r_i^N be country i's share of extraction under cooperation and non-cooperation, respectively. Hence, from (8) and (11), for $i, j = 1, 2, i \neq j$,

$$r_i^N = \frac{Q_i^N(y, g_i, g_j)}{Q_1^N(y, g_i, g_j) + Q_2^N(y, g_i, g_j)} = \frac{g_i A_j^N}{g_i A_j^N + g_j A_i^N}, \tag{19}$$

$$r_i^C = \frac{Q_i^C(y, g_i, g_j)}{Q_1^C(y, g_i, g_j) + Q_2^C(y, g_i, g_j)} = \frac{g_i}{g_i + g_j}. \tag{20}$$

Under cooperation, an increase in the demand size of a country increases his own share of extraction, and thus reduces the other country's share of extraction. These effects hold under non-cooperation as well. In other words, Proposition 4.3 states that, under non-cooperation, in response to an increase in the demand size of country i, country j increases its extraction, his share decreases due to the greater increase in the demand size of his rival, i.e., country i. Under the cooperative solution, an increase in the future demand size of a country does not affect the countries' present shares of extractions. However, under non-cooperation, the share of the country whose future demand size has increased decreases due to the strategic reaction of his rival.

Remark 4.4. *For $i, j = 1, 2$, $i \neq j$,*

1. *Under non-cooperation, from* (19),
   (a) $\frac{\partial r_i^N}{\partial g_i} > 0$, $\frac{\partial r_j^N}{\partial g_i} < 0$,
   (b) $\frac{\partial r_i^N}{\partial \hat{g}_i} < 0$, $\frac{\partial r_j^N}{\partial \hat{g}_i} > 0$.
2. *Under cooperation, from* (20),
   (a) $\frac{\partial r_i^C}{\partial g_i} > 0$, $\frac{\partial r_j^C}{\partial g_i} < 0$,
   (b) $\frac{\partial r_i^C}{\partial \hat{g}_i} = 0$, $\frac{\partial r_j^C}{\partial \hat{g}_i} = 0$.

5 Stackelberg

In this section, we extend our analysis to the Stackelberg environment.[6] Proposition 5.1 characterizes the Stackelberg solution. The subscript L stands for *leader* whereas the subscript F stands for *follower*.

Proposition 5.1. *Under Stackelberg, the leader extracts*

$$Q_L(y, g_L, g_F) = \frac{g_L/A_L}{g_L/A_L + \alpha\delta} y \tag{21}$$

and the follower extracts

$$Q_F(y, g_L, g_F) = \frac{\alpha\delta g_F/A_F}{(\alpha\delta + g_F/A_F)(\alpha\delta + g_L/A_L)} y, \tag{22}$$

where, for $z \in \{L, F\}$, $A_z = \frac{\lambda_z g_z + \frac{\theta_z}{1-\alpha\delta}}{1-\alpha\delta\lambda_z}$.

Proof. See Appendix B. □

Proposition 5.2 provides the condition under which the leader extracts more than the follower.

[6] We wish to thank a reviewer for suggesting this analysis. See Levhari and Mirman (1980) for a definition as well as an interpretation of leader-follower as sophisticated-naive.

Proposition 5.2. *Suppose that $\lambda_i \in [0,1)$ and $\theta_i > 0$. Then, under Stackelberg, from (21) and (22), $Q_L(y, g_L, g_F) > Q_F(y, g_L, g_F)$, if and only if*

$$\frac{g_L}{A_L} > \frac{\alpha\delta \frac{g_F}{A_F}}{\alpha\delta + \frac{g_F}{A_F}}, \tag{23}$$

or

$$\frac{A_F}{g_F} + \frac{1}{\alpha\delta} > \frac{A_L}{g_L}. \tag{24}$$

Proposition 5.3 states the effect of an increase in the size of demand on extractions. Comparing Proposition 5.3 with Proposition 4.3 provides interesting insights about the difference between Cournot-Nash and Stackelberg. In particular, the follower reduces his consumption along with an increase in the leader's present demand. The reason is that the follower does not react strategically to an increase in the leader's demand. Instead, by observing the increase in the leader's demand, the follower reduces his extraction to preserve the resource for the future. However, an increase in follower's demand does not affect the leader's extraction.

Proposition 5.3. *1. Under Stackelberg, from* (21) *and* (22)*,*

(a) $\frac{\partial Q_L(y, g_L, g_F)}{\partial g_L} > 0$, $\frac{\partial Q_F(y, g_L, g_F)}{\partial g_L} < 0$,

(b) $\frac{\partial Q_L(y, g_L, g_F)}{\partial g_F} = 0$, $\frac{\partial Q_F(y, g_L, g_F)}{\partial g_F} > 0$.

Remark 5.4 presents the effect of a change in one countries demand on the country's share of extraction. Let r_L and r_F be leader's and follower's share of extraction, respectively. Hence, from (21) and (22),

$$r_L = \frac{\left(\alpha\delta + \frac{g_F}{A_F}\right)\frac{g_L}{A_L}}{\frac{g_F}{A_F}\frac{g_L}{A_L} + \alpha\delta\left(\frac{g_L}{A_L} + \frac{g_F}{A_F}\right)}, \tag{25}$$

$$r_F = \frac{\alpha\delta\frac{g_F}{A_F}}{\frac{g_F}{A_F}\frac{g_L}{A_L} + \alpha\delta\left(\frac{g_L}{A_L} + \frac{g_F}{A_F}\right)}. \tag{26}$$

An increase in the demand size of a country increases his own share of extraction, and thus reduces the other country's share of extraction.

Remark 5.4. *Under Stackelberg, from* (25) *and* (26),

1. $\frac{\partial r_L}{\partial g_L} > 0$, $\frac{\partial r_F}{\partial g_L} < 0$,
2. $\frac{\partial r_F}{\partial g_F} > 0$, $\frac{\partial r_L}{\partial g_F} < 0$.

6 Final Remark

We study the effect of heterogeneous and exogenous growth in demand on extraction of a common pool resource. Consideration of heterogeneous demand growth in exploitation of resources is a relevant line of research as historical data shows that the consumption of resources is growing over time and this consumption growth is highly heterogeneous among different countries. See Appendix A.

We extend the Great Fish War framework of Levhari and Mirman (1980) by accounting for heterogeneous growth in demand. We compare the non-cooperative solution (Cournot and Stackelberg) with the cooperative solution. Our results suggest that heterogeneity in demand growth has a profound effect on both cooperative and non-cooperative solutions. Moreover, the presence of heterogeneous growth in demand may exacerbate the tragedy of the commons because the anticipation of higher demand from rivalrous countries induces each country to increase extraction. Finally, although the cooperative solution allocates more resources toward the largest country, the non-cooperative solution ignores such differences, i.e., each country extracts the same amount regardless of demand size.

Our analysis is an exploratory attempt to account for heterogeneity in demand growth in the most parsimonious setting. We acknowledge that some of our results are reminiscent of our assumption of considering an exogenous linear demand growth rule that appears in the countries' welfare as a multiplicative term. Some extensions can be envisioned in future work to enhance our understanding of the impact of demand growth on resource extraction. Specifically, we could extend the present analysis to an endogenous growth model by linking the growth rate of demand with past and present extractions. That is, as countries extract more, they develop new sectors of production of goods and services, which in turn requires higher demand for natural resources and energy-related resources.

A Figures

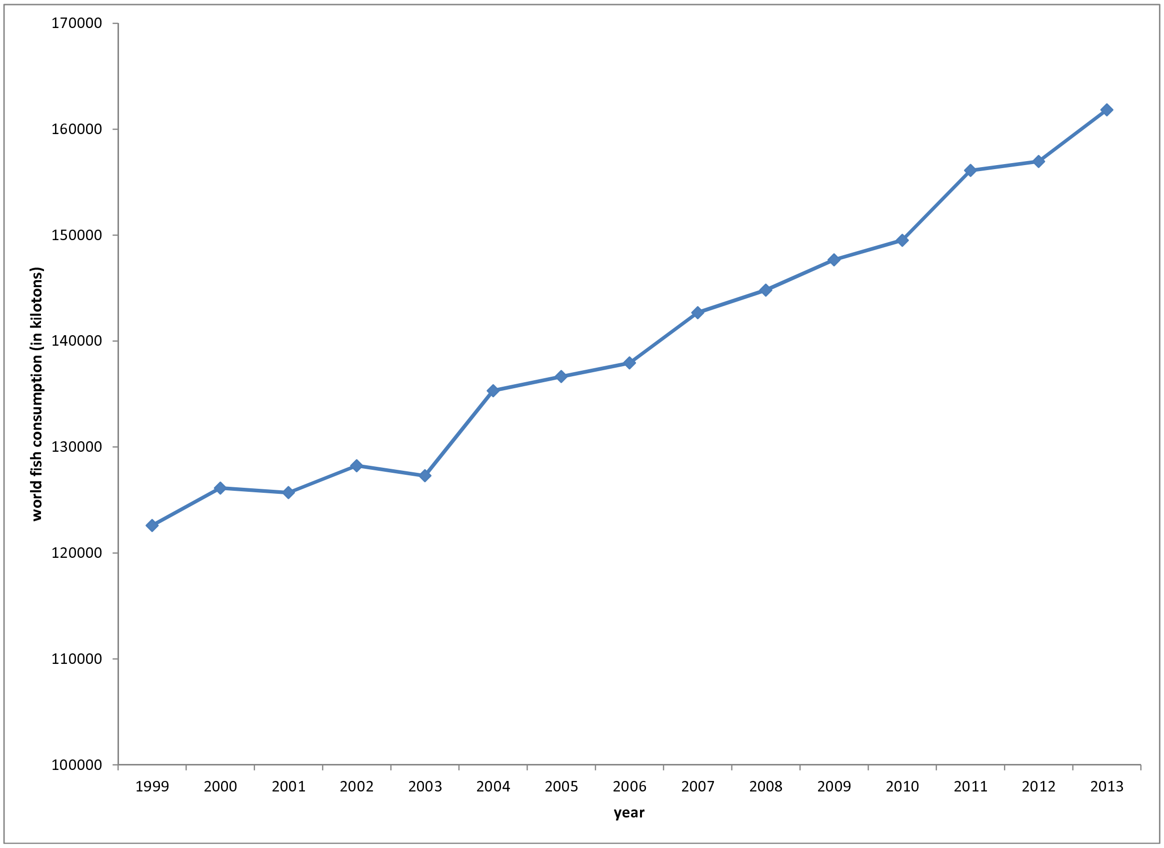

Figure 1: World Fish Consumption. Source: OECD/FAO (2013),"OECD-FAO Agricultural Outlook: Highlights 2013," OECD Agriculture Statistics (database).

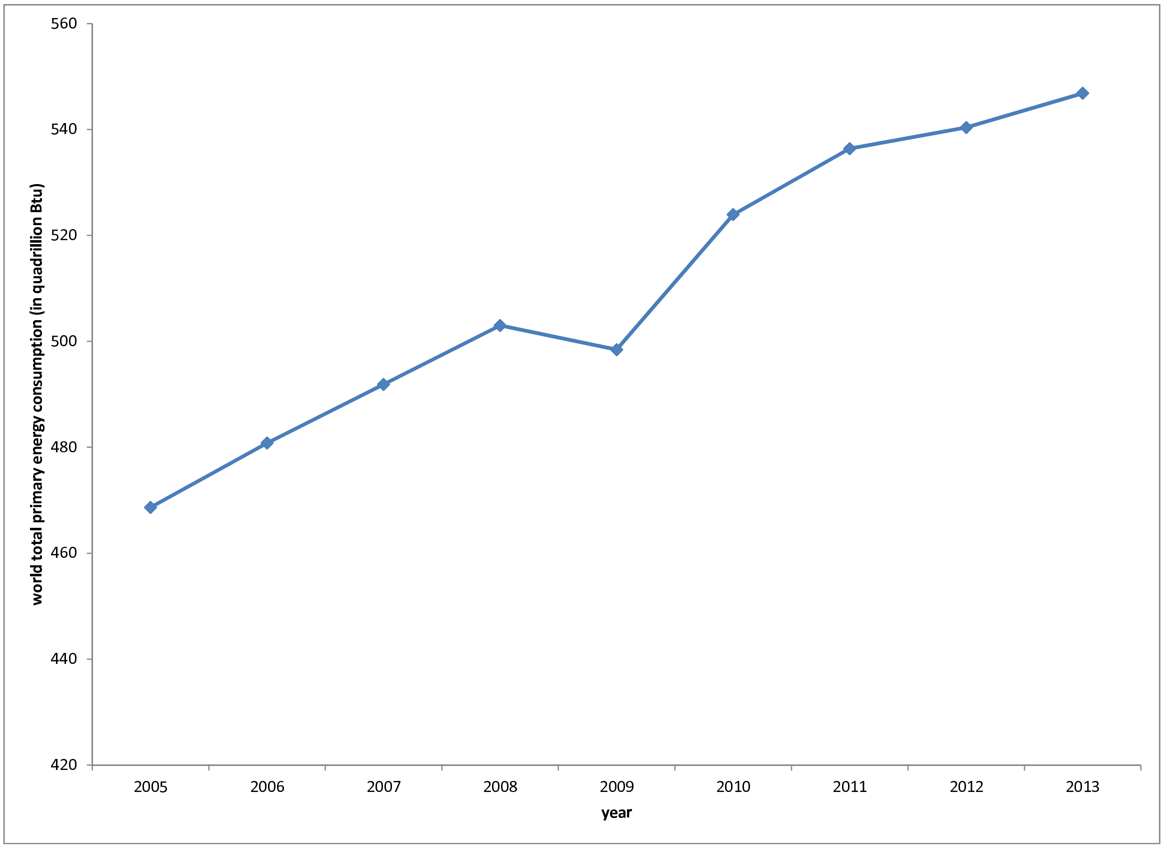

Figure 2: World Total Primary Energy Consumption. Source: U.S. Energy Information Administration (EIA); International Energy Statistics database (as of November 2012); and International Energy Agency, "Balances of OECD and Non-OECD Statistics" (2012).

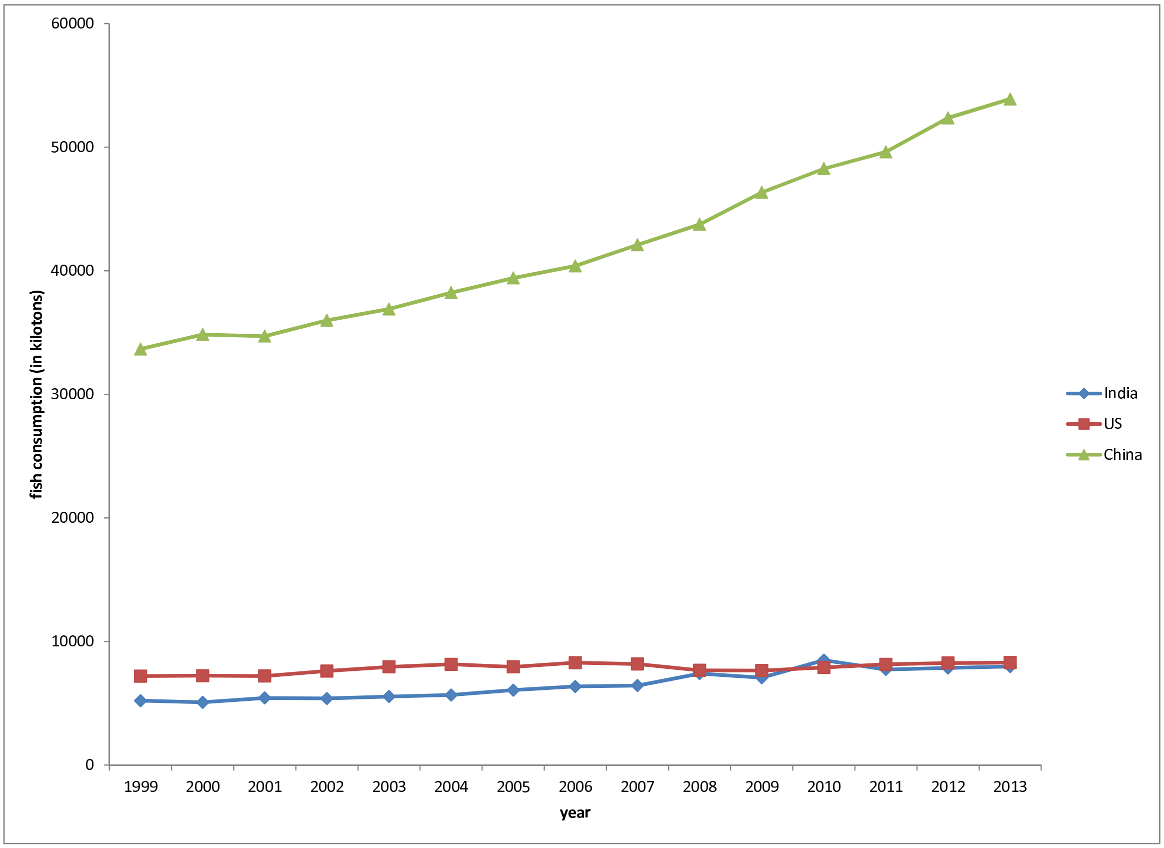

Figure 3: Fish Consumption by Country. Source: OECD/FAO (2013), "OECD-FAO Agricultural Outlook: Highlights 2013," OECD Agriculture Statistics (database).

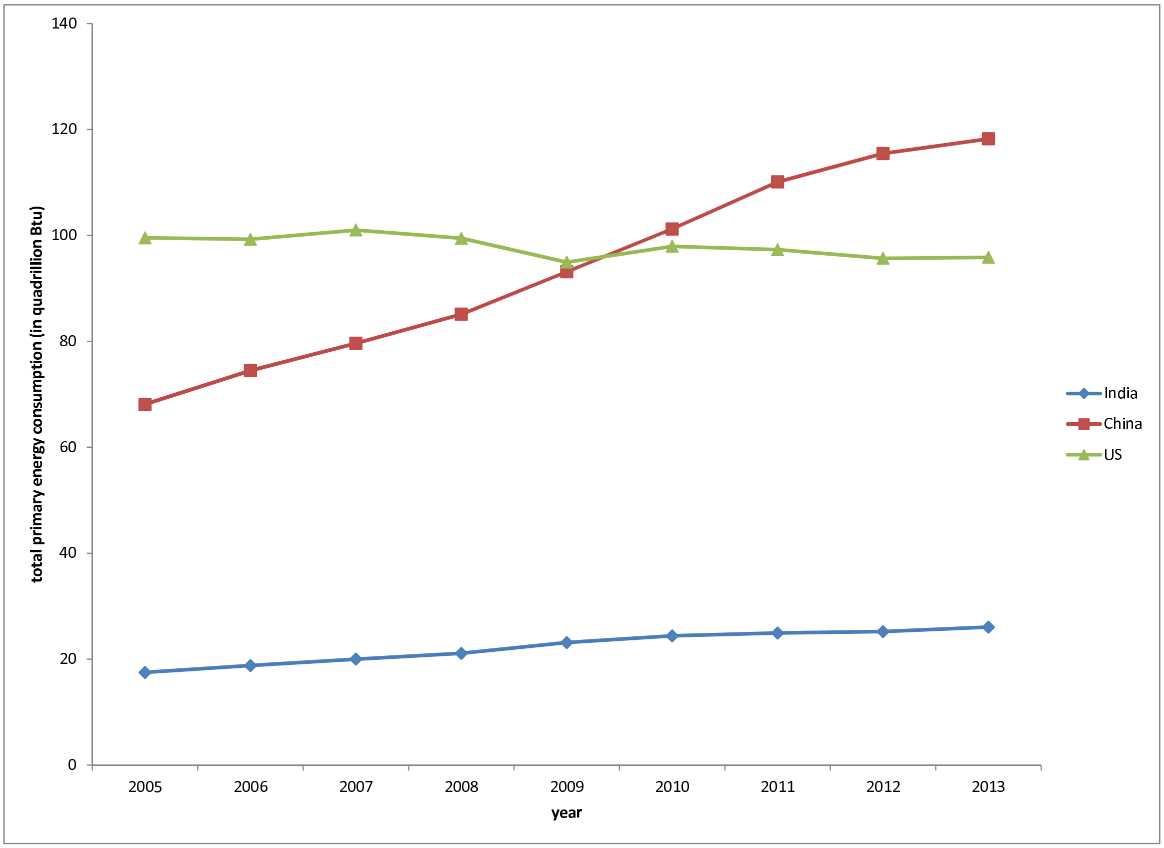

Figure 4: Total Primary Energy Consumption by Country. Source: U.S. Energy Information Administration (EIA); International Energy Statistics database (as of November 2012); and International Energy Agency, "Balances of OECD and Non-OECD Statistics" (2012).

B Proofs

Proof of Proposition 3.2. We conjecture that country i's value function has the form,[7]

$$V_i^N(y, g_i, g_j) = (X_i^N g_i + Y_i^N) \ln y + \varphi_i^N(g_i, g_j). \tag{27}$$

Plugging (27) into the objective function of (6) yields the dynamic maximization problem

$$\max_{q_i} \{g_i \ln q_i + \delta(A_i \ln(y - q_1 - q_2)^\alpha + \varphi_i^N(\hat{g}_i, \hat{g}_j)\} \tag{28}$$

where $A_i^N = X_i^N \hat{g}_i + Y_i^N$. For $i, j = 1, 2, i \neq j$, given $q_j = Q_j^N(y, g_j, g_i)$, country i's first-order condition is

$$\frac{g_i}{q_i} = \frac{\alpha\delta A_i^N}{y - q_i - Q_j^N(y, g_j, g_i)}, \tag{29}$$

which yields $Q_i^N(y, g_i, g_j) = \omega_i^N y$ where

$$\omega_i^N = \frac{g_i / A_i^N}{g_i / A_i^N + g_j / A_j^N + \alpha\delta}. \tag{30}$$

Plugging $Q_i^N(y, g_i, g_j) = \omega_i^N y$, $A_i^N = X_i^N \hat{g}_i + Y_i^N$, and (27) into the objective function of (28) yields the value function

$$\begin{aligned} V_i^N(y, g_i, g_j) = g_i \ln \omega_i^N + g_i \ln y + \delta\alpha(X_i^N \hat{g}_i + Y_i^N) \ln(1 - \omega_i^N - \omega_j^N)) \\ + \delta\alpha(X_i^N \hat{g}_i + Y_i^N) \ln y + \varphi_i^N(\hat{g}_i, \hat{g}_j), \end{aligned} \tag{31}$$

which needs to agree with the conjecture as defined by (27), i.e.,

$$X_i^N g_i + Y_i^N = g_i + \delta\alpha \left(X_i^N (\lambda_i g_i + \theta_i) + Y_i^N \right) \tag{32}$$

[7] The conjecture can be inferred by solving the problem recursively as done in Levhari and Mirman (1980). By solving recursively, one realizes that the value function is always linear in $\ln y$. Moreover, the limit of the solution for the t-period game as t goes to infinity is the solution to the infinite-horizon game that we consider. Hence, there is a unique feedback-Nash equilibrium *in the class of strategies that are linear in the stock.*

and

$$\varphi_i^N(g_i, g_j) = g_i \ln \omega_i^N + \delta\alpha \left(X_i^N \hat{g}_i + Y_i^N \right) \ln(1 - \omega_i^N - \omega_j^N) + \delta\varphi_i^N(\hat{g}_i, \hat{g}_j). \quad (33)$$

Given that $\alpha\delta\lambda_i < 1$, equation (32) implies that

$$X_i^N = \frac{1}{1 - \alpha\delta\lambda_i}, \quad (34)$$

$$Y_i^N = \frac{\alpha\delta\theta_i}{1 - \alpha\delta} \frac{1}{1 - \alpha\delta\lambda_i}, \quad (35)$$

which, using (30) and the fact that $A_i^N = X_i^N \hat{g}_i + Y_i^N$, yields (8) and (9).

Proof of Proposition 3.3. We conjecture that the cooperative value function has the form,

$$V^C(y, g_1, g_2) = \left(X_1^C g_1 + X_2^C g_2 + Y^C \right) \ln y + \varphi^C(g_1, g_2). \quad (36)$$

Plugging (27) into (10) yields

$$\begin{aligned} V^C(y, g_1, g_2) = \max_{0<q_1,q_2<y} \{ & g_1 \ln q_1 + g_2 \ln q_2 \\ & + \delta(A^C \ln(y - q_1 - q_2)^\alpha + \varphi^C(\hat{g}_1, \hat{g}_2))\}. \end{aligned} \quad (37)$$

where $A^C = X_1^C \hat{g}_1 + X_2^C \hat{g}_2 + Y^C$. For $i, j = 1, 2, i \neq j$, the first-order condition for i yields

$$q_i = g_i \frac{y - q_j}{g_i + \alpha\delta A^C} \quad (38)$$

so that $Q_i^C(y, g_1, g_2) = \omega_i^C y$ where

$$\omega_i^C = \frac{g_i}{g_i + g_j + \alpha\delta A^C}. \quad (39)$$

Plugging $Q_i^C(y, g_i, g_j) = \omega_i^C y$, $A^C = X_1^C \hat{g}_1 + X_2^C \hat{g}_2 + Y^C$ and (36) into (37) yields the value function

$$\begin{aligned} V^C(y, g_1, g_2) = (g_1 + g_2) \ln y + g_1 \ln \omega_1^C + g_2 \ln \omega_2^C \\ + \delta\alpha \left(X_1^C \hat{g}_1 + X_2^C \hat{g}_2 + Y^C \right) \left(\ln y + \ln(1 - \omega_1^C - \omega_2^C) \right) \\ + \delta\varphi^C(\hat{g}_1, \hat{g}_2), \end{aligned} \tag{40}$$

which needs to agree with the conjecture as defined by (36), i.e.,

$$X_1^C g_1 + X_2^C g_2 + Y^C = g_1 + g_2 + \alpha\delta \left(X_1^C \hat{g}_1 + X_2^C \hat{g}_2 + Y^C \right) \tag{41}$$

and

$$\begin{aligned} \varphi^C(g_1, g_2) = g_1 \ln \omega_1^C + g_2 \ln \omega_2^C + \\ \delta\alpha \left(X_1^C \hat{g}_1 + X_2^C \hat{g}_2 + Y^C \right) \left(\ln(1 - \omega_1^C - \omega_2^C) \right) + \delta\varphi^C(\hat{g}_1, \hat{g}_2). \end{aligned} \tag{42}$$

Solving equation (41) for X_1^C, X_2^C and Y^C yields

$$X_i^C = \frac{1}{1 - \alpha\delta\lambda_i}, \tag{43}$$

$$\begin{aligned} Y^C &= \frac{\alpha\delta}{1 - \alpha\delta} \left(X_1^{SP}\theta_1 + X_2^{SP}\theta_2 \right) \\ &= \frac{\alpha\delta}{1 - \alpha\delta} \left(\frac{\theta_1}{1 - \alpha\delta\lambda_1} + \frac{\theta_2}{1 - \alpha\delta\theta_1} \right), \end{aligned} \tag{44}$$

which, using (39) and the fact that $A^C = X_1^C \hat{g}_1 + X_2^C \hat{g}_2 + Y^C$, yields (11) and (12).

Proof of Proposition 5.1. We conjecture that the value function of the follower has the form,

$$V_F(y, g_F, g_L) = (X_F g_F + Y_F) \ln y + \varphi_F(g_F, g_L). \tag{45}$$

Using (45), the value function is rewritten as

$$V_F(y, g_F, g_L) = \max_{q_F} \{ g_F \ln q_F + \delta(A_F \ln(y - q_F - q_L)^\alpha + \varphi_F(\hat{g}_F, \hat{g}_L) \} \tag{46}$$

where $A_F = X_F \hat{g}_F + Y_F$. Then, the first-order condition is

$$\frac{g_F}{q_F} = \frac{\alpha\delta A_F}{y - q_F - q_L}, \tag{47}$$

which yields

$$q_F = \frac{g_F(y - q_L)}{\alpha\delta A_F + g_F}. \tag{48}$$

Given (48), the leader's maximization problem is

$$\max_{q_L} \left\{ g_L \ln q_L + \delta V_L \left(\left(y - q_L - \frac{g_F(y - q_L)}{\alpha\delta A_F + g_F} \right)^\alpha, \lambda_L g_L + \theta_L, \lambda_F g_F + \theta_F \right) \right\} \tag{49}$$

We conjecture that the value function of the leader has the form,

$$V_L(y, g_F, g_L) = (X_L g_L + Y_L) \ln y + \varphi_L(g_F, g_L). \tag{50}$$

Plugging (50) into the objective function of (49) yields

$$V_L(y, g_F, g_L) = \max_{q_L} \left\{ g_L \ln q_L + \delta \left(A_L \ln \left(y - q_L - \frac{g_F\,(y - q_L)}{\alpha\delta A_F + g_F} \right)^\alpha + \varphi_L(\hat{g}_F, \hat{g}_L) \right) \right\} \tag{51}$$

where $A_L = X_L \hat{g}_L + Y_L$. Then, the first-order condition is

$$\frac{g_L}{q_L} = \frac{\frac{\alpha^2\delta^2 A_F A_L}{\alpha\delta A_F + g_F}}{y - q_L - \frac{g_F(y - q_L)}{\alpha\delta A_F + g_F}}, \tag{52}$$

which yields

$$Q_L\,(y, g_L, g_F) = \omega_L y, \tag{53}$$

where $\omega_L = \frac{g_L/A_L}{g_L/A_L + \alpha\delta}$. Plugging (53) into (48) yields (22)

$$Q_F\,(y, g_L, g_F) = \omega_F y, \tag{54}$$

where $\omega_F = \frac{\alpha\delta\frac{g_F}{A_F}}{\left(\alpha\delta + \frac{g_F}{A_F}\right)\left(\alpha\delta + \frac{g_L}{A_L}\right)}$. Plugging (45) and (54) into the objective function of (46) yields the value function

$$(X_F g_F + Y_F)\ln y + \varphi_F(g_F, g_L) = g_F \ln \omega_F + g_L \ln y + \delta\alpha(X_F \hat{g}_F + Y_F)\ln(1-\omega_F-\omega_L)) + \delta\alpha(X_F\hat{g}_F + Y_F)\ln y + \delta\varphi_F(\hat{g}_F, \hat{g}_L), \tag{55}$$

which needs to agree with the conjecture as defined by (45), i.e.,

$$X_F g_F + Y_F = g_F + \delta\alpha\left(X_F(\lambda_F g_F + \theta_F) + Y_F\right) \tag{56}$$

and

$$\varphi_F(g_F, g_L) = g_F \ln\omega_F + \delta\alpha(X_F\hat{g}_F + Y_F)\ln(1-\omega_F-\omega_L)) + \delta\varphi_F(\hat{g}_F, \hat{g}_L). \tag{57}$$

Given that $\alpha\delta\lambda_i < 1$, equation (56) implies that

$$X_F = \frac{1}{1-\alpha\delta\lambda_F}, \tag{58}$$

$$Y_F = \frac{\alpha\delta\theta_F}{1-\alpha\delta}\frac{1}{1-\alpha\delta\lambda_F}, \tag{59}$$

and thus $A_F = \frac{\lambda_F g_F + \frac{\theta_F}{1-\alpha\delta}}{1-\alpha\delta\lambda_F}$. Similarly, it can be shown that

$$X_L = \frac{1}{1-\alpha\delta\lambda_L}, \tag{60}$$

$$Y_L = \frac{\alpha\delta\theta_L}{1-\alpha\delta}\frac{1}{1-\alpha\delta\lambda_L}, \tag{61}$$

so that $A_L = \frac{\lambda_L g_L + \frac{\theta_L}{1-\alpha\delta}}{1-\alpha\delta\lambda_L}$.

C Steady State

This section compares the steady state between Cournot-Nash non-cooperation and cooperation. We first consider the effect of heterogeneity in demand growth on extraction and stock in the steady state. We then study the effect of demand growth on the welfare of individual countries as well as the global welfare.[8]

C.1 Extraction and Stock

Although demand growth (and heterogeneity) has a profound effect on the equilibrium values for extraction, the non-cooperative steady state turns out to be unaffected by the level of demand and the exogenous parameters governing demand growth. Interestingly, growth in demand does have an effect on the cooperative solution.

Remark C.1. *In the non-cooperative steady state, for* $i, j = 1, 2, i \neq j$

$$Q_i^N(\overline{y}^N, \overline{g}_i, \overline{g}_j) = \frac{1-\alpha\delta}{2-\alpha\delta}\overline{y}^N. \tag{62}$$

where

$$\overline{y}^N = \left(\frac{\alpha\delta}{2-\alpha\delta}\right)^{\frac{\alpha}{1-\alpha}} \tag{63}$$

is the steady state stock.

Unlike the non-cooperative case, heterogeneity in demand growth has an effect on the steady state of the cooperative solution. However, the effect is present only in the allocation of the stock since the steady state stock remains unaffected by growth in demand.

Remark C.2. *In the cooperative steady state, for* $i, j = 1, 2, i \neq j$

$$Q_i^C(\overline{y}^C, \overline{g}_i, \overline{g}_j) = \frac{\overline{g}_i\,(1-\alpha\delta)}{\overline{g}_i + \overline{g}_j}\overline{y}^C. \tag{64}$$

where $\overline{g}_1, \overline{g}_2$ *are given by* (5) *and*

$$\overline{y}^C = (\alpha\delta)^{\frac{\alpha}{1-\alpha}} \tag{65}$$

is the steady state stock.

[8] We wish to thank a reviewer for suggesting this analysis.

C.2 Welfare

Note first that because of the log utility function, utility levels are negative when consumption is below one. Although negative utility has no bearing for our previous analysis on extraction policies, it does have an effect when studying the effect of demand growth on welfare. Indeed, holding everything else constant, an increase in demand size must increase instantaneous payoffs, which is not possible for consumption below one. The law of motion is thus modified in order to proceed with the welfare analysis. Specifically, (1) is rewritten as

$$y_{t+1} = \eta y_t^{\alpha} \tag{66}$$

where $\eta > 1$ allows the stock of the resource to be larger than one at the steady state. Specifically,

$$\overline{y}^N = \eta^{\frac{1}{1-\alpha}} \left(\frac{\alpha\delta}{2-\alpha\delta} \right)^{\frac{\alpha}{1-\alpha}}, \tag{67}$$

$$\overline{y}^C = \eta^{\frac{1}{1-\alpha}} (\alpha\delta)^{\frac{\alpha}{1-\alpha}}. \tag{68}$$

We also assume that η is large enough to make consumption at the steady state larger than one. This assumption has no effect on the non-cooperative and cooperative solutions because the parameter η is only found in the constant term of the value function.

In the non-cooperative case, a higher growth rate in demand for one country leads to higher welfare for that same country, but it has no effect on the welfare of the rivalrous country. On the other hand, under cooperation, a higher growth rate in demand for any country has a positive effect on welfare. One of the reasons is that the steady state stock is unaffected by growth in demand. Hence, in the Great Fish War model, a higher demand has no effect on the steady state availability of the resource.

Proposition C.3. *In the steady state,*

1. *Under non-cooperation,* $\frac{\partial V^N(\overline{y}^C, \overline{g}_i, \overline{g}_j)}{\partial \overline{g}_i} > 0$ *and* $\frac{\partial V^N(\overline{y}^C, \overline{g}_i, \overline{g}_j)}{\partial \overline{g}_j} = 0$,
2. *Under cooperation,* $\frac{\partial V^N(\overline{y}^C, \overline{g}_i, \overline{g}_j)}{\partial \overline{g}_i} > 0$.

Proof. Under non-cooperation, $V_i^N(\overline{y}^N, \overline{g}_i, \overline{g}_j) = \overline{g}_i \ln Q_i^N(\overline{y}^N, \overline{g}_i, \overline{g}_j) + \delta V_i^N(\overline{y}^N, \overline{g}_i, \overline{g}_j)$, which simplifies to

$$V_i^N(\overline{y}^N, \overline{g}_i, \overline{g}_j) = \frac{\overline{g}_i \ln Q_i^N(\overline{y}^N, \overline{g}_i, \overline{g}_j)}{1-\delta} \tag{69}$$

where $Q_i^N(\overline{y}^N, \overline{g}_i, \overline{g}_j) = \frac{1-\alpha\delta}{2-\alpha\delta}\overline{y}^N$. Hence,

$$\frac{\partial V_i^N(\overline{y}^N, \overline{g}_i, \overline{g}_j)}{\partial \overline{g}_i} = \frac{\ln Q_i^N(\overline{y}^N, \overline{g}_i, \overline{g}_j)}{1-\delta} > 0 \tag{70}$$

since we assume that η is large enough to yield a consumption above one, i.e., $\frac{1-\alpha\delta}{2-\alpha\delta}\overline{y}^N > 1$. Moreover, $\frac{\partial V_i^N(\overline{y}^N, \overline{g}_i, \overline{g}_j)}{\partial \overline{g}_j} = 0$.

Next, under cooperation,

$$V^C(\overline{y}^C, \overline{g}_i, \overline{g}_j) = \overline{g}_i \ln Q_i^C(\overline{y}^C, \overline{g}_i, \overline{g}_j) + \overline{g}_j \ln Q_j^C(\overline{y}^C, \overline{g}_j, \overline{g}_i) + \delta V^C(\overline{y}^C, \overline{g}_i, \overline{g}_j), \tag{71}$$

that is

$$V^C(\overline{y}^C, \overline{g}_i, \overline{g}_j) = \frac{\overline{g}_i \ln Q_i^C(\overline{y}^C, \overline{g}_i, \overline{g}_j) + \overline{g}_j \ln Q_j^C(\overline{y}^C, \overline{g}_j, \overline{g}_i)}{1-\delta} \tag{72}$$

where, for $i = 1, 2, i \neq j$, $Q_i^C(\overline{y}^C, \overline{g}_i, \overline{g}_j) = \frac{\overline{g}_i(1-\alpha\delta)\overline{y}^C}{\overline{g}_i+\overline{g}_j} > 1$ since we assume that η is large enough to yield consumption above one. Using (72),

$$\begin{aligned}
\frac{\partial V^C(\overline{y}^C, \overline{g}_i, \overline{g}_j)}{\partial \overline{g}_i} &= \frac{\ln Q_i^C(\overline{y}^C, \overline{g}_i, \overline{g}_j) + \frac{\overline{g}_i \frac{\partial Q_i^C(\overline{y}^C, \overline{g}_i, \overline{g}_j)}{\partial \overline{g}_i}}{Q_i^C(\overline{y}^C, \overline{g}_i, \overline{g}_j)} + \frac{\overline{g}_j \frac{\partial Q_j^C(\overline{y}^C, \overline{g}_j, \overline{g}_i)}{\partial \overline{g}_i}}{Q_j^C(\overline{y}^C, \overline{g}_j, \overline{g}_i)}}{1-\delta}, && (73) \\
&= \frac{\ln Q_i^C(\overline{y}^C, \overline{g}_i, \overline{g}_j) + \frac{\overline{g}_i \frac{\overline{g}_j(1-\alpha\delta)\overline{y}^C}{\left(\overline{g}_i+\overline{g}_j\right)^2}}{\frac{\overline{g}_i(1-\alpha\delta)\overline{y}^C}{\overline{g}_i+\overline{g}_j}} + \frac{-\overline{g}_j \frac{\overline{g}_j(1-\alpha\delta)\overline{y}^C}{\left(\overline{g}_i+\overline{g}_j\right)^2}}{\frac{\overline{g}_j(1-\alpha\delta)\overline{y}^C}{\overline{g}_i+\overline{g}_j}}}{1-\delta}, && (74) \\
&= \ln Q_i^C(\overline{y}^C, \overline{g}_i, \overline{g}_j) > 0 && (75)
\end{aligned}$$

since $Q_i^C(\overline{y}^C, \overline{g}_i, \overline{g}_j) = \frac{\overline{g}_i(1-\alpha\delta)\overline{y}^C}{\overline{g}_i+\overline{g}_j} > 1$. □

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