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Noisy Learning and Price Discrimination: Implications for Information Dissemination and Profits

Catherine Gendron-Saulnier

Marc Santugini

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Gendron-Saulnier: Analysis Group Montreal, Canada.
catherine.gendronsaulnier@analysisgroup.com.
Santugini: Department of Applied Economics and CIRPÉE, HEC Montréal
marc.santugini@hec.ca

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Abstract:
We study third-degree price discrimination in the presence of uninformed buyers who extract noisy information from observing prices. In a noisy learning environment, price discrimination can be detrimental to the firm and beneficial to the consumers. On the one hand, discriminatory pricing reduces consumers' uncertainty, i.e., the variance of posterior beliefs upon observing prices is reduced. Specifically, observing two prices under discriminatory pricing provides more information than one price under uniform pricing even when discriminatory pricing reduces the amount of information contained in each price. On the other hand, it is not always optimal for the firm to use discriminatory pricing since the presence of uninformed buyers provides the firm with the incentive to engage in noisy price signaling. Indeed, if the benefit from price flexibility (through discriminatory pricing) is offset by the cost signaling quality through two distinct prices, then it is optimal to integrate markets and thus to use uniform pricing.

Keywords: Discriminatory pricing, Market segmentation, Monopoly, Quality of information, Learning, Uniform pricing, Third-degree price discrimination.

JEL Classification: D82, D83, L12, L15.

1 Introduction

As long as there is some easily observable characteristic (e.g., age, income, or geographic location) by which a firm can group buyers and arbitrage can be prevented, it is possible for the firm to segment markets and engage in (third-degree) price discrimination. An important question is whether market segmentation is beneficial for society. The welfare analysis on market segmentation has generally been undertaken under the assumption of complete information on the part of consumers.[1] In this case, the welfare effects are ambiguous. One has to weight the losses of consumers in low-elasticity markets against the gains of those in high-elasticity markets and of the firm. Moreover, one has to consider that discriminatory pricing may lead to the opening of new markets.

In the case of incomplete information, little is known about the effect of market segmentation on welfare and especially on consumers' well-being. This is relevant since the differences among the segmented groups might concern not only tastes, but also information regarding the quality of the good. For instance, with the spread of online commerce, it becomes easier for a firm to introduce a product in a new market. Consumers in the new market might have tastes for the product that differ from consumers' tastes in the original market and they might have less information because of the novelty of the brand. Another example is the case of a prescription drug readily available in the US which is introduced in a developing country. In addition of being able to pay less for the drug, consumers in the developing country might be less informed about the effectiveness of the prescription drug.[2]

The introduction of asymmetric information among buyers leads naturally to the issue of the informative role of prices. Indeed, prices have been

[1]See Armstrong (2006) for a survey on price discrimination. See Schmalensee (1981) and Tirole (1988) for a detailed discussion on third-degree price discrimination.

[2]The implementation of drug information centers is a primary concern in many developing countries (Flores Vidotti, 2004). Proper sources of information on drugs are not easily accessible in developing countries. There are several reasons for the absence of information: inadequate translation in local languages, prohibitive cost to acquire information, and even customers' unawareness on how to obtain information.

shown to be instrumental in disseminating information to market participants (Grossman, 1989).[3] One of the purpose of this paper is to study the effect of market segmentation on the informational content of prices. Specifically, does discriminatory pricing provide less or more information to the uninformed buyers? If it were not for the endogeneity of prices, it could be argued that an increase in the number of price-signals due to market segmentation yields more information to the uninformed buyers (i.e., more precise posterior beliefs). However, since the firm sets prices, the distribution of the price-signals (i.e., the informational content) does depend on whether the firm uses discriminatory pricing. There is thus a trade-off. Price discrimination generates more price-signals, but each of these signals might be less precise.

To study the effect of discriminatory pricing on the dissemination of information via market prices, we consider the simplest model of third-degree price discrimination of a monopoly selling a homogeneous good to two separate markets. In one of the markets, some buyers do not know the quality of the good. Yet, the presence of informed buyers makes it possible for prices to disseminate information. Under noisy demand, we show that market segmentation alters the informational content of price-signals received by the uninformed buyers. Specifically, discriminatory pricing have informational benefits over uniform pricing, i.e., the posterior beliefs of the uninformed buyers have a smaller bias and a lower variance.

The introduction of incomplete information also raises questions on the profitability of third-degree price discrimination from the firm's perspective. It is the second purpose of this paper to address this issue. In an environment of complete information, it is always profitable for a monopoly to segment markets with different demands and to engage in third-degree price discrimination. The reason is that setting different prices – a lower price

[3]Several studies have provided conditions under which privately-held information by firms becomes public through prices, beginning with perfectly competitive markets (Kihlstrom and Mirman, 1975; Grossman, 1976, 1978; Grossman and Stiglitz, 1980) and continuing with imperfectly competitive markets (Wolinsky, 1983; Riordan, 1986; Bagwell and Riordan, 1991; Judd and Riordan, 1994; Daughety and Reinganum, 1995, 2005, 2007, 2008a,b; Janssen and Roy, 2010; Daher et al., 2012).

in the market segments with greater price elasticity and a higher price in those with lower price elasticity – allows the firm to capture more of the consumer surplus in each market. However, it is not known whether market segmentation is systematically profitable under incomplete information.

In this paper, by endogenizing the firm's decision to segment or integrate the market, we show that, when confronted with uninformed buyers, market segmentation is not necessarily the more profitable pricing strategy. This is because the firm faces a trade-off when choosing to segment or integrate the markets. On the one hand, market segmentation yields more flexibility and the ability to capture a greater share of the consumer surplus. On the other hand, market segmentation implies that the firm signals quality with two prices instead of one. Hence, two prices are distorted from their complete information counterpart, whereas only one price is when markets are integrated. If the signaling cost (due to the distortion in the prices) is higher under market segmentation than under market integration, then it is possible that the loss due to signaling outweighs the benefit from price flexibility. We find that the higher the number of informed buyers, the more similar the market segments have to be for market integration to be the more profitable option. We also find that it is more likely that market integration be optimal when uninformed buyers are numerous and originate from the market segment with the higher willingness to pay.

The remainder of the paper is organized as follows. Section 2 surveys the literature. Section 3 presents the informational benefit of discriminatory pricing for the uninformed buyers. Section 4 studies the profitability of discriminatory pricing. Finally, Section 5 concludes.

2 Literature

We contribute to a large literature on third degree price discrimination, starting with the classic work of Pigou (1920). The reminiscent question with market segmentation is related to the conditions under which price discrimination raises welfare. For instance, Schmalensee (1981) and Varian (1985) identify when an increase in output is necessary for an increase in welfare, whereas Nahata et al. (1990), instead on focusing on output, concentrate on the price effects of discrimination. By contrast, Aguirre et al. (2010) and Cowan (2013) identify sufficient conditions for price discrimination to increase welfare. More recently, Bergemann et al. (2015) show that there is always a way to segment the market such that the combination of consumer and producer surplus satisfy the following: (i) consumer surplus is nonnegative, (ii) producer surplus is at least as high as profits under the uniform monopoly price, and (iii) total surplus does not exceed the efficient surplus.

We depart from this literature in two ways. First, by introducing price discrimination in an environment where some buyers have incomplete information. Second, by concentrating on the signaling aspect of prices instead of on the welfare effects of price discrimination.

We are the first, to our knowledge, to analyze the issue of market segmentation in a stochastic environment with learning. There is, however, a small but growing literature on learning in a stochastic setting. Matthews and Mirman (1983) study a limit pricing environment. Judd and Riordan (1994) and Mirman et al. (2014b) study noisy learning in the monopoly case whereas Mirman et al. (2014a, 2015) study the informational role of prices in competitive markets. Gordon and Nöldeke (2013) study how the communication strategy in a sender-receiver game affects the amount of information that is transmitted given that communication is inherently noisy. It is interesting to note that the use of noisy signaling models is growing, maybe partly because recent experimental work suggests that the stochastic environment in signaling maps better into the behavior of experimental subjects (de Haan et al., 2011; Jeitschko and Norman, 2012).

In this paper, we contribute to the literature on noisy signaling by studying the informational role of prices in the presence of market segmentation. The noisy environment enables us to study thoroughly the effect that market segmentation has on the informational content of prices. In a noiseless environment, the firm reacts to the informational externality, but has limited control over the flow of information. In other words, in equilibrium, either the unknown quality is not revealed and learning buyers revert to their prior beliefs, or it is fully revealed. Hence, under noiseless demand, whether the firm uses discriminatory pricing has no particularly meaningful effect on the posterior beliefs.

The second issue we tackle in this paper relates to the profitability of market segmentation for the firm. Whether market integration is optimal is closely related to the question of whether uniform pricing for differentiated goods is optimal. In both problems, the benefits of the increased price flexibility need to be compared to the costs of charging different prices. Some recent papers (McMillan, 2007; Orbach and Einav, 2007; Chen, 2009; Chen and Cui, 2013; Richardson and Stähler, 2013) study such question in the context of differentiated goods. The present paper contributes partially to this strand of the literature by identifying a cost to charging different prices in a signaling context. Hence, we provide a glimpse to what incomplete information can yield when goods are differentiated.

At last, our work is related to several papers in international economics investigating the non-optimality of charging different prices for different markets (Friberg, 2001; Asplund and Friberg, 2000). However, these papers do not study the optimality of market segmentation (or integration) in a noisy signaling environment. Friberg (2001) studies whether a firm selling in regions with different currencies should segment the markets with an emphasis on the impact of the exchange rate, whereas Asplund and Friberg (2000) focus on the transportation cost from one region to another.[4] We do not explore these issues here, but rather provide an information-based reason for the profitability of market integration.

3 Information Dissemination

In this section, we study the effect of discriminatory pricing on information dissemination, i.e., how much buyers learn about quality from observing price(s). To that end, we present a model in which a firm sells a good to two segmented markets. We first solve for the equilibrium under discriminatory pricing as well as the benchmark equilibrium of uniform pricing. We then

[4] Other papers such as Friberg (2003), Friberg and Martensen (2001) and Gallo (2010) study the profitability of market segmentation in the context of a duopoly.

study the effect of discriminatory pricing on the dissemination of information.[5]

3.1 Set Up

Consider a firm selling a good of quality $\mu > 0$ in two markets: market A and market B. The firm chooses at which prices she sells the good in each market. That is she chooses P_A and P_B. We assume arbitrage can be prevented such that it is possible to segment the two markets using third-degree price discrimination. In this section, we assume that the decision to segment the markets or not is exogenous. In other words, we abstract from the question of whether market segmentation is profitable.

In market A, all buyers are informed, i.e., they know μ. Aggregate demand in market A is given by

$$Q_A(P_A, \mu, \eta_A) = \mu - P_A + \eta_A \tag{1}$$

where η_A is a demand shock that is unobserved by the buyers. The difference in demand between markets A and B is two-fold. A first difference concerns information. Unlike market A, market B is composed of both informed and uninformed buyers. Specifically, a fraction $\lambda \in [0, 1]$ of the buyers knows μ and thus a fraction $1 - \lambda$ does not know μ. Although the uninformed buyers have prior beliefs about μ, they also extract partial information about quality from observing prices, i.e., prices are noisy signals. That is, upon observing prices, the uninformed buyers' posterior mean for quality is $\int x\hat{\xi}(x|P_A, P_B)\mathrm{d}x$ where $\hat{\xi}(\cdot|P_A, P_B)$ is the posterior p.d.f. of $\tilde{\mu}$ given P_A and P_B.[6] A second difference is that conditional on μ, the buyers in market B have a reservation price $\gamma\mu$ where $\gamma > 0$ reflects the disparity in demand between the two

[5] In this section, market segmentation is set exogenously. The profitability of market segmentation on the firm is discussed in Section 4 when market segmentation is endogenized, i.e., the firm decides whether to segment or to integrate the markets.

[6] Note that $\hat{\xi}(\cdot|P_A, P_B)$ is the general expression for posterior beliefs upon observing two signals. If there is no market segmentation, then the uninformed buyers receive two *identical* signals, i.e., $P \equiv P_A = P_B$. In that case, posterior beliefs can be simplified to $\hat{\xi}(\cdot|P)$.

markets (unless $\gamma = 1$).[7] Aggregate demand in market B is thus given by

$$Q_B(P_B, \mu, \hat{\xi}(\cdot|P_A, P_B), \eta_B) = \lambda(\gamma\mu - P_B) + (1-\lambda)\left(\gamma \int x\hat{\xi}(x|P_A, P_B)\mathrm{d}x - P_B\right) + \eta_B \tag{2}$$

where η_B is a demand shock that is unobserved by the buyers and $\int x\hat{\xi}(x|P_A, P_B)\mathrm{d}x$ is the posterior mean of μ. The updating of beliefs reflects the learning activity of the uninformed buyers.

Next, we describe the firm's maximization problem. For simplicity, the firm's marginal cost is assumed to be zero. In addition of knowing the quality μ, the firm has complete information about demand, i.e., both η_A and η_B are known to the firm. This informational asymmetry between the buyers and the firm conveys the idea that the firm knows more about demand than the buyers do.

We consider two cases. First, under discriminatory pricing, using (1) and (2), the firm's maximization problem is

$$\max_{P_A, P_B} \left\{ P_A \cdot Q_A(P_A, \mu, \eta_A) + P_B \cdot Q_B(P_B, \mu, \hat{\xi}(\cdot|P_A, P_B), \eta_B) \right\}. \tag{3}$$

Second, under uniform pricing, the firm sets one price, i.e., $P \equiv P_A = P_B$, and the uninformed buyers receive only one signal. Using (1) and (2), the firm's maximization problem is

$$\max_{P} \left\{ P \cdot \left(Q_A(P, \mu, \eta_A) + Q_B(P, \mu, \hat{\xi}(\cdot|P), \eta_B) \right) \right\}. \tag{4}$$

Before proceeding with the definition and characterization of the equilibrium, we discuss the distributional assumption for prior beliefs and the random demand shocks.

Assumption 3.1. *Prior beliefs are $\tilde{\mu} \sim N(\rho, \sigma_\mu^2)$, with $\rho > 0$. Distributions of demand shocks are $\tilde{\eta}_A \sim N(0, \sigma_\eta^2), \tilde{\eta}_B \sim N(0, \sigma_\eta^2)$ such that $\mathbb{E}[\tilde{\eta}_A \tilde{\eta}_B] = 0$.*

[7] The assumption that market A has only informed buyers is without loss of generality. All results continue to hold if we assume a fraction λ_A of buyers is uninformed in market A and a fraction λ_B in market B.

The demand shocks are known to the firm, but unobserved by the buyers, which implies that the prices cannot fully reveal quality since they also depend on unobserved demand shocks. We rely on the fact that the family of normal distributions with an unknown mean is a conjugate family for samples from a normal distribution.[8] With the normality assumption, we obtain a unique linear equilibrium, i.e., an equilibrium in which the uninformed buyers' updating rule is linear in the price-signals. Although negative demand shocks can yield a negative price or a negative posterior mean, the values of the parameters of the model can be restricted to ensure that the probability of such events be arbitrarily close to zero. Moreover, it turns out that, for any parameters, equilibrium values for mean prices are always positive.

3.2 Equilibrium

First, consider the situation in which the firm uses discriminatory pricing ($\mathcal{D}$). The equilibrium consists of the firm's price strategies, $P^*_{\mathcal{D},A}(\mu, \eta_A, \eta_B)$ and $P^*_{\mathcal{D},B}(\mu, \eta_B, \eta_A)$; the distribution of the price-signals conditional on any quality x, $\phi^*_{\mathcal{D}}(P_A, P_B|x)$; and the uninformed buyers' posterior beliefs about the quality upon observing any prices $\{P_A, P_B\}$, $\hat{\xi}^*_{\mathcal{D}}(x|P_A, P_B)$.[9] In equilibrium, the posterior beliefs are consistent with Bayes' rule and the equilibrium distribution of prices.

Definition 3.2. *For any $\mu > 0$, the tuple $\left\{P^*_{\mathcal{D},A}(\mu, \eta_A, \eta_B), P^*_{\mathcal{D},B}(\mu, \eta_B, \eta_A), \phi^*_{\mathcal{D}}(P_A, P_B|\cdot), \hat{\xi}^*_{\mathcal{D}}(\cdot|P_A, P_B)\right\}$ is a noisy signaling equilibrium with discriminatory pricing if,*

[8]Normal assumption combined with linear demand yields closed-form equilibrium values and makes the analysis tractable by focusing on the mean and variance of price and posterior beliefs. See Grossman and Stiglitz (1980), Kyle (1985), Judd and Riordan (1994), Mirman et al. (2014a,b, 2015) for the use of normal distributions to study the informational role of prices in problems without market segmentation.

[9]The variable μ refers to the true quality whereas x is used as a dummy variable for quality.

1. *Given* $\hat{\xi}^*_{\mathcal{D}}(\cdot|P_A, P_B)$*, and for any* η_A *and* η_B*, the firm's price strategies are*

$$\begin{aligned}\left\{P^*_{\mathcal{D},A}(\mu, \eta_A, \eta_B), P^*_{\mathcal{D},B}(\mu, \eta_B, \eta_A)\right\} = \arg\max_{P_A, P_B} \Big\{ & P_A \cdot Q_A(P_A, \mu, \eta_A) \\ & + P_B \cdot Q_B(P_B, \mu, \hat{\xi}^*_{\mathcal{D}}(\cdot|P_A, P_B), \eta_B)\Big\}.\end{aligned} \tag{5}$$

2. *Given the distribution of* $\{\tilde{\eta}_A, \tilde{\eta}_B\}$*,* $\phi^*_{\mathcal{D}}(P_A, P_B|x)$ *is the p.d.f. of the random price-signals* $\left\{P^*_{\mathcal{D},A}(x, \tilde{\eta}_A, \tilde{\eta}_B), P^*_{\mathcal{D},B}(x, \tilde{\eta}_B, \tilde{\eta}_A)\right\}$ *conditional on any quality* x*.*

3. *Given* $\phi^*_{\mathcal{D}}(P_A, P_B|\cdot)$ *and prior beliefs* $\xi(\cdot)$*, the uninformed buyers' posterior beliefs about quality upon observing* P_A *and* P_B *is* $\tilde{\mu}^*_{\mathcal{D}}|P_A, P_B$ *with the p.d.f.*

$$\hat{\xi}^*_{\mathcal{D}}(x|P_A, P_B) = \frac{\xi(x)\phi^*_{\mathcal{D}}(P_A, P_B|x)}{\int_{x' \in \mathbb{R}} \xi(x')\phi^*_{\mathcal{D}}(P_A, P_B|x')\mathrm{d}x'}, \quad \forall\ x \in \mathbb{R}. \tag{6}$$

Using Definition 3.2, Proposition 3.3 characterizes the noisy signaling equilibrium when the firm engages in third-degree price discrimination. Specifically, the price strategies and the posterior beliefs (as a function of the price-signals) are provided. The joint distribution of the price-signals is immediate from Assumption 3.1, (7), and (8).

Proposition 3.3. *Suppose that markets are segmented. For $\lambda \in [0, 1)$, there exists a noisy signaling equilibrium with discriminatory pricing.*[10] *In equilibrium,*

1. *Given quality μ and demand shocks $\{\eta_A, \eta_B\}$, the firm sets prices*

$$P^*_{\mathcal{D},A}(\mu, \eta_A, \eta_B) = \frac{\delta^*_0\delta^*_1\gamma^2(1-\lambda)^2 + (2 - 2\delta^*_2\gamma(1-\lambda) + \delta^*_1\gamma^2\lambda(1-\lambda))\mu}{4 - \delta^{*2}_1\gamma^2(1-\lambda)^2 - 4\delta^*_2\gamma(1-\lambda)} + \frac{(2 - 2\delta^*_2\gamma(1-\lambda))\eta_A + \delta^*_1\gamma(1-\lambda)\eta_B}{4 - \delta^{*2}_1\gamma^2(1-\lambda)^2 - 4\delta^*_2\gamma(1-\lambda)} \tag{7}$$

and

$$P^*_{\mathcal{D},B}(\mu, \eta_B, \eta_A) = \frac{2\delta^*_0\gamma(1-\lambda) + (\delta^*_1\gamma(1-\lambda) + 2\gamma\lambda)\mu + \delta^*_1\gamma(1-\lambda)\eta_A + 2\eta_B}{4 - \delta^{*2}_1\gamma^2(1-\lambda)^2 - 4\delta^*_2\gamma(1-\lambda)}. \tag{8}$$

2. *Given any observation $\{P_A, P_B\}$, the uninformed buyers' posterior beliefs are*

$$\tilde{\mu}^*_{\mathcal{D}}|P_A, P_B \sim N\left(\delta^*_0 + \delta^*_1 P_A + \delta^*_2 P_B, \frac{\sigma^2_\eta\sigma^2_\mu}{\sigma^2_\eta + (1+\gamma^2\lambda^2)\sigma^2_\mu}\right). \tag{9}$$

Here,

$$\delta^*_0 = \frac{\rho\sigma^2_\eta}{\sigma^2_\eta + \sigma^2_\mu(1+\gamma^2\lambda)}, \tag{10}$$

$$\delta^*_1 = \frac{2\sigma^2_\mu}{\sigma^2_\eta + \sigma^2_\mu(1+\gamma^2\lambda)}, \tag{11}$$

$$\delta^*_2 = \frac{2\gamma(\lambda\sigma^2_\mu(\sigma^2_\eta + 2\sigma^2_\mu) - \sigma^4_\mu(1-\gamma^2\lambda^2))}{(\sigma^2_\eta + \sigma^2_\mu(1+\gamma^2\lambda))(\sigma^2_\eta + \sigma^2_\mu(1+\gamma^2\lambda(2-\lambda)))}. \tag{12}$$

Proof. See Appendix A. □

[10] When $\lambda = 1$, all buyers are informed and no updating rule needs to be specify. In this case, there exists an equilibrium with equilibrium prices $P^*_{\mathcal{D},A}(\mu, \eta_A, \eta_B) = (\mu + \eta_A)/2$ and $P^*_{\mathcal{D},B}(\mu, \eta_B, \eta_A) = (\gamma\mu + \eta_B)/2$, which are equal to (7) and (8) evaluated at $\lambda = 1$, respectively.

Next, we define and characterize the noisy signaling equilibrium in the benchmark model in which the firm uses uniform pricing ($\mathcal{U}$).

Definition 3.4. *For any $\mu > 0$, the tuple $\left\{P^*_{\mathcal{U}}(\mu, \eta_A, \eta_B), \phi^*_{\mathcal{U}}(P|\cdot), \hat{\xi}^*_{\mathcal{U}}(\cdot|P)\right\}$ is a noisy signaling equilibrium with uniform pricing if,*

1. *Given $\hat{\xi}^*_{\mathcal{U}}(\cdot|P)$, and for any η_A and η_B, the firm's price strategy is*

$$P^*_{\mathcal{U}}(\mu, \eta_A, \eta_B) = \arg\max_P \left\{P \cdot \Big(Q_A(P, \mu, \eta_A) + Q_B(P, \mu, \hat{\xi}^*_{\mathcal{U}}(\cdot|P), \eta_B)\Big)\right\}. \tag{13}$$

2. *Given the distribution of $\{\tilde{\eta}_A, \tilde{\eta}_B\}$, $\phi^*_{\mathcal{U}}(P|x)$ is the p.d.f. of the random price-signal $P^*_{\mathcal{U}}(x, \tilde{\eta}_A, \tilde{\eta}_B)$ conditional on any quality x.*

3. *Given $\phi^*_{\mathcal{U}}(P|\cdot)$ and prior beliefs $\xi(\cdot)$, the uninformed buyers' posterior beliefs upon observing any P is $\tilde{\mu}^*|P$ with p.d.f.*

$$\hat{\xi}^*_{\mathcal{U}}(x|P) = \frac{\xi(x)\phi^*_{\mathcal{U}}(P|x)}{\int_{x' \in \mathbb{R}} \xi(x')\phi^*_{\mathcal{U}}(P|x')\mathrm{d}x'}, \quad \forall\ x \in \mathbb{R}. \tag{14}$$

Proposition 3.5 characterizes the noisy signaling equilibrium when the firm does not price discriminate. The distribution of the price-signal is immediate from Assumption 3.1 and (15).

Proposition 3.5. *Suppose that markets are not segmented. For $\lambda \in [0, 1)$, there exists a noisy signaling equilibrium with uniform pricing.*[11] *In equilibrium,*

1. *Given quality μ and demand shocks $\{\eta_A, \eta_B\}$, the firm sets the price*

$$P^*_{\mathcal{U}}(\mu, \eta_A, \eta_B) = \frac{\beta^*_0\gamma(1-\lambda) + (1+\gamma\lambda)\mu + \eta_A + \eta_B}{4 - 2\beta^*_1\gamma(1-\lambda)} \tag{15}$$

   in markets A and B.

[11] When $\lambda = 1$, all buyers are informed and no updating rule needs to be specify. In this case, there exists an equilibrium with $P^*_{\mathcal{U}}(\mu, \eta_A, \eta_B) = ((1+\gamma)\mu + \eta_A + \eta_B)/4$ which is equal to (15) evaluated at $\lambda = 1$.

2. Given any observation P, the uninformed buyers' posterior beliefs are

$$\tilde{\mu}^*_{\mathcal{U}}|P \sim N\left(\beta_0^* + \beta_1^* P, \frac{2\sigma_\eta^2\sigma_\mu^2}{2\sigma_\eta^2 + (1+\lambda\gamma)^2\sigma_\mu^2}\right). \tag{16}$$

Here,

$$\beta_0^* = \frac{2\rho\sigma_\eta^2}{2\sigma_\eta^2 + \sigma_\mu^2(1+\gamma+\gamma\lambda+\gamma^2\lambda)}, \tag{17}$$

$$\beta_1^* = \frac{4(1+\gamma\lambda)\sigma_\mu^2}{2\sigma_\eta^2 + \sigma_\mu^2(1+2\gamma+2\gamma^2\lambda-\gamma^2\lambda^2)}. \tag{18}$$

Proof. See Appendix A. □

From (7), (8), and (15), equilibrium prices are linear functions of the quality μ as well as demand shocks η_A and η_B. Although prices are informative about quality, the presence of unknown demand shocks prevents the buyers from learning the exact value of quality, i.e., price is partially revealing of quality. Hence, noise in demand allows us to study the impact of pricing strategies (discriminatory vs. uniform) on the dissemination of information.[12]

As a first step, one can notice that the amount of information conveyed by the price(s) depends on the pricing strategy adopted by the firm.

Remark 3.6. *The pricing strategy chosen by the firm alters the amount of information conveyed by the price-signal. More specifically, for $s \in \{A, B\}$*

1. $\mathbb{V}ar[P^*_{\mathcal{U}}(\tilde{\mu}, \eta_A, \eta_B)|\, \eta_A, \eta_B] \geq \mathbb{V}ar[P^*_{\mathcal{D},s}(\tilde{\mu}, \eta_A, \eta_B)|\, \eta_A, \eta_B]$,
2. $\mathbb{V}ar[P^*_{\mathcal{U}}(\mu, \tilde{\eta}_A, \tilde{\eta}_B)|\, \mu] < \mathbb{V}ar[P^*_{\mathcal{D},s}(\mu, \tilde{\eta}_A, \tilde{\eta}_B)|\, \mu]$.

Remark 3.6 is important since it means that the pieces of information from which the buyers learn can have a lower quality when the firm uses discriminatory pricing. Indeed, on the one hand, observation 1. establishes that, conditional on demand shocks η_A and η_B, the price-signal under uniform

[12] In our model, if demand shocks are known to buyers, then quality is perfectly inferred from observing the price(s) under both uniform and discriminatory pricing.

pricing is more sensitive to quality μ than the price-signal in segment $s \in \{A, B\}$ under discriminatory pricing. It is thus easier to distinguish between different qualities under uniform pricing. On the other hand, observation 2. establishes that, conditional on quality μ, the price-signal under uniform pricing is less sensitive to the demand shocks (η_A, η_B) than the price-signal in segment s under discriminatory pricing. In other words, there is less noise in the price-signal under uniform pricing. Taken together, these observations imply that the price-signal with uniform pricing is a better signal than the price-signal from market segment s under discriminatory pricing.

Yet, one cannot already conclude that uniform pricing is better for the dissemination of information to uninformed buyers. The reason is that when the firm segments the market, uninformed buyers obtain two price-signals instead of one. Hence, there is a trade-off: the uniform price-signal is better, but there are two signals when markets are segmented. In the next section, we show that, in spite of this trade-off, discriminatory pricing induces more learning.

3.3 Comparison of Pricing Strategies

We now compare discriminatory and uniform pricing for the dissemination of information. We consider two aspects to measure the amount of information conveyed by prices: the bias of the posterior mean and the posterior variance.

3.3.1 Bias of the Posterior Mean

The bias of the posterior mean measures the distance between the true quality μ and the uninformed buyers' mean posterior evaluation of this quality. For instance, consider that discriminatory pricing is used by the firm and that μ is the true quality. A posteriori, upon seeing P_A and P_B, uninformed buyers believe that quality has a mean $\mathbb{E}[\tilde{\mu}^*_{\mathcal{D}}|P_A, P_B]$. Then, on average, the posterior mean is $\int_{P_A,P_B} \mathbb{E}[\tilde{\mu}^*_{\mathcal{D}}|P_A, P_B]\phi^*_{\mathcal{D}}(P_A, P_B|\mu)\mathrm{d}P_A\mathrm{d}P_B$ and the expected bias denoted by $\mathcal{B}^*_{\mathcal{D}}$ is the average distance between the true quality μ and the uninformed buyers' mean evaluation of this quality.

Using Proposition 3.3, the expected bias under discriminatory pricing is the absolute value of the difference between the unconditional posterior mean for quality and the true quality μ, i.e.,

$$\mathcal{B}^*_{\mathcal{D}} \equiv \Big| \int_{P_A,P_B} \mathbb{E}[\tilde{\mu}^*_{\mathcal{D}}|P_A,P_B]\phi^*_{\mathcal{D}}(P_A,P_B|\mu)\mathrm{d}P_A\mathrm{d}P_B - \mu \Big|. \tag{19}$$

Consider next that uniform pricing is used by the firm. Then, the expected bias under uniform pricing follows similarly, from Proposition 3.5, and is defined as

$$\mathcal{B}^*_{\mathcal{U}} \equiv \Big| \int_{P} \mathbb{E}[\tilde{\mu}^*_{\mathcal{U}}|P]\phi^*_{\mathcal{U}}(P|\mu)\mathrm{d}P - \mu \Big|. \tag{20}$$

Proposition 3.7 establishes that the expected bias of the posterior mean is always smaller under discriminatory pricing than under uniform pricing. In other words, with third-degree price discrimination, the buyers are, on average, closer to the truth.[13]

Proposition 3.7. *From* (19) *and* (20),$|\mathcal{B}^*_{\mathcal{D}}| \leq |\mathcal{B}^*_{\mathcal{U}}|$.

Proof. From (7), (8), and (9),

$$\int_{P_A,P_B} \mathbb{E}[\tilde{\mu}^*_{\mathcal{D}}|P_A,P_B]\phi^*_{\mathcal{D}}(P_A,P_B|\mu)\mathrm{d}P_A\mathrm{d}P_B = \frac{\rho\sigma^2_\eta + \mu(1+\gamma^2\lambda^2)\sigma^2_\mu}{\sigma^2_\eta + (1+\gamma^2\lambda^2)\sigma^2_\mu}. \tag{21}$$

From (15) and (16),

$$\int_{P} \mathbb{E}[\tilde{\mu}^*_{\mathcal{U}}|P]\phi^*_{\mathcal{U}}(P|\mu)\mathrm{d}P = \frac{2\rho\sigma^2_\eta + \mu(1+\gamma\lambda)^2\sigma^2_\mu}{2\sigma^2_\eta + (1+\gamma\lambda)^2\sigma^2_\mu}. \tag{22}$$

Plugging (21) into (19) and (22) into (20) yields the results. □

From Proposition 3.7, observing two signals instead of one reduces the expected bias of the posterior beliefs. However, for specific realization of demand shocks $\{\eta_A, \eta_B\}$, posterior beliefs might closer to the true quality with

[13] In the particular case of $\rho = \mu$, the posterior mean of quality is on average unbiased regardless of the pricing regime, i.e., $|\mathcal{B}^*_{\mathcal{D}}| = |\mathcal{B}^*_{\mathcal{U}}| = 0$.

uniform pricing than with discriminatory pricing. Thus, in order to compare the quality of information dissemination under the two pricing regimes, we need to discuss the effect of price discrimination on the variance of the posterior beliefs.

3.3.2 Posterior Variance

Upon observing the price-signal(s), uninformed buyers believe that quality is normally distributed with a mean and a variance as given in (9) or in (16). We now study the effect of the firm's pricing strategy on this posterior variance.[14]

The size of the posterior variance is related to the uninformed buyers' learning speed. More specifically, the smaller the posterior variance is, the faster is the learning speed and the more information is conveyed to uninformed buyers. The reason is that the posterior variance tells us how likely it is that values far from the posterior mean be the actual true quality. For instance, if one were to construct a 95% confidence interval for the value of the true quality, the size of this interval would be determined by the posterior variance.

Proposition 3.8 states that the posterior variance for quality is always greatest under uniform pricing. Hence, price discrimination provides more information to the uninformed buyers, i.e., the posterior beliefs for quality are less variable. Equation (23) characterizes the variance differential. Note that the presence of both demand uncertainty and prior uncertainty are necessary for market segmentation to have an effect on the informational content of prices. If there is no prior uncertainty (i.e., $\sigma_\mu^2 \to 0$), then there is no reason to learn from prices. Moreover, if there is no unknown demand shock (i.e., $\sigma_\eta^2 \to 0$), then observing more prices does not provide more information to the uninformed buyers since the uninformed buyers can infer exactly the true quality regardless of the pricing strategy.

[14]Note that the variance of the posterior beliefs, the object studied in this section, is different from the posterior beliefs' variance where the posterior beliefs is taken as a random variable.

Proposition 3.8. *For* $\lambda \in [0,1)$*, from* (9) *and* (16)*,*

$$\mathbb{V}[\tilde{\mu}^*_{\mathcal{U}}] - \mathbb{V}[\tilde{\mu}^*_{\mathcal{D}}] = \frac{(1-\gamma\lambda)^2\sigma_\eta^2\sigma_\mu^4}{(\sigma_\eta^2 + (1+\gamma^2\lambda^2)\sigma_\mu^2)(2\sigma_\eta^2 + (1+\gamma\lambda)^2\sigma_\mu^2)} \geq 0. \tag{23}$$

Before discussing Proposition 3.8, it is worth considering three special cases of (23). Consider first the benchmark case of full information with two identical markets, i.e., $\gamma = \lambda = 1$. Hence, for an uninformed *outsider*, market segmentation yields no gain in precision of the posterior beliefs. Next, consider two special cases for which discriminatory prices provide more precise posterior beliefs. If $\lambda = 1$ and $\gamma \in [0,1)$, then discriminatory prices provide better information to an uninformed *outsider*. The fact that two signals about two fully informed markets be available makes the posterior beliefs more precise. In other words, the market price is more informative to outsiders.[15] Finally, if $\gamma = 1$ and $\lambda \in (0,1)$, then preferences over the good are the same between the two markets, but some buyers in market B are uninformed. In the presence of uninformed buyers, segmenting a market into two identical markets provides more precise information to the uninformed buyers.[16]

We now discuss Proposition 3.8. This discussion builds in part on Remark 3.6 outlining the fact that the variance of the price-signals changes with the pricing regime. Yet, in spite of changes in the variances of the price-signals, the gain in precision due to discriminatory prices always holds. On the one hand, discriminatory pricing (compared to uniform pricing) implies that the buyers receive two signals instead of one. Hence, holding everything else constant, price discrimination provides more signals and increases the precision of the posterior beliefs. On the other hand, since the firm sets prices, the variance of the price-signals are endogenous. In particular, it is possible for the variance of the price-signals to increase as a consequence of market segmentation. Hence, there is a trade-off. Price discrimination offers

[15]Given $\lambda = 1$, from (9) and (16) we have $\mathbb{V}[\tilde{\mu}^*] = \sigma_\eta^2\sigma_\mu^2/(\sigma_\eta^2 + (1+\gamma^2)\sigma_\mu^2)$ and $\mathbb{V}[\tilde{\mu}^*] = 2\sigma_\eta^2\sigma_\mu^2/(2\sigma_\eta^2 + (1+\gamma)^2\sigma_\mu^2)$ respectively.

[16]Given $\gamma = 1$ and $\lambda \in (0,1)$, from (9) and (16), we have $\mathbb{V}[\tilde{\mu}^*] = \sigma_\eta^2\sigma_\mu^2/(\sigma_\eta^2 + (1+\lambda^2)\sigma_\mu^2)$ and $\mathbb{V}[\tilde{\mu}^*] = 2\sigma_\eta^2\sigma_\mu^2/(2\sigma_\eta^2 + (1+\lambda)^2\sigma_\mu^2)$, respectively.

more signals but each of these signals might be less precise. Although there is a trade-off, it turns out that third-degree price discrimination always reduces the variance of the posterior mean for quality even when each price-signal becomes less precise.

We finish this section with a comparative analysis on (23). Specifically, we show how the parameters of the model mitigate or reinforce the positive effect of discriminatory pricing on the variance of the posterior beliefs. Remark 3.9 presents the effect of noise on the variance differential.

Remark 3.9. *From* (23)*, for* $\lambda \in [0, 1)$,

$$\frac{\partial(\mathbb{V}[\tilde{\mu}^*_{\mathcal{U}}] - \mathbb{V}[\tilde{\mu}^*_{\mathcal{D}}])}{\partial \sigma_\mu} \geq 0. \tag{24}$$

An increase in the variance of the prior beliefs always increases the variance differential. Specifically, from (24), if the prior beliefs are very precise, then the differential in the posterior variance stemming from the observation of two price-signals instead of one is relatively small. Since the uninformed buyers are quite certain that the true quality lies within a constraint interval, the firm's signaling activity does not play a prominent role as the informational reaction to the price-signals is small, i.e., β^*_1 and $\{\delta^*_1, \delta^*_2\}$ are small. On the other hand, if the prior beliefs are very diffuse, the firm's signaling activity matters a lot and the differential in information that two price-signals convey instead of one is significantly more important.

Next, consider the effect of the proportion of informed buyers and the differential in demand on (23). From (23), notice that only the product $\lambda\gamma$ matters for the variance differential. Since $\gamma > 0$ and $\lambda \in [0, 1)$, two cases can occur, that is, $\lambda\gamma < 1$ and $\lambda\gamma \geq 1$. Remark 3.10 concentrates on the case $\lambda\gamma < 1$ and states that the larger is the proportion of informed buyers and the lesser is the differential in buyers' valuation, then the smaller is the variance differential coming from market segmentation.

Remark 3.10. *From* (23)*, for* $\lambda \in [0, 1)$ *and* $\gamma \in (0, 1]$,

$$\frac{\partial \mathbb{V}[\tilde{\mu}^*_{\mathcal{U}}] - \mathbb{V}[\tilde{\mu}^*_{\mathcal{D}}]}{\partial \lambda} \leq 0 \tag{25}$$

and

$$\frac{\partial \mathbb{V}[\tilde{\mu}^*_{\mathcal{U}}] - \mathbb{V}[\tilde{\mu}^*_{\mathcal{D}}]}{\partial \gamma} \leq 0. \tag{26}$$

From (25), as λ increases, under both uniform pricing and third-price discrimination, the posterior beliefs become less volatile as the price-signals incorporate more information from the mere fact that a larger proportion of buyers knows the true quality μ. However, the posterior variance decreases more rapidly when the uninformed buyers observe a single price-signal rather than two price-signals. This means that the benefit on the flow of information arising from a signal of a better quality is subject to some form of decreasing return. From (9) and (16), $\beta_1^* > \delta_1^* + \delta_2^*$. Hence, the uninformed buyers are always more sensitive to an improvement in the quality of a price-signal (due to more informed buyers) under uniform pricing than under discriminatory pricing. This higher sensibility translates into a higher decay rate of the posterior variance.

Next, we investigate the effect of the parameter γ on the variance differential in (23). From (26), an increase in γ reduces the benefit from observing two price-signals rather than one. Recall that γ is an indicator of how elastic is market B relative to market A, i.e., as $\gamma \to 1$, the two market segments are more similar to each other. Hence an increase in γ, by homogenizing the two markets, implies that P_A and P_B incorporate and convey a more similar content to the uninformed buyers.[17] Therefore, as $\gamma \to 1$, the uninformed buyers gain less from observing a second price-signal as it contains little supplementary informative content.[18]

[17]Using the criterion of mutual information $MI(\tilde{P}_A, \tilde{P}_B) = -log(1-\rho^2)/2$ where ρ is the correlation coefficient between $\tilde{P}_A$ and $\tilde{P}_B$, then we have $\partial MI(\tilde{P}_A, \tilde{P}_B)/\partial\gamma > 0$ such that the mutual information of $\tilde{P}_A$ and $\tilde{P}_B$ increases with γ.

[18]Comparative static in the case $\lambda\gamma \geq 1$ is more complicated. Overall, results of Remark 3.10 continue to hold if $\lambda\gamma$ is sufficiently high.

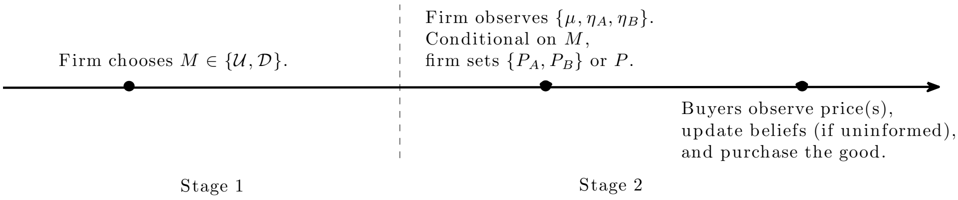


Figure 1: Timeline

4 On the Profitability of Discriminatory Pricing

We now extend the model by allowing the firm to either segment or integrate the two markets.[19] Our model has now two stages. In a nutshell, at the first stage, the firm decides whether or not to split the market into two separate markets. Then, at the second stage, the firm uses uniform pricing if there is market integration and discriminatory price if there is market segmentation. In either case, the firm takes into account the fact that prices can provide partial information about quality to the uninformed buyers.

4.1 Preliminaries

The demand side is unchanged, that is, demands in market A and B are given by (1) and (2), respectively. Except for the quality parameter μ and the demand shocks η_A and η_B, all the other parameters of the model (including the uninformed buyers' prior beliefs and the distribution of the demand shocks) are common knowledge.

The timing is as follows. At the first stage, the firm does not know the quality, nor the demand shocks and decides whether or not to segment the markets by comparing the expected profit under discriminatory pricing with the expected profit under uniform pricing, rationally anticipating quality, the

[19]Since we are studying third-degree price discrimination, arbitrage is assumed to be impossible.

demand shocks as well as the learning activity of the uninformed buyers.[20] Formally, let $M \in \{\mathcal{U}, \mathcal{D}\}$ be the firm's decision in the first stage. If $M = \mathcal{U}$, then the markets are integrated and pricing is uniform. If $M = \mathcal{D}$, then the markets are segmented leading to discriminatory pricing. At the second stage, after observing quality and the demand shocks, the firm sets the price(s). The uninformed buyers observe the segmentation decision, but do not know the quality nor the demand shocks.[21] Upon observing the price(s), the buyers update beliefs (if uninformed), and purchase the good. Figure 1 summarizes the timeline.

We now describe formally the behavior of the firm at each stage.[22] We begin with the second stage. If the markets are not segmented, then the firm sets one price. Using (1) and (2) evaluated at $P \equiv P_A = P_B$, stage-2 maximization problem (given $M = \mathcal{U}$) is the same as in (4). If the markets are segmented, then the firm sets a price in each market. Using (1) and (2), given that the firm has decided to segment the market at stage 1, stage-2 maximization problem (given $M = \mathcal{D}$) is the same as in (3). Note that from (3) and (4), the firm's expected profits are influenced by the uninformed buyers' posterior mean or equivalently by the p.d.f. $\hat{\xi}_{\mathcal{U}}(\cdot|P)$ and $\hat{\xi}_{\mathcal{D}}(\cdot|P_A, P_B)$.

At the first stage, the firm decides whether to use discriminatory or uniform pricing strategies. Formally, let the tuple $\{\{P_{\mathcal{U}}(\mu, \eta_A, \eta_B), \{P_{\mathcal{D},A}(\mu, \eta_A, \eta_B), P_{\mathcal{D},B}(\mu, \eta_B, \eta_A)\}\}, \{\hat{\xi}_{\mathcal{U}}(\cdot|P_A, P_B), \hat{\xi}_{\mathcal{D}}(\cdot|P)\}\}$ define a strategy at the second stage. Specifically, $P_{\mathcal{U}}(\mu, \eta_A, \eta_B)$ is the firm's price strategy under uniform pricing and $\{P_{\mathcal{D},A}(\mu, \eta_A, \eta_B), P_{\mathcal{D},B}(\mu, \eta_B, \eta_A)\}$ is the firm's price strategy under discriminatory. The terms $\hat{\xi}_{\mathcal{U}}(\cdot|P)$ and $\hat{\xi}_{\mathcal{D}}(\cdot|P_A, P_B)$ are the uninformed buyers' posterior beliefs under uniform pricing and discriminatory pricing, respec-

[20]This reflects the idea that the firm faces some uncertainty in demand before making a decision about market segmentation. If the firm knows μ before making the segmentation decision, then its choice, if not independent of μ, signals information to the uninformed buyers. In order to abstract for the increased complexity of having both prices and segmentation decision acting as signals, we assume that the firm learns μ only after her segmentation decision is made.

[21]The fact that the buyers do not observe the demand shocks conveys the idea that the firm knows more about demand than the buyers do. Moreover, this informational asymmetry enables prices to provide partial (noisy) information about the quality of the good.

[22]A definition of the perfect Bayesian equilibrium is provided in Appendix B.

tively. Given these strategies and posterior beliefs at stage-2, the expected profits of the firm under uniform pricing and discriminatory pricing are

$$\mathbb{E}[\Pi_{\mathcal{U}}(\tilde{\mu},\tilde{\eta}_A,\tilde{\eta}_B)] = \mathbb{E}\Big[P_{\mathcal{U}}(\tilde{\mu},\tilde{\eta}_A,\tilde{\eta}_B)\cdot\Big(Q_A(P_{\mathcal{U}}(\tilde{\mu},\tilde{\eta}_A,\tilde{\eta}_B),\tilde{\mu},\tilde{\eta}_A) + Q_B(P_{\mathcal{U}}(\tilde{\mu},\tilde{\eta}_A,\tilde{\eta}_B),\tilde{\mu},\hat{\xi}_{\mathcal{U}}(\cdot|P_{\mathcal{U}}(\tilde{\mu},\tilde{\eta}_A,\tilde{\eta}_B)),\tilde{\eta}_B)\Big)\Big] \quad (27)$$

and

$$\mathbb{E}[\Pi_{\mathcal{D}}(\tilde{\mu},\tilde{\eta}_A,\tilde{\eta}_B)] = \mathbb{E}[P_{\mathcal{D},A}(\tilde{\mu},\tilde{\eta}_A,\tilde{\eta}_B)\cdot Q_A(P_{\mathcal{D},A}(\tilde{\mu},\tilde{\eta}_A,\tilde{\eta}_B),\tilde{\mu},\tilde{\eta}_A)] + \mathbb{E}[P_{\mathcal{D},B}(\tilde{\mu},\tilde{\eta}_B,\tilde{\eta}_A) \cdot Q_B(P_{\mathcal{D},B}(\tilde{\mu},\tilde{\eta}_B,\tilde{\eta}_A),\tilde{\mu},\hat{\xi}_{\mathcal{D}}(\cdot|P_{\mathcal{D},A}(\tilde{\mu},\tilde{\eta}_A,\tilde{\eta}_B),P_{\mathcal{D},B}(\tilde{\mu},\tilde{\eta}_B,\tilde{\eta}_A)),\tilde{\eta}_B)], \quad (28)$$

respectively. Here, $\mathbb{E}[\cdot]$ is the expectation operator over $\{\tilde{\mu},\tilde{\eta}_A,\tilde{\eta}_B\}$ where a tilde sign is used to distinguish a random variable from its realization.

Let $\{M^*,\{P^*_{\mathcal{U}}(\mu,\eta_A,\eta_B),\{P^*_{\mathcal{D},A}(\mu,\eta_A,\eta_B),P^*_{\mathcal{D},B}(\mu,\eta_B,\eta_A)\}\},\{\hat{\xi}^*_{\mathcal{U}}(\cdot|P_A,P_B),\hat{\xi}^*_{\mathcal{D}}(\cdot|P)\}\}$ be a perfect Bayesian equilibrium. Then, using (27) and (28), at the first stage the firm chooses not to split the two markets (i.e., $M^* = \mathcal{U}$) if and only if

$$\mathbb{E}[\Pi^*_{\mathcal{U}}(\tilde{\mu},\tilde{\eta}_A,\tilde{\eta}_B)] > \mathbb{E}[\Pi^*_{\mathcal{D}}(\tilde{\mu},\tilde{\eta}_A,\tilde{\eta}_B)]. \quad (29)$$

4.2 Comparisons of Profits

We now provide conditions under which (29) holds. Specifically, we show that the presence of uninformed buyers (inducing the firm to engage in noisy signaling) makes it possible for the firm to obtain higher expected profits by not segmenting the market.

In order to do so, we need to obtain the equilibrium profits for each possible state in stage-2 (i.e., $M \in \{\mathcal{U},\mathcal{D}\}, \forall\ (\mu,\eta_A,\eta_B)$) and then take the expectation with respect to $(\tilde{\mu},\tilde{\eta}_A,\tilde{\eta}_B)$ to obtain stage-1 expected profits. Proposition 4.1 gives the expected profits for $M = \mathcal{U}$ and $M = \mathcal{D}$.

Proposition 4.1. *Given $M = \mathcal{U}$ and given $M = \mathcal{D}$, there exists an equilibrium in the second stage characterized by Proposition 3.3 and Proposition 3.5, respectively. Then, stage-1 expected profits are*

1. *For $M = \mathcal{U}$,*

$$\mathbb{E}[\Pi^*_{\mathcal{U}}(\tilde{\mu}, \tilde{\eta}_A, \tilde{\eta}_B)] = \frac{(1+\gamma)^2(\rho + \sigma^2_\mu) + 2\sigma^2_\eta}{8} - (1-\lambda)^2\Psi^*_{\mathcal{U}}, \tag{30}$$

where

$$\Psi^*_{\mathcal{U}} = \frac{\gamma^2\sigma^2_\mu}{8}\left(1 + \frac{(1+\gamma)^2(1+\gamma\lambda)^2\rho^2\sigma^2_\mu}{(2\sigma^2_\eta + (1+\gamma)(1+\gamma\lambda)\sigma^2_\mu)^2}\right) \tag{31}$$

2. *For $M = \mathcal{D}$,*

$$\mathbb{E}[\Pi^*_{\mathcal{D}}(\tilde{\mu}, \tilde{\eta}_A, \tilde{\eta}_B)] = \frac{(1+\gamma^2)(\rho^2 + \sigma^2_\mu) + 2\sigma^2_\eta}{4} - (1-\lambda)^2\Psi^*_{\mathcal{D}}, \tag{32}$$

where

$$\Psi^*_{\mathcal{D}} = \frac{\gamma^2\sigma^2_\mu(\sigma^4_\eta + 2(1+\gamma^2\lambda)\sigma^2_\eta\sigma^2_\mu + \gamma^2(1+\gamma^2\lambda^2)\sigma^2_\mu(\rho^2+\sigma^2_\mu))}{4\left(\sigma^4_\eta + 2(1+\gamma^2\lambda)\sigma^2_\eta\sigma^2_\mu + (1+\gamma^2)(1+\gamma^2\lambda^2)\sigma^4_\mu\right)}. \tag{33}$$

Proof. It follows from $\left\{P^*_{\mathcal{D},A}(\mu,\eta_A,\eta_B), P^*_{\mathcal{D},B}(\mu,\eta_B,\eta_A), \phi^*_{\mathcal{D}}(P_A,P_B|\cdot), \hat{\xi}^*_{\mathcal{D}}(\cdot|P_A,P_B)\right\}$ in Proposition 3.3 and from $\left\{P^*_{\mathcal{U}}(\mu,\eta_A,\eta_B), \phi^*_{\mathcal{U}}(P|\cdot), \hat{\xi}^*_{\mathcal{U}}(\cdot|P)\right\}$ in Proposition 3.5. □

Proposition 4.1 shows that regardless of the firm's decision to segment or integrate the markets, the first-stage expected profits are the sum of two components. The first component is the *full-information* expected profits, i.e., when all buyers are informed (i.e., $\lambda = 1$). The second component is a distortion that emanates from the firm's need to signal quality via prices. Indeed, in order to signal the quality of the good to the uninformed buyers, the firm alters prices. This distortion in prices translates into a loss in expected profits. Formally, from (31) and (33), $-\Psi^*_{\mathcal{U}} \leq 0$ and $-\Psi^*_S \leq 0$ denote the loss in expected profits (due to signaling) under no market segmentation and market segmentation, respectively.

Using Proposition 4.1, we show that when there is noisy signaling, it is possible that the firm prefers *not* to segment the market. Taking the difference in profits between (32) and (30) yields

$$\frac{(1-\gamma)^2(\rho^2+\sigma_\mu^2)}{8}+\frac{\sigma_\eta^2}{4}-(1-\lambda)^2(\Psi_S^*-\Psi_{\mathcal{U}}^*). \tag{34}$$

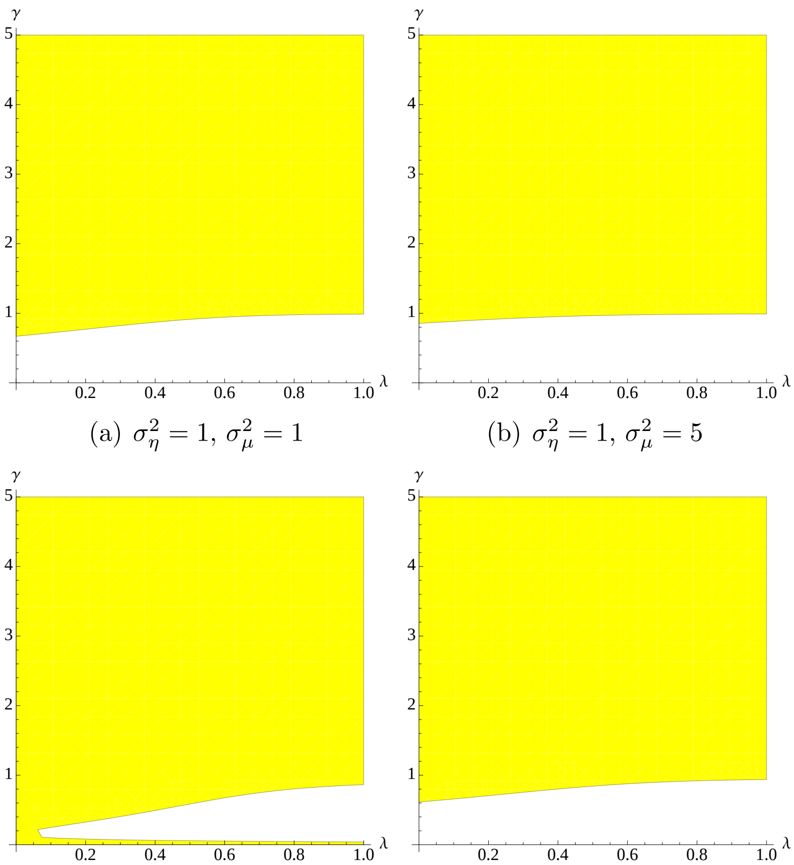
When (34) is positive (negative), the firm prefers to (not to) segment the markets. Consider first the benchmark case of full information when all buyers are informed, i.e., $\lambda = 1$. If every buyer is informed, then it is always profitable for a firm to segment the market. Indeed, in that case, there is no loss in expected profits due to signaling. Using two prices instead of one always yields higher expected profits for two reasons: a) because using two prices allows the firm to set prices that are more appropriate for each market segments, this is captured by the first term in (34), and b) because, by setting two prices instead of one, the firm accounts for the specific demand shocks in each market segment and not only for the average shock, this is captured by the second term in (34).

Remark 4.2. *If* $\lambda = 1$, *then*

$$\mathbb{E}[\Pi_{\mathcal{U}}^*(\tilde{\mu},\tilde{\eta}_A,\tilde{\eta}_B)] < \mathbb{E}[\Pi_{\mathcal{D}}^*(\tilde{\mu},\tilde{\eta}_A,\tilde{\eta}_B)].$$

Remark 4.2 implies that a necessary condition for the firm to prefer <u>not to</u> segment the market is the presence of uninformed buyers, which is related to the loss (due to signaling) in expected profits. Indeed, in order to offset the benefit from price flexibility (by segmenting the market), it is necessary (but not sufficient) for the loss in expected profits under market segmentation to be greater than the loss in expected profits under no market segmentation. For $\lambda \in [0,1)$, it is possible that $\Psi_{\mathcal{D}}^* > \Psi_{\mathcal{U}}^*$. Figure 2 depicts the region of the parameters space $\{\lambda,\gamma\}$ corresponding to $\Psi_{\mathcal{D}}^* > \Psi_{\mathcal{U}}^*$.

Proposition 4.3 establishes the condition under which the firm chooses not to segment the market. Condition (35) compares the gains and losses in expected profits from integrating the markets. Intuitively, the firm faces a trade-off. On the one hand, market segmentation yields more flexibility and the ability to capture more of the consumer surplus. On the other hand, the firm also has to incur a signaling cost, i.e., the distortion needed to signal quality via prices depends on whether the market is integrated or segmented.



(a) $\sigma_\eta^2 = 1$, $\sigma_\mu^2 = 1$ (b) $\sigma_\eta^2 = 1$, $\sigma_\mu^2 = 5$

(c) $\sigma_\eta^2 = 5$, $\sigma_\mu^2 = 1$ (d) $\sigma_\eta^2 = 5$, $\sigma_\mu^2 = 5$

Figure 2: Shaded area indicates $\Psi_{\mathcal{U}}^* \leq \Psi_D^*$ with $\rho = 10$.

Specifically, the firm does not segment the market if there is a reduction in cost due to signaling which is greater than the loss from price flexibility. While there is always a loss from giving up price flexibility, whether there is a reduction in cost due to signaling depends on the parameter values.

Proposition 4.3. *If $\lambda \in [0,1)$, then, at the first stage, it is optimal for the firm not to segment the market (i.e., $M^* = \mathcal{U}$) if and only if*

$$\Psi^*_{\mathcal{D}} - \Psi^*_{\mathcal{U}} \geq \frac{(1-\gamma)^2(\rho^2+\sigma^2_\mu)+2\sigma^2_\eta}{8(1-\lambda)^2} \tag{35}$$

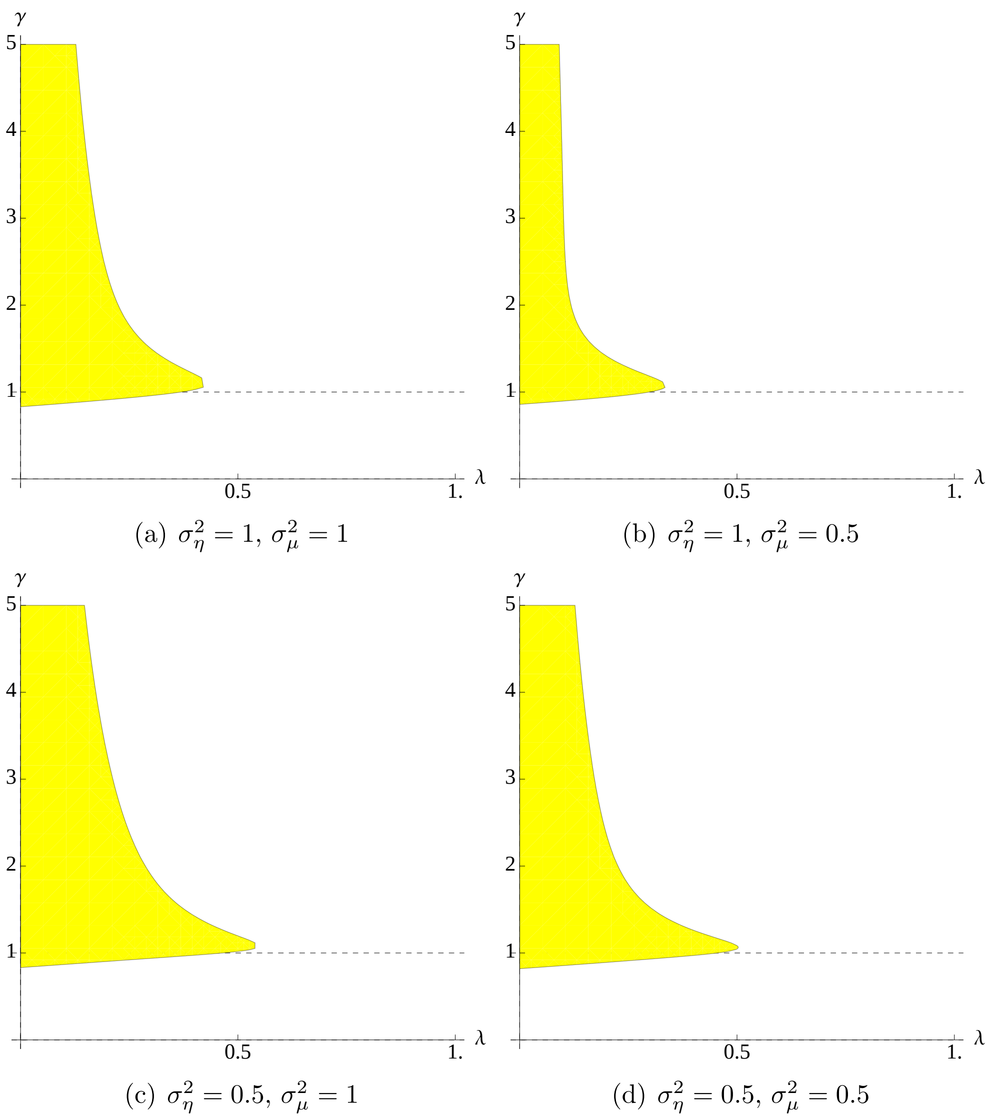
where $\Psi^*_{\mathcal{U}}$ and $\Psi^*_{\mathcal{D}}$ are given by (31) *and* (33), *respectively.*

One question is why the firm under market segmentation cannot price uniformly in order to reach the higher profits generated under market integration. The reason is that the firm's decision to segment or integrate the market in stage 1 impacts the firm's stage-2 profit function through the uninformed buyers' posterior beliefs. Specifically, the uninformed buyers' posterior beliefs at stage 2 depend on the firm's decision to segment the market at stage 1, i.e., $\hat{\xi}_{\mathcal{U}}(\cdot|P) \neq \hat{\xi}_{\mathcal{D}}(\cdot|P)$. Since these beliefs impose an informational externality on the firm, they alter the firm's maximization problem. See (3) and (4). Indeed, the uninformed buyers process the information differently. That is, conditional on observing the same price signals the posterior mean of quality under market segmentation -with or without two identical prices- is different from the posterior mean under no market segmentation. In particular, mimicking the non-segmentation price strategy does not yield the same expected profits.[23]

[23]Formally, plugging the non-segmentation price strategy into the market segmentation expected profits, i.e., plugging $P^*_{\mathcal{U}}(\mu,\eta_A,\eta_B)$ into the objective function in (3) does not yield the expected profits in (27), i.e.,

$$\begin{aligned}
&\mathbb{E}\Big[P^*_{\mathcal{U}}(\mu,\tilde{\eta}_A,\tilde{\eta}_B)\cdot\Big(Q_A(P^*_{\mathcal{U}}(\mu,\tilde{\eta}_A,\tilde{\eta}_B),\mu,\tilde{\eta}_A)\\
&+Q_B(P^*_{\mathcal{U}}(\mu,\tilde{\eta}_A,\tilde{\eta}_B),\mu,\hat{\xi}^*_{\mathcal{U}}(\cdot|P^*_{\mathcal{U}}(\mu,\tilde{\eta}_A,\tilde{\eta}_B)),\tilde{\eta}_B)\Big)\Big]\\
&\neq \mathbb{E}[P^*_{\mathcal{U}}(\mu,\tilde{\eta}_A,\tilde{\eta}_B)\cdot Q_A(P^*_{\mathcal{U}}(\mu,\tilde{\eta}_A,\tilde{\eta}_B),\mu,\tilde{\eta}_A)]+\mathbb{E}[P^*_{\mathcal{U}}(\mu,\tilde{\eta}_A,\tilde{\eta}_B)\\
&\cdot Q_B(P^*_{\mathcal{U}}(\mu,\tilde{\eta}_A,\tilde{\eta}_B),\mu,\hat{\xi}^*_{\mathcal{D}}(\cdot|P^*_{\mathcal{U}}(\mu,\tilde{\eta}_A,\tilde{\eta}_B),P^*_{\mathcal{U}}(\mu,\tilde{\eta}_A,\tilde{\eta}_B),\tilde{\eta}_B)],
\end{aligned} \tag{36}$$

because the act of segmenting or integrating the market influences ex ante the type of price signals the uninformed buyers receive, which alters the updating rule, i.e., $\hat{\xi}^*_{\mathcal{D}} \neq \hat{\xi}^*_{\mathcal{U}}$.



(a) $\sigma_\eta^2 = 1$, $\sigma_\mu^2 = 1$

(b) $\sigma_\eta^2 = 1$, $\sigma_\mu^2 = 0.5$

(c) $\sigma_\eta^2 = 0.5$, $\sigma_\mu^2 = 1$

(d) $\sigma_\eta^2 = 0.5$, $\sigma_\mu^2 = 0.5$

Figure 3: The shaded area shows $M^* = \mathcal{U}$ for $\rho = 10$.

It is convenient to depict the condition stated in Proposition 4.3. Figure 3 illustrates Proposition 4.3 by showing regions of the parameters space $\{\lambda, \gamma\}$ corresponding to $M^* = \mathcal{U}$ for some σ_η^2 and σ_μ^2. The firm chooses not to segment the markets when the fraction of informed buyers is low enough and the reservation price on market B is either almost similar to the one of market A, or higher. In terms of the parameters, this implies that λ is low and γ is not too low. This is consistent with the decomposition of expected profits provided in Proposition 4.1. Indeed, as noted, the firm faces a trade-off between a benefit from price flexibility and a cost from having to signal quality from prices.

The farther γ is from 1, the greater the gain from price flexibility and splitting the market since markets A and B are very different. That is, the first component in (34) is increasing in $(1-\gamma)^2$. The firm will prefer not to segment the market and avoiding the signaling cost when the price-signals are worthy for a large mass of buyers (for low values of λ).

When the two markets are identical, i.e., $\gamma = 1$, market integration can turn out to be the optimal pricing regime for the firm.[24] In fact, this is the situation in which market integration is the most susceptible to be profitable since the benefit from price flexibility is then at its lowest.

5 Conclusion

In this paper, we have studied the common commercial practice of third-degree price discrimination in the presence of consumer learning, using prices as informative signals of quality. Understanding the implications of price discrimination is particularly important since the practice has gained in popularity with the shift from brick-and-mortar stores to the online marketplace as it is easier for firms to accumulate information on consumers and thus, easier to charge different prices to different consumer segments.

In this particular paper, we study the effect of market segmentation on the informational content of prices and highlight it can be beneficial for con-

[24]There is still a difference between the two markets because there are some uninformed buyers in market B.

sumers and detrimental for the firm. More specifically, we find that market segmentation improves the informational content of the price-signals, which benefits the uninformed buyers by yielding more precise posterior beliefs. Since the introduction of noise precludes complete learning, the uninformed buyers continue to face uncertainty about the product's quality. In future work, it would be interesting to study the effect of risk-aversion under incomplete learning.

One caveat is however in order. That is, our analysis implicitly assumes that both markets are served whether pricing is discriminatory or not. In general, price discrimination makes it profitable to serve markets that would otherwise not be served with uniform pricing. In order words, discriminatory pricing may lead to the opening of new markets. In the presence of uninformed buyers, uniform pricing might make it more likely to exclude the buyers of one of the markets. The reason is that the informational externality generally leads to an increase in the mean prices. Hence, the benefit of market segmentation (in terms of accessibility of the good) is enhanced by the presence of uninformed buyers. See Appendix C.

Our second contribution is to show that market segmentation is not necessarily optimal for the firm. Under complete information on the demand side, a monopoly obtains a higher expected profit by charging different prices for market segments having different price elasticities. We show that this conclusion does not hold because of the firm's need to engage in price signaling. Therefore, we outline an important difference regarding the effect of market segmentation between complete and incomplete information environments.

An extension of the model would be to assume the firm already knows μ when choosing whether to segment the market or not. In this case, her best-response would be to segment the market only when μ is above some function of the parameters. Consequently, her choice together with the price-signal(s) will convey information to uninformed buyers. Instead of following a normal distribution, the uninformed buyers' posterior beliefs will now follow a truncated normal distribution. Whether an equilibrium exists in this case is a question we leave for future research.

A Proofs

Let $\mathbb{E}$ and $\mathbb{V}$ be the *expectation* and *variance* operators, respectively.

Proof of Proposition 3.3. Using Definition 3.2, we proceed in three steps. First, given the uninformed buyers' updating rule, we solve for the firm's optimal price strategies. Second, we derive the distribution of the posterior beliefs that follows from the firm's price strategies and the prior beliefs. Finally, we check that the uninformed buyers's updating rule and the distribution of the price-signals are mutually consistent.

1. Given (9), $\mathbb{E}[\tilde{\mu}^*_{\mathcal{D}}|P_A, P_B] = \delta^*_0 + \delta^*_1 P_A + \delta^*_2 P_B$. Plugging (1), (2), and $\mathbb{E}[\tilde{\mu}^*_{\mathcal{D}}|P_A, P_B]$ into (3) yields

$$\max_{P_A,P_B} \{P_A \cdot (\mu - P_A + \eta_A) \\ +P_B \cdot (\lambda(\gamma\mu - P_B) + (1-\lambda)(\gamma(\delta^*_0 + \delta^*_1 P_A + \delta^*_2 P_B) - P_B) + \eta_B)\} . \tag{37}$$

Taking the first-order conditions with respect to prices yields

$$P_A : \mu - 2P_A + \eta_A + (1-\lambda)\gamma\delta^*_1 P_B = 0, \tag{38}$$

$$P_B : \lambda(\gamma\mu - 2P_B) + (1-\lambda)(\gamma(\delta^*_0 + \delta^*_1 P_A + 2\delta^*_2 P_B) - 2P_B) + \eta_B = 0. \tag{39}$$

Given the expressions for δ^*_1 and δ^*_2 given in (11) and (12), the Hessian matrix is negative definite. Solving (38) and (39) for the price strategies yields

$$P^*_{\mathcal{D},A}(\mu, \eta_A, \eta_B) = \frac{\delta^*_0\delta^*_1\gamma^2(1-\lambda)^2 + (2 - 2\delta^*_2\gamma(1-\lambda) + \delta^*_1\gamma^2\lambda(1-\lambda))\mu}{4 - \delta^{*2}_1\gamma^2(1-\lambda)^2 - 4\delta^*_2\gamma(1-\lambda)} \\ + \frac{(2 - 2\delta^*_2\gamma(1-\lambda))\eta_A + \delta^*_1\gamma(1-\lambda)\eta_B}{4 - \delta^{*2}_1\gamma^2(1-\lambda)^2 - 4\delta^*_2\gamma(1-\lambda)} \tag{40}$$

and

$$P^*_{\mathcal{D},B}(\mu,\eta_B,\eta_A) = \frac{2\delta_0^*\gamma(1-\lambda) + (\delta_1^*\gamma(1-\lambda)+2\gamma\lambda)\mu + \delta_1^*\gamma(1-\lambda)\eta_A + 2\eta_B}{4-\delta_1^{*2}\gamma^2(1-\lambda)^2 - 4\delta_2^*\gamma(1-\lambda)}. \tag{41}$$

2. Next, given the firm's price strategies, we solve for the buyers' posterior beliefs. Specifically, using the expressions for $P^*_{\mathcal{D},A}$ and $P^*_{\mathcal{D},B}$, let

$$\tilde{z}_A \equiv 2\left(P^*_{\mathcal{D},A}(\mu,\tilde{\eta}_A,\tilde{\eta}_B) - \frac{\delta_0^*\delta_1^*\gamma^2(1-\lambda)^2}{D_0}\right)\cdot\left(\frac{D_0}{D_0+\delta_1^{*2}\gamma^2(1-\lambda)^2+2\delta_1^*\gamma^2\lambda(1-\lambda)}\right) \tag{42}$$

$$= \mu + \left(\frac{2-2\delta_2^*\gamma(1-\lambda)}{D_1}\right)\tilde{\eta}_A + \left(\frac{\delta_1^*\gamma(1-\lambda)}{D_1}\right)\tilde{\eta}_B, \tag{43}$$

and

$$\tilde{z}_B \equiv \left(P^*_{\mathcal{D},B}(\mu,\tilde{\eta}_B,\tilde{\eta}_A) - \frac{2\delta_0^*\gamma(1-\lambda)}{D_0}\right)\left(\frac{D_0}{\delta_1^*\gamma(1-\lambda)+2\gamma\lambda}\right) \tag{44}$$

$$= \mu + \left(\frac{\delta_1^*\gamma(1-\lambda)}{\delta_1^*\gamma(1-\lambda)+2\gamma\lambda}\right)\tilde{\eta}_A + \left(\frac{2}{\delta_1^*\gamma(1-\lambda)+2\gamma\lambda}\right)\tilde{\eta}_B, \tag{45}$$

where

$$D_0 \equiv 4 - 4\delta_2^*\gamma(1-\lambda) - \delta_1^{*2}\gamma^2(1-\lambda)^2, \tag{46}$$

$$D_1 \equiv 2 - 2\delta_2^*\gamma(1-\lambda) + \delta_1^*\gamma^2\lambda(1-\lambda). \tag{47}$$

From (43) and (45), $\tilde{\mathbf{z}}|\mu \equiv [\tilde{z}_A,\tilde{z}_B]'|\mu$ is jointly normally distributed. Hence, given the prior distribution $\tilde{\mu} \sim N(\rho,\sigma_\mu^2)$, the posterior distribution of quality μ upon observing $\mathbf{z}$ (i.e., upon observing $\{P_A,P_B\}$) is

$$\tilde{\mu}^*_{\mathcal{D}}|\mathbf{z} \sim N(\rho+\sigma_\mu^2\mathbb{1}\mathbf{\Sigma}^{-1}(\mathbf{z}-\rho\mathbb{1}'),\sigma_\mu^2-\sigma_\mu^4\mathbb{1}\mathbf{\Sigma}^{-1}\mathbb{1}') \tag{48}$$

where $\mathbb{1}$ is a 1×2 vector of ones and

$$\mathbf{\Sigma} \equiv \left[\begin{array}{cc} \sigma_\mu^2 + \sigma_\eta^2 \frac{4(1+\delta_2\gamma(\lambda-1))^2+\delta_1^2\gamma^2(1-\lambda)^2}{D_1^2} & \sigma_\mu^2 + \sigma_\eta^2 \frac{2\delta_1\gamma(1-\lambda)(2+\delta_2\gamma(\lambda-1))}{D_1(\delta_1\gamma(1-\lambda)+2\gamma\lambda)} \\ \sigma_\mu^2 + \sigma_\eta^2 \frac{2\delta_1\gamma(1-\lambda)(2+\delta_2\gamma(\lambda-1))}{D_1(\delta_1\gamma(1-\lambda)+2\gamma\lambda)} & \sigma_\mu^2 + \sigma_\eta^2 \left(\frac{\delta_1^2\gamma^2(1-\lambda)^2+4}{(\delta_1\gamma(1-\lambda)+2\gamma\lambda)^2} \right) \end{array} \right]. \tag{49}$$

Simplifying (48) yields

$$\begin{aligned} \mathbb{E}[\tilde{\mu}_{\mathcal{D}}^*|P_A, P_B] = & \frac{\rho\sigma_\eta^2 - \delta_0^*\gamma^2\lambda(1-\lambda)\sigma_\mu^2 + (2-\delta_1^*\gamma^2\lambda(1-\lambda))\sigma_\mu^2 P_A}{\sigma_\eta^2 + (1+\gamma^2\lambda^2)\sigma_\mu^2} \\ & + \frac{(2\gamma\lambda(1-\delta_2^*\gamma(1-\lambda)) - \delta_1^*\gamma(1-\lambda))\sigma_\mu^2 P_B}{\sigma_\eta^2 + (1+\gamma^2\lambda^2)\sigma_\mu^2}, \end{aligned} \tag{50}$$

and

$$\mathbb{V}[\tilde{\mu}_{\mathcal{D}}^*|P_A, P_B] = \frac{\sigma_\eta^2\sigma_\mu^2}{\sigma_\eta^2 + (1+\gamma^2\lambda^2)\sigma_\mu^2}. \tag{51}$$

3. Setting (50) equal to $\delta_0^* + \delta_1^* P_A + \delta_2^* P_B$ and solving for δ_0^*, δ_1^* and δ_2^* yields (10), (11), and (12). Since δ_0^*, δ_1^* and δ_2^* uniquely exist, the posterior beliefs are normally distributed as defined by (9) and are consistent with (50) and (51). Moreover, from (7) and (8), the price-signals are jointly normally distributed.

Proof of Proposition 3.5. The proof of Proposition 3.5 follows the same steps of the proof of Proposition 3.3. Using Definition 3.4, we proceed as follows.

1. Given (16), $\mathbb{E}[\tilde{\mu}_{\mathcal{U}}^*|P] = \beta_0^* + \beta_1^* P$. Plugging (1), (2), and $\mathbb{E}[\tilde{\mu}_{\mathcal{U}}^*|P]$ into (4) yields

$$\max_P \; \{P \cdot ((\mu - P + \eta_A) + \lambda(\gamma\mu - P) + (1-\lambda)(\gamma(\beta_0^* + \beta_1^* P) - P) + \eta_B)\}. \tag{52}$$

Taking the first-order condition with respect to P yields

$$(1+\gamma\lambda)\mu + \eta_A + \eta_B + \beta_0^*\gamma(1-\lambda) - 2P(2 - \beta_1^*\gamma(1-\lambda)) = 0. \tag{53}$$

Given the expression for β_1^* given in (18), the second-order condition holds, i.e., $-2\left(2-\beta_1^*\gamma(1-\lambda)\right) < 0$. Then, solving (53) for the price strategy yields

$$P_{\mathcal{U}}^*(\mu,\eta_A,\eta_B) = \frac{\beta_0^*\gamma(1-\lambda)+(1+\gamma\lambda)\mu+\eta_A+\eta_B}{4-2\beta_1^*\gamma(1-\lambda)}. \tag{54}$$

2. Next, given the firm's price strategy, we solve for the buyers' posterior beliefs. Specifically, using P^*, let

$$\tilde{z} \equiv \frac{2(2-\beta_1^*\gamma(1-\lambda))P_{\mathcal{U}}^*(\mu,\tilde{\eta}_A,\tilde{\eta}_B)-\beta_0^*\gamma(1-\lambda)}{1+\gamma\lambda} \tag{55}$$

$$= \mu + \frac{\tilde{\eta}_A+\tilde{\eta}_B}{1+\gamma\lambda} \tag{56}$$

such that $\tilde{z}|\mu$ is normally distributed with mean μ and variance $\sigma_z^2 \equiv 2\sigma_\eta^2/(1+\gamma\lambda)^2$. Given the prior distribution, $\tilde{\mu} \sim N(\rho,\sigma_\mu^2)$, the posterior belief upon observing z (i.e., upon observing P) is

$$\tilde{\mu}_{\mathcal{U}}^*|z \sim N\left(\frac{\rho\sigma_z^2+z\sigma_\mu^2}{\sigma_z^2+\sigma_\mu^2}, \frac{1}{1/\sigma_z^2+1/\sigma_\mu^2}\right). \tag{57}$$

Hence, simplifying (57), the posterior mean and variance are

$$\mathbb{E}[\tilde{\mu}_{\mathcal{U}}^*|P] = \frac{2\rho\sigma_\eta^2+((4-2\beta_1^*\gamma(1-\lambda))P-\beta_0^*\gamma(1-\lambda))(1+\gamma\lambda)\sigma_\mu^2}{2\sigma_\eta^2+(1+\lambda\gamma)^2\sigma_\mu^2} \tag{58}$$

and

$$\mathbb{V}[\tilde{\mu}_{\mathcal{U}}^*|P] = \frac{2\sigma_\eta^2\sigma_\mu^2}{2\sigma_\eta^2+(1+\lambda\gamma)^2\sigma_\mu^2}. \tag{59}$$

3. Setting (58) equal to $\beta_0^*+\beta_1^*P$ and solving for β_0^* and β_1^* yields (17) and (18). Since β_0^* and β_1^* uniquely exist, the posterior beliefs are normally distributed as defined by (16) and are consistent with (58) and (59). Finally, from (15), the price-signal is normally distributed.

B Equilibrium Definition

Definition B.1 states the Perfect Bayesian Equilibrium. The equilibrium consists of the firm's strategy (a segmentation decision at stage 1 and prices at stage 2), the distribution of the price-signals conditional on any quality x, and the uninformed buyers' posterior beliefs about the quality upon observing any prices.[25] In equilibrium, the posterior beliefs are consistent with Bayes' rule and the equilibrium distribution of prices.

Definition B.1. *The tuple* $\{\{M^*, \{\{P^*_{\mathcal{U}}(\mu, \eta_A, \eta_B), \{P^*_{\mathcal{D},A}(\mu, \eta_A, \eta_B), P^*_{\mathcal{D},B}(\mu, \eta_B, \eta_A)\}\}\}\}$, $\{\hat{\xi}^*_{\mathcal{U}}(\cdot|P_A, P_B), \hat{\xi}^*_{\mathcal{D}}(\cdot|P)\}\}$ *is a perfect Bayesian equilibrium if, for all* $\mu > 0$,

1. *At stage 2,*

   (a) *For* $M^* = \mathcal{U}$,

      i. *Given* $\hat{\xi}^*_{\mathcal{U}}(\cdot|P)$, *and for any* η_A *and* η_B, *the firm's price strategy is*

      $$P^*_{\mathcal{U}}(\mu, \eta_A, \eta_B) = \arg\max_P \left\{ P\cdot\Big(Q_A(P, \mu, \eta_A) + Q_B(P, \mu, \hat{\xi}^*_{\mathcal{U}}(\cdot|P), \eta_B)\Big) \right\}. \tag{60}$$

      ii. *Given the distribution of* $\{\tilde{\eta}_A, \tilde{\eta}_B\}$, $\phi^*_{\mathcal{U}}(P|x)$ *is the p.d.f. of the random price-signal* $P^*_{\mathcal{U}}(x, \tilde{\eta}_A, \tilde{\eta}_B)$ *conditional on any quality* x.

      iii. *Given* $\phi^*_{\mathcal{U}}(P|\cdot)$ *and prior beliefs* $\xi(\cdot)$, *the uninformed buyers' posterior beliefs upon observing any* P *is* $\tilde{\mu}^*|P$ *with p.d.f.*

      $$\hat{\xi}^*_{\mathcal{U}}(x|P) = \frac{\xi(x)\phi^*_{\mathcal{U}}(P|x)}{\int_{x'\in\mathbb{R}} \xi(x')\phi^*_{\mathcal{U}}(P|x')\mathrm{d}x'}, \quad \forall x \in \mathbb{R}. \tag{61}$$

   (b) *For* $M^* = \mathcal{D}$,

[25] The variable μ refers to the true quality whereas x is used as a dummy variable for quality.

i. Given $\hat{\xi}^*_{\mathcal{D}}(\cdot|P_A, P_B)$, and for any η_A and η_B, the firm's price strategies are

$$\left\{P^*_{\mathcal{D},A}(\mu, \eta_A, \eta_B), P^*_{\mathcal{D},B}(\mu, \eta_B, \eta_A)\right\} = \arg\max_{P_A, P_B} \Big\{P_A \cdot Q_A(P_A, \mu, \eta_A) + P_B \cdot Q_B(P_B, \mu, \hat{\xi}^*_{\mathcal{D}}(\cdot|P_A, P_B), \eta_B)\Big\}. \tag{62}$$

ii. Given the distribution of $\{\tilde{\eta}_A, \tilde{\eta}_B\}$, $\phi^*_{\mathcal{D}}(P_A, P_B|x)$ is the p.d.f. of the random price-signals $\left\{P^*_{\mathcal{D},A}(x, \tilde{\eta}_A, \tilde{\eta}_B), P^*_{\mathcal{D},B}(x, \tilde{\eta}_B, \tilde{\eta}_A)\right\}$ conditional on any quality x.

iii. Given $\phi^*_{\mathcal{D}}(P_A, P_B|\cdot)$ and prior beliefs $\xi(\cdot)$, the uninformed buyers' posterior beliefs about quality upon observing P_A and P_B is $\tilde{\mu}^*_{\mathcal{D}}|P_A, P_B$ with the p.d.f.

$$\hat{\xi}^*_{\mathcal{D}}(x|P_A, P_B) = \frac{\xi(x)\phi^*_{\mathcal{D}}(P_A, P_B|x)}{\int_{x' \in \mathbb{R}} \xi(x')\phi^*_{\mathcal{D}}(P_A, P_B|x')\mathrm{d}x'}, \quad \forall x \in \mathbb{R}. \tag{63}$$

2. At stage 1,

$$M^* = \arg\max_{M \in \{\mathcal{U}, \mathcal{D}\}} \mathbb{1}_{[M=\mathcal{U}]} \cdot \mathbb{E}[\Pi^*_{\mathcal{U}}(\tilde{\mu}, \tilde{\eta}_A, \tilde{\eta}_B)] + \mathbb{1}_{[M=\mathcal{D}]} \cdot \mathbb{E}[\Pi^*_{\mathcal{D}}(\tilde{\mu}, \tilde{\eta}_A, \tilde{\eta}_B)] \tag{64}$$

where

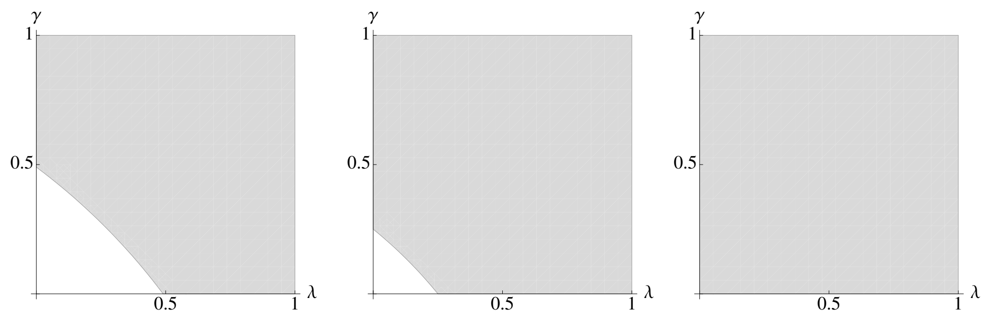
$$\mathbb{E}[\Pi^*_{\mathcal{U}}(\tilde{\mu}, \tilde{\eta}_A, \tilde{\eta}_B)] = \mathbb{E}\Big[P^*_{\mathcal{U}}(\tilde{\mu}, \tilde{\eta}_A, \tilde{\eta}_B) \cdot \Big(Q_A(P^*_{\mathcal{U}}(\tilde{\mu}, \tilde{\eta}_A, \tilde{\eta}_B), \tilde{\mu}, \tilde{\eta}_A) + Q_B(P^*_{\mathcal{U}}(\tilde{\mu}, \tilde{\eta}_A, \tilde{\eta}_B), \tilde{\mu}, \hat{\xi}^*_{\mathcal{U}}(\cdot|P^*_{\mathcal{U}}(\tilde{\mu}, \tilde{\eta}_A, \tilde{\eta}_B)), \tilde{\eta}_B)\Big)\Big] \tag{65}$$

and

$$\mathbb{E}[\Pi^*_{\mathcal{D}}(\tilde{\mu}, \tilde{\eta}_A, \tilde{\eta}_B)] = \mathbb{E}\Big[P^*_{\mathcal{D},A}(\tilde{\mu}, \tilde{\eta}_A, \tilde{\eta}_B) \cdot Q_A(P^*_{\mathcal{D},A}(\tilde{\mu}, \tilde{\eta}_A, \tilde{\eta}_B), \tilde{\mu}, \tilde{\eta}_A)\Big] + \mathbb{E}\Big[P^*_{\mathcal{D},B}(\tilde{\mu}, \tilde{\eta}_B, \tilde{\eta}_A) \cdot Q_B(P^*_{\mathcal{D},B}(\tilde{\mu}, \tilde{\eta}_B, \tilde{\eta}_A), \tilde{\mu}, \hat{\xi}^*_{\mathcal{D}}(\cdot|P^*_{\mathcal{D},A}(\tilde{\mu}, \tilde{\eta}_A, \tilde{\eta}_B), P^*_{\mathcal{D},B}(\tilde{\mu}, \tilde{\eta}_B, \tilde{\eta}_A)), \tilde{\eta}_B)\Big]. \tag{66}$$

C Probability of Exclusion

In this appendix, we study whether the presence of uninformed buyers (or the informational externality) decreases or increases the probability of excluding market B under uniform pricing. In market B, the informed buyers and the uninformed buyers with unbiased prior beliefs do not buy the good if the price is above the reservation price, i.e., $P > \gamma\mu$. Using (15), we compare the probability of such an event under the two scenarios of complete and incomplete information, i.e., $P^*_{\mathcal{U}}(\mu, \tilde{\eta}_A, \tilde{\eta}_B)|_{\lambda=1}$ and $P^*_{\mathcal{U}}(\mu, \tilde{\eta}_A, \tilde{\eta}_B)|_{\lambda\in(0,1)}$. In Figure 4, the shaded area encompasses the points $\{\gamma, \lambda\}$ for which the presence of uninformed buyers increases the probability of exclusion, i.e., $\mathbb{P}[\gamma\mu < P^*_{\mathcal{U}}(\mu, \tilde{\eta}_A, \tilde{\eta}_B)|_{\lambda=1}] < \mathbb{P}[\gamma\mu < P^*_{\mathcal{U}}(\mu, \tilde{\eta}_A, \tilde{\eta}_B)|_{\lambda\in(0,1)}]$.[26] An increase in the variance of the demand shock increases the size of the shaded area.



(a) $\sigma_\eta = 0.1$ (b) $\sigma_\eta = 0.5$ (c) $\sigma_\eta = 1$

Figure 4: Shaded area indicates $\mathbb{P}[\gamma\mu < P^*_{\mathcal{U}}(\mu, \tilde{\eta}_A, \tilde{\eta}_B)|_{\lambda=1}] < \mathbb{P}[\gamma\mu < P^*_{\mathcal{U}}(\mu, \tilde{\eta}_A, \tilde{\eta}_B)|_{\lambda\in(0,1)}]$.

[26] To generate Figure 4, we set $\mu = 1$ and $\sigma^2_\mu = 1$.

References

Aguirre, I., Cowan, S., and Vickers, J. (2010). Monopoly Price Discrimination and Demand Curvature. *Amer. Econ. Rev.*, 100(4):1601–1615.

Armstrong, M. (2006). Recent Developments in the Economics of Price Discrimination. In Blundell, R., Newey, W., and Persson, W., editors, *Advances in Economics and Econometrics: Theory and Applications, Ninth World Congress of the Econometric Society*, volume 2. Cambridge University Press.

Asplund, M. and Friberg, R. (2000). The Law of One Price in Scandinavian Duty-Free Stores. SSE/EFI Working Paper no 351.

Bagwell, K. and Riordan, M. (1991). High and Declining Prices Signal Product Quality. *Amer. Econ. Rev.*, 81(1):224–239.

Bergemann, D., Brooks, B., and Morris, S. (2015). The Limits of Price Discrimination. *Amer. Econ. Rev.*, 105(3):921–957.

Chen, C. (2009). A Puzzle or a Choice: Uniform Pricing for Motion Pictures at the Box. *Atl. Econ. J.*, 37(1):73–85.

Chen, Y. and Cui, T. (2013). The Benefit of Uniform Price for Branded Variants. *Market. Sci.*, 32(1):36–50.

Cowan, S. (2013). Welfare-Increasing Thirs-Degree Price Discriminaion. University of Oxford, Economics Series Working Paper 652.

Daher, W., Mirman, L., and Santugini, M. (2012). Information in Cournot: Signaling with Incomplete Control. *Int. J. Ind. Organ.*, 30(4):361–370.

Daughety, A. and Reinganum, J. (1995). Product Safety: Liability, R&D, and Signaling. *Amer. Econ. Rev.*, 85(5):1187–1206.

Daughety, A. and Reinganum, J. (2005). Secrecy and Safety. *Amer. Econ. Rev.*, 95(4):1074–1091.

Daughety, A. and Reinganum, J. (2007). Competition and Confidentiality: Signaling Quality in a Duopoly when there is Universal Private Information. *Games Econ. Behav.*, 58(1):94–120.

Daughety, A. and Reinganum, J. (2008a). Communicating Quality: A Unified Model of Disclosure and Signalling. *RAND J. Econ.*, 39(4):973–989.

Daughety, A. and Reinganum, J. (2008b). Imperfect Competition and Quality Signalling. *RAND J. Econ.*, 39(1):163–183.

de Haan, T., Offerman, T., and Sloof, R. (2011). Noisy Signaling: Theory and Experiment. *Games Econ. Behav.*, 73(2):402–428.

Flores Vidotti, C. (2004). Drug information centers in developing countries and the promotion of rational use of drugs: A viewpoint about challenges and perspectives. *Int. Pharm. J.*, 18(1):21–23.

Friberg, R. (2001). Two Monies, Two Markets?: Variability and the Option to Segment. *J. Int. Econ.*, 55(2):317–327.

Friberg, R. (2003). Common Currency, Common Market? *J. Eur. Econ. Assoc.*, 1(2-3):650–661.

Friberg, R. and Martensen, K. (2001). Endogenous Market Segmentation and the Law of One Price. Working Paper.

Gallo, F. (2010). To Segment or Not to Segment Markets? A Note on the Profitability of Market Segmentation for an International Oligopoly. Working Paper.

Gendron-Saulnier, C. and Santugini, M. (2013). The Informational Benefit of Price Discrimination. Cahiers de Recherche 13-02, HEC Montréal, Institut d'économie appliquée.

Gordon, S. and Nöldeke, G. (2013). Figures of Speech and Informative Communication. Mimeo.

Grossman, S. (1976). On the Efficiency of Competitive Stock Markets where Traders Have Different Information. *J. Finance*, 31(2):573–585.

Grossman, S. (1978). Further Results on the Informational Efficiency of Competitive Stock Markets. *J. Econ. Theory*, 18(1):81–101.

Grossman, S. (1989). *The Informational Role of Prices.* MIT Press.

Grossman, S. and Stiglitz, J. (1980). On the Impossibility of Informationally Efficient Markets. *Amer. Econ. Rev.*, 70(3):393–408.

Janssen, M. and Roy, S. (2010). Signaling Quality through Prices in an Oligopoly. *Games Econ. Behav.*, 68(1):192–207.

Jeitschko, T. and Norman, H.-T. (2012). Signaling in Deterministic and Stochastic Settings. *J. Econ. Behav. Organ.*, 82(1):39–55.

Judd, K. and Riordan, M. (1994). Price and Quality in a New Product Monopoly. *Rev. Econ. Stud.*, 61(4):773–789.

Kihlstrom, R. and Mirman, L. (1975). Information and Market Equilibrium. *Bell. J. Econ.*, 6(1):357–376.

Kyle, A. (1985). Continuous Auctions and Insider Trading. *Econometrica*, 53(6):1315–1335.

Matthews, S. and Mirman, L. (1983). Equilibrium Limit Pricing: The Effects of Private Information and Stochastic Demand. *Econometrica*, 51(4):981–996.

McMillan, R. (2007). Different Flavor, Same Price: The Puzzle of Uniform Pricing for Differentiated Products. Working Paper.

Mirman, L., Salgueiro, E., and Santugini, M. (2014a). Learning in a Perfectly Competitive Market. CIRPEE working paper 1423.

Mirman, L., Salgueiro, E., and Santugini, M. (2014b). Noisy Signaling in Monopoly. *Int. Rev. Econ. Finance*, 29(C):504–511.

Mirman, L., Salgueiro, E., and Santugini, M. (2015). Noisy Learning in a Competitive Market with Risk Aversion. CIRPEE working paper 1502.

Nahata, B., Ostaszewski, K., and Sahoo, P. (1990). Direction of Price Changes in Third-Degree Price Discrimination. *Amer. Econ. Rev.*, 80(5):1254–1258.

Orbach, B. and Einav, L. (2007). Uniform Prices for Differentiated Goods: The Case of the Movie-Theater Industry. *Int. Rev. Law Econ.*, 27(2):129–153.

Pigou, A. (1920). *The Economics of Welfare*. London: Macmillan.

Richardson, M. and Stähler, F. (2013). On the "uniform pricing puzzle" in recorded music. ANU Working Papers in Economics and Econometrics no. 612.

Riordan, M. (1986). Monopolistic Competition with Experience Goods. *Quart. J. Econ.*, 101(2):265–279.

Schmalensee, R. (1981). Output and Welfare Implications of Monopolistic Third-Degree Price Discrimination. *Amer. Econ. Rev.*, 71(1):242–247.

Tirole, J. (1988). *The Theory of Industrial Organization*. MIT press.

Varian, H. (1985). Price Discrimination and Social Welfare. *Amer. Econ. Rev.*, 75(4):970–875.

Wolinsky, A. (1983). Prices as Signals of Product Quality. *Rev. Econ. Stud.*, 50(4):647–658.