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Screening with Congestion

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Abstract:
We study the effect of congestion on monopoly second-degree price discrimination. We provide three results. First, with congestion, the firm does not always provide distinct contracts (i.e., it is not always optimal to price discriminate) and it is more likely for the low-valuation buyer to be excluded. Second, the presence of congestion implies that no buyer receives an efficient allocation. In particular, the high-valuation buyer might be offered a higher or a lower quality (relative to the first-degree price discrimination offer). Finally, congestion might be beneficial to buyers. Specifically, for values of the parameters for which all types are serviced, consumer surplus under second-degree price discrimination may be greater than consumer surplus under no price discrimination.

Keywords: Congestion, Second-degree price discrimination, Screening

JEL Classification: D40, D62, D86, L14

1 Introduction

The benefits that consumers derive from many services depend not only on the intrinsic quality, but also on the platform (or the network) used by the firm to dispense services in a timely and effective manner. Under capacity constraints about the size of the platform (or network), over-utilization of these services by some consumers leads to significant congestion, which results in delays and thus a degraded experience for all consumers. Consider for instance the market for residential broadband service (RBB). If the network is inadequately provisioned for content requests, congestion may arise, which leads to saturation of critical links in the network, and thus delays in the transmission of data packets.

In the presence of vertical differentiation for the intrinsic quality of the service, firms offering different packages for the provision of the service might allow high-valuation consumers to experience a better service.[1] Yet, it is those high-valuation consumers who experience the greatest disutility from congestion due to their greater utilization. This leads to the following question. Is it beneficial for buyers to be screened when there is congestion? More specifically, what is the effect of congestion on second-degree price discrimination? The interdependence of individual demands, via the existence of congestion externalities, limits the applicability of standard results on (monopoly) second-degree price discrimination (Mussa and Rosen, 1978; Maskin and Riley, 1984).[2]

Our purpose is thus to study the impact of congestion on second-degree price discrimination. To that end, we embed congestion into a model à la Mussa and Rosen (1978) in which a monopoly offers packages of different qualities of service and different fees to consumers who are differentiated with respect to preferences for the service. Specifically, we present an analytically tractable model in which consumers derive utility from the quality of the service they have purchased but experience disutility from the overall

[1] For instance, in the market for RBB, firms offer different packages of quality (e.g., a provisioned speed or a usage allowance) at different fees.

[2] For surveys on price discrimination, see Tirole (1988, ch. 3), Varian (1989), Armstrong (2006), and Stole (2007).

provision of the service across consumers. After characterizing the unique optimal contract under second-degree price discrimination with congestion, we study the effect of second-degree price discrimination on behavior and welfare.

Our results are as follows. We first show that congestion affects the seller's ability to offer distinct packages. If buyers are *similar* (but not identical) in their valuation of the good, then the seller cannot discriminate and offers the same package to both buyers. That is, congestion may yield a bunching equilibrium whereas in the absence of congestion, the seller always offers different packages. We then show that congestion results in inefficient allocations for *both* buyers. In particular, the well-known result of "no distortion at the top" no longer holds once congestion externalities are introduced. The direction of the distortion depends on the values of the parameters. Sometimes the distortion is upward, sometimes it is downward. We distinguish ourselves from Jebsi and Thomas (2005) which shows that a congestion cost that is identical across consumers always leads to upward distortions in consumption for the high-valuation users.

We then show that second-degree price discrimination may improve consumer surplus relative to a baseline of no price discrimination for certain values of the parameters. While this is true without congestion in situations in which the lower buyer is excluded under no price discrimination and is offered a package with price discrimination, we show that this is true even when both buyers are serviced. This runs counter to most arguments against price discrimination that focus on the redistribution of consumer surplus to the firm. This result arises due to the monopolist's willingness to degrade quality for low-valuation users so as to limit the effects of congestion experienced by high-valuation users.

The paper is organized as follows. Section 2 presents and discusses a model of second-degree price discrimination in which buyers face a congestion cost. Section 3 studies the effect of congestion on efficiency and consumer surplus. Section 4 concludes the paper.

2 Model and Optimal Behavior

Consider a seller who can offer a service at any level of quality. Specifically, the seller offers a package for a quality of service $q \geq 0$ and charges a flat fee $t \geq 0$ for access. However, due to capacity constraint on the part of the seller, the buyer's utility derived from any quality of service may be reduced by congestion. For instance, if the seller provides residential broadband, a buyer's utility depends positively on quality, but negatively on the congestion that arises when the network is over-utilized.

To provide a clear analysis of screening under congestion, we suppose that there are two heterogeneous buyers.[3] Buyer L has a lower valuation for the service than buyer H. Each buyer derives utility from access to the service, but also receives disutility from congestion that arises as a result of providing several levels of quality. If buyers L and H accept packages $\{q_L, t_L\}$ and $\{q_H, t_H\}$, respectively, net utility of buyer L is[4]

$$u_L = \alpha q_L - q_L^2 - c(q_L + q_H)q_L, \tag{1}$$

while net utility of buyer H is

$$u_H = q_H - q_H^2 - c(q_L + q_H)q_H, \tag{2}$$

where the parameter $\alpha \in (0, 1)$ reflects heterogeneity among buyers. The quadratic specification for utility is similar to that of Lambrecht et al. (2007). The average congestion cost $c(q_L + q_H)$ is increasing in total quality $q_L + q_H$, and the parameter $c \geq 0$ measures the degree of disutility due to congestion.

Because the seller is often unable to perfectly price discriminate, we consider optimal behavior of the seller under second-degree price discrimination. The seller proposes two packages so that each type of buyer prefers the package intended for him. Definition 2.1 presents the seller's maximization problem under second-degree price discrimination. There are two sorts of

[3] Alternatively, consider two equally-sized groups of buyers, group L and group H. The relative size of each group has no effect on the essence of our results. Hence, we consider the case of two heterogeneous buyers for the sake of simplicity.

[4] It is assumed that $0 \leq q_L, q_H < 1/2$.

constraints. The incentive compatibility constraints state that each buyer prefers the package designed for him. The individual rationality constraints ensure that each buyer accepts his designated package.

Definition 2.1. *Under second-degree price discrimination, the seller solves the following program:*

$$\Pi^{**}(\alpha, c) = \max_{q_L, q_H, t_L, t_H \geq 0} t_L + t_H \tag{3}$$

subject to the incentive compatibility (IC) and individual rationality (IR) constraints, i.e.,

$$(\underline{IC}) : \alpha q_L - q_L^2 - c(q_L + q_H)q_L - t_L \geq \alpha q_H - q_H^2 - c(q_H + q_H)q_H - t_H, \tag{4}$$
$$(\overline{IC}) : q_H - q_H^2 - c(q_L + q_H)q_H - t_H \geq q_L - q_L^2 - c(q_L + q_L)q_L - t_L. \tag{5}$$

and

$$(\underline{IR}) : \alpha q_L - q_L^2 - c(q_L + q_H)q_L \geq t_L, \tag{6}$$
$$(\overline{IR}) : q_H - q_H^2 - c(q_L + q_H)q_H \geq t_H. \tag{7}$$

To ensure the existence of an interior solution, we assume that the congestion cost is not too high.[5] Assumption 2.2 holds for the remainder of the paper.

Assumption 2.2. $c \in \left[0, \frac{2}{9}(1 + \sqrt{10})\right]$, *such that* $4(1 + c) - 9c^2 > 0$.

Proposition 2.3 provides optimal packages under second-degree price discrimination. The types of packages offered to the buyers depend on the relative valuation of buyer L and the congestion cost parameter. The optimal packages stated in Proposition 2.3 embed the standard case with no congestion, i.e., $c = 0$. Throughout the paper, we study the effect of congestion by comparing our results to the benchmark case $c = 0$.

[5]See Footnote 16 in Appendix A.

Proposition 2.3. *Under second-degree price discrimination,*

1. *For* $\alpha \in \left(0, \frac{2+5c}{4(1+c)}\right]$, *the seller offers* $\{q_L^{**}, t_L^{**}\} = \{0, 0\}$ *and*

$$\{q_H^{**}, t_H^{**}\} = \left\{\frac{1}{2(1+c)}, \frac{1}{4(1+c)}\right\}. \tag{8}$$

2. *For* $\alpha \in \left(\frac{2+5c}{4(1+c)}, \frac{2(1+2c)}{2+5c}\right)$, *the seller offers*[6]

$$q_L^{**} = \frac{2(2\alpha - 1) - (5 - 4\alpha)c}{4(1+c) - 9c^2}, \tag{9}$$

and

$$q_H^{**} = \frac{2 - 3(2\alpha - 1)c}{4(1+c) - 9c^2}. \tag{10}$$

3. *For* $\alpha \in \left[\frac{2(1+2c)}{2+5c}, 1\right)$, $c > 0$ *the seller offers the same package to both buyers,*

$$\{q^{**}, t^{**}\} = \left\{\frac{\alpha}{2(1+2c)}, \frac{\alpha^2}{4(1+2c)}\right\}, \tag{11}$$

Proof. See Appendix A. □

To see more clearly how optimal levels of quality are influenced by the parameters of the model, we plot q_L^{**} and q_H^{**} as functions of α and c. Figure 1 provides both a contour plot and a three-dimensional view of q_L^{**} and q_H^{**}. The three-dimensional view gives information about the magnitude of the effects of the parameters on optimal levels of quality. For any buyer, regardless of congestion (and as long as a buyer is offered a positive level of quality), the level of quality is decreasing in the congestion cost. Moreover, an increase in buyer L's valuation increases his level of quality.

[6]The expressions for t_L^{**} and t_H^{**} are cumbersome and omitted for this case. See Appendix A.

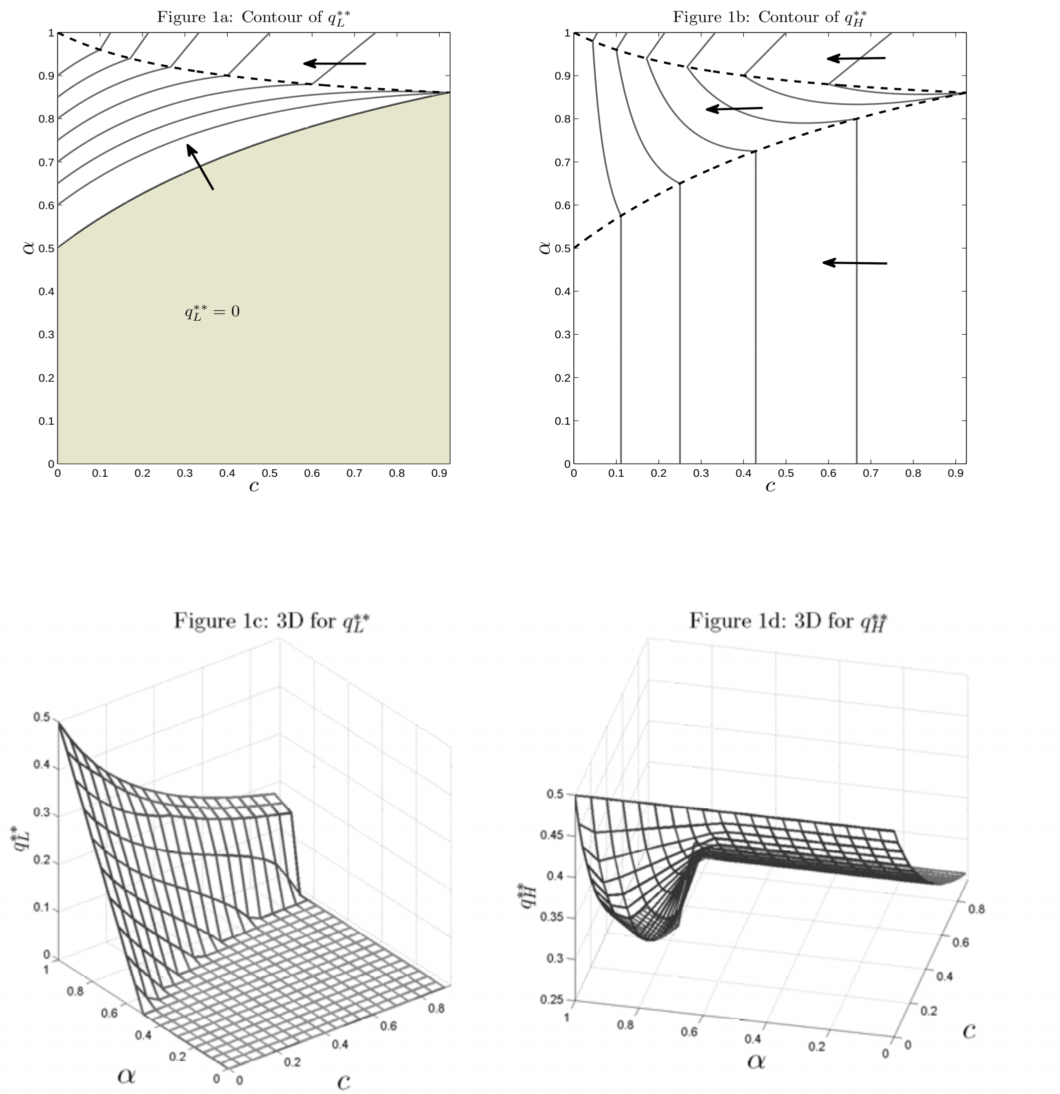

Figure 1: Optimal Levels of Quality. For Figures 1a and 1b, a contour plot reads similarly to an indifference curve in a utility graph. A curve on the graph regroups the set of pairs $\{\alpha, c\}$ that yields identical optimal quality of service. The arrows indicates the direction for an increase in the value of levels of quality. Figures 1c and 1d complement Figures 1a and 1b by providing a three-dimensional view of the levels of quality.

Congestion adds a link between buyer L's valuation and the level of quality received by buyer H. Indeed, when both buyers are offered positive levels of quality, then an increase in buyer L's valuation reduces the level of quality offered to buyer H. Indeed, while an increase in α induces the seller to offer a higher level of quality to buyer L, it also increases the congestion cost for both buyers, which reduces the level of quality offered to buyer H.

Remark 2.4. *From Proposition 2.3, for $c > 0$ and $\alpha \in \left(\frac{2+5c}{4(1+c)}, \frac{2(1+2c)}{2+5c}\right)$, $\frac{\partial q_H^{**}}{\partial \alpha} < 0$.*

Having characterized analytically and graphically the optimal packages offered by the seller, we now discuss how congestion affects the seller's decision to effectively price discriminate (i.e., to offer distinct packages) and to service both buyers. Graphically, from Figure 2, the set of values for α for which the seller offers different and positive levels of quality is shrinking in c and vanishes for large values of c.

First, Remark 2.5 states that the presence of congestion (i.e., $c > 0$) affects the seller's ability to offer distinct packages. Although the seller never offers a greater level of quality to the low-valuation buyer, it could be the same.[7] Consistent with Proposition 2.3, under second-degree price discrimination with a congestion cost, the seller cannot offer two distinct qualities for higher values of α, and hence $q_L^{**} = q_H^{**}$. If buyers are *similar* (but not identical) in their valuation of the good, the seller cannot discriminate and offers the same package to both buyers. For intermediate values of α, the seller does offer two different packages while servicing both buyers, i.e., $q_H^{**} > q_L^{**}$. Finally, for low values of α, the seller excludes the low-valuation buyer.

[7] As depicted in Figures 6 and 7 in Appendix D, an increase in the parameter for congestion cost or an increase in buyer L's valuation reduces the difference between the levels of quality. While the seller never offers a better level of quality to buyer L, the distance between the two levels of quality depends on buyer L's valuation and the congestion cost parameter.

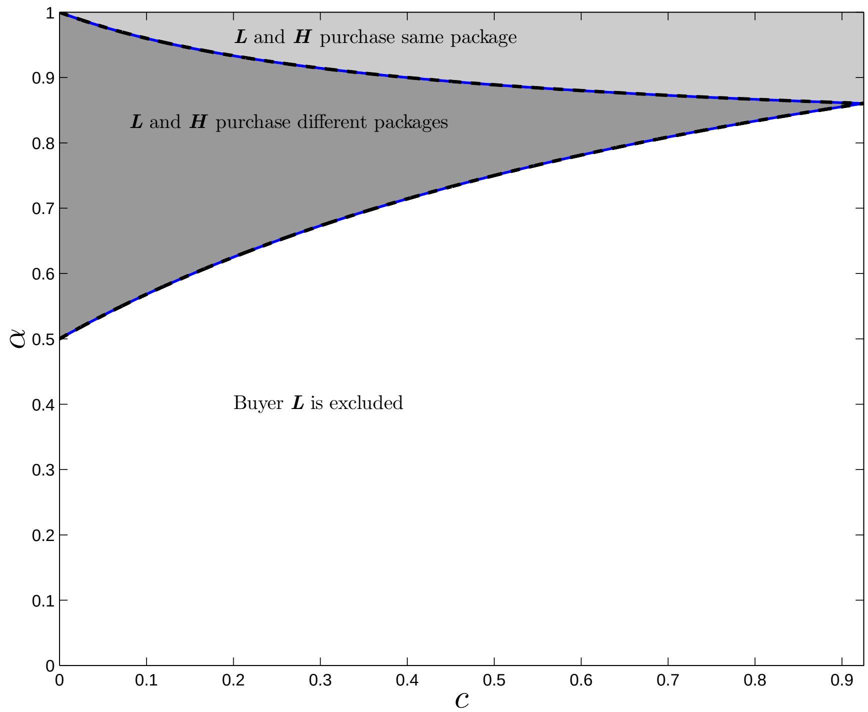

Figure 2: Types of Packages. The seller offers different types of packages depending on the values for the pair $\{\alpha, c\}$. The dotted lines separate the three cases. Consistent with Proposition 2.3, the increasing concave dotted line is $\alpha = \frac{2+5c}{4(1+c)}$, while the decreasing convex dotted line is $\alpha = \frac{2(1+2c)}{2+5c}$. The area **buyer** L **is excluded** regroups the set of pairs $\{\alpha, c\}$ for which $q_H^{**} > q_L^{**} = 0$. The area L **and** H **consume different packages** regroups the set of pairs $\{\alpha, c\}$ for which $q_H^{**} > q_L^{**} > 0$. The area L **and** H **consume same package** regroups the set of pairs $\{\alpha, c\}$ for which $q_H^{**} = q_L^{**} > 0$.

Remark 2.5. *From Proposition 2.3,* $0 \leq q_L^{**} \leq q_H^{**} < 1/2$*. Moreover, for* $c > 0$*,*

1. *If* $\alpha \in \left(0, \frac{2(1+2c)}{2+5c}\right)$*, then two different levels of quality are offered, i.e.,* $q_L^{**} < q_H^{**}$*.*
2. *If* $\alpha \in \left[\frac{2(1+2c)}{2+5c}, 1\right)$*, then only one type of quality is offered to both buyers, i.e.,* $q_L^{**} = q_H^{**}$*.*

To explain the seller's decision to offer the same package with heterogeneous buyers, recall Remark 2.4. When there is congestion and both buyers are serviced, the level of quality offered to buyer H is decreasing in buyer L's valuation. For high enough values of α, this negative externality on buyer H implies that the levels of quality are arbitrarily close to each other so that the seller offers only one type of package to both buyers.[8] See also Figure 6 in Appendix D.

Remark 2.6 states that the seller's ability to serve both buyers depends on the congestion cost. An increase in the congestion cost c increases the threshold for buyer L to be excluded. See also Figure 7 in Appendix D.

Remark 2.6. *From Proposition 2.3, an increase in the congestion cost makes it more likely for buyer L to be excluded, i.e., it increases the range of values for* α *for which buyer L is excluded.*

3 Effect of Congestion

Having characterized and discussed how the seller's behavior changes with congestion, we now study the effect of congestion cost on efficiency and consumer surplus.

3.1 Efficiency

We show that a positive cost of congestion has a profound effect on efficiency, i.e., in general no buyer receives the efficient allocation. We begin by

[8]Offering a higher quality to buyer L than to buyer H is not optimal for the seller. See Appendix A.

providing the optimal levels of quality under first-degree (perfect) price discrimination in Proposition 3.1. We then compare perfect and second-degree price discrimination.

Proposition 3.1. *Under perfect price discrimination,* $q_H^* > q_L^* \geq 0$ *such that*

1. *For* $\alpha \in \left(0, \frac{c}{1+c}\right]$, $q_L^* = 0$ *and*

$$q_H^* = \frac{1}{2(1+c)}. \tag{12}$$

2. *For* $\alpha \in \left(\frac{c}{1+c}, 1\right)$,

$$q_L^* = \frac{\alpha - (1-\alpha)c}{2(1+2c)}, \tag{13}$$

$$q_H^* = \frac{1 + (1-\alpha)c}{2(1+2c)}. \tag{14}$$

Proof. See Appendix B for a full-characterization and the proof for the case of perfect price discrimination. □

Proposition 3.2 states that when the seller services both buyers under perfect and second-degree price discrimination, there is in general a distortion for buyer H.[9] The inefficiency due to congestion can either increase or decrease quality depending on the values of the parameters.

[9]When buyer L is excluded under second-degree price discrimination, then $q_H^{**} > q_H^*$ and $0 = q_L^{**} \leq q_L^*$ with equality in some cases.

Proposition 3.2. *Suppose that the seller services both buyer under perfect and second-degree price discrimination, i.e.,* $\alpha \in \left(\frac{2+5c}{4(1+c)}, 1\right)$. *Then, for* $c > 0$,[10]

1. *For* $\alpha \in \left(\frac{2+5c}{4(1+c)}, \frac{17c+9c^2+6}{20c+9c^2+8}\right)$, $q_H^{**} > q_H^*$.
2. *For* $\alpha = \frac{17c+9c^2+6}{20c+9c^2+8}$, $q_H^{**} = q_H^*$.
3. *For* $\alpha \in \left(\frac{17c+9c^2+6}{20c+9c^2+8}, 1\right)$, $q_H^{**} < q_H^*$.

Proof. From Propositions 2.3 and 3.1, comparing (10) and (14) yields the conditions stated in Proposition 3.2. □

Proposition 3.3 states the direction of the inefficiency for buyer L's level of quality. When the seller offers the same package to both buyers under second-degree price discrimination, the distortion is downward (i.e., $q_L^{**} < q_L^*$). However, when the seller offers different packages under second-degree price discrimination, the distortion is upward (i.e., $q_L^{**} > q_L^*$).

Proposition 3.3. *Suppose that the seller services both buyers under perfect and second-degree price discrimination, i.e.,* $\alpha \in \left(\frac{2+5c}{4(1+c)}, 1\right)$. *Then, for* $c > 0$,[11]

1. *For* $\alpha \in \left(\frac{2+5c}{4(1+c)}, \frac{2*1+2c)}{2+5c}\right)$, $q_L^{**} < q_L^*$.
2. *For* $\alpha \in \left(\frac{2*1+2c)}{2+5c}, 1\right)$, $q_L^{**} > q_L^*$.

Proof. From Propositions 2.3 and 3.1, comparing (9) and (13) yields the conditions stated in Proposition 3.3. □

The conditions stated in Propositions 3.2 and 3.3 are depicted in Figures 3 and 4. Specifically, Figures 3 and 4 orders the pair $\{q_H^{**}, q_H^*\}$ and $\{q_H^{**}, q_H^*\}$, respectively for different values of the pair $\{\alpha, c\}$.[12] As in Figure 2, the dotted lines represent the boundaries for the different types of packages offered

[10] The conditions stated below are not valid when evaluated at $c = 0$.

[11] The conditions stated below are not valid when evaluated at $c = 0$.

[12] See Figures 8 and 9 in Appendix D for the size of these differences for buyer H and buyer L, respectively.

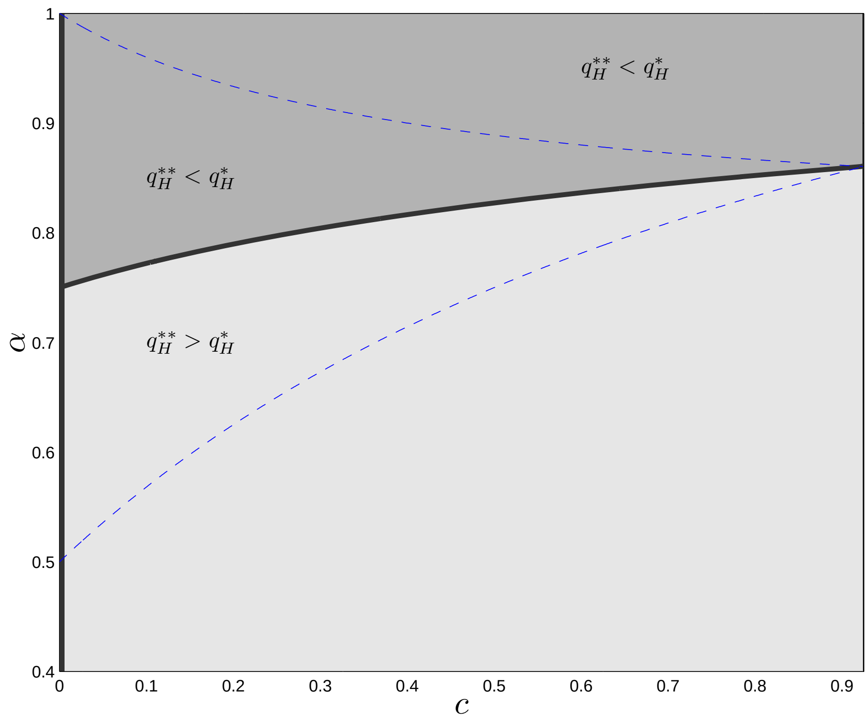

Figure 3: Efficiency for Buyer H's Level of Quality. The area $q_H^{**} < q_H^*$ regroups the set of pairs $\{\alpha, c\}$ for which the inefficiency decreases buyer H's level of quality. The area $q_H^{**} > q_H^*$ regroups the set of pairs $\{\alpha, c\}$ for which the inefficiency increases buyer H's level of quality. The thick solid increasing concave line represents the case in which there is efficiency with congestion. There is also efficiency for the benchmark case of no congestion at $c = 0$.

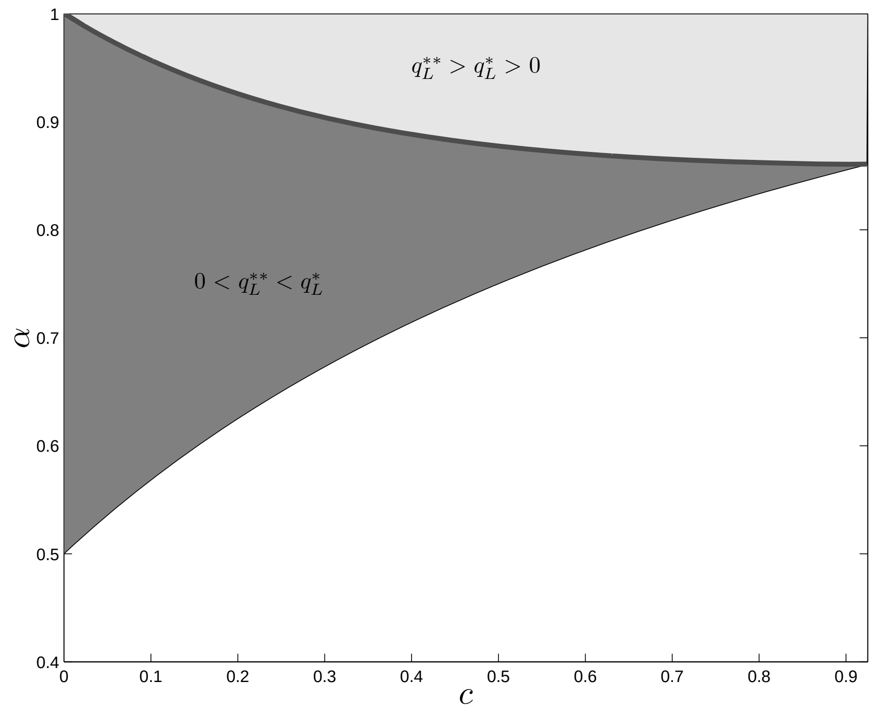

Figure 4: Efficiency for Buyer L's Level of Quality. The area $q_L^{**} > q_L^* > 0$ regroups the set of pairs $\{\alpha, c\}$ for which the inefficiency increases buyer L's level of quality. The area $0 < q_L^{**} < q_L^*$ regroups the set of pairs $\{\alpha, c\}$ for which the inefficiency decreases buyer L's level of quality including the benchmark case of no congestion for $c = 0$, and $\alpha \in (1/2, 1)$. The thick solid decreasing convex line represents the case in which there is efficiency

under second-degree price discrimination. Above the dotted increasing convex line, both buyers are offered positive levels of quality under perfect and second-degree price discrimination. Past the dotted decreasing concave line, as stated earlier, second-degree price discrimination implies that the seller offers the same package. The thick solid line represents the case in which the outcome is efficient.

3.2 Consumer Surplus

Having discussed the issue of efficiency by comparing first-degree and second-degree price discrimination, we now examine whether price discrimination can enhance consumer surplus when there is congestion. To that end, we compare second-degree price discrimination with the case in which the seller does not price discriminate. See Appendix C for the full characterization and discussion of optimal packages under no price discrimination. We show that consumer surplus (through an increase in buyer H's consumer surplus since buyer L's consumer surplus is always zero regardless of price discrimination) may increase when the seller price discriminates. Figure 5 depicts the effect of price discrimination on consumer surplus for different values of α and c. The hat sign refers to the case of no price discrimination. Let $CS^{**} = u_H^{**} - t_H^{**}$ and $\widehat{CS} = \hat{u}_H - \hat{t}_H$ be the consumer surplus under second-degree price discrimination and no price discrimination, respectively.[13]

Remark 3.4 states that when the seller offers different packages to both buyers under second-degree price discrimination (recall Figure 2) and both buyers are serviced under no price discrimination (i.e., areas $\mathcal{A}_2$ and $\mathcal{A}_3$ in Figure 5), the effect on consumer surplus may be positive or negative. In areas $\mathcal{A}_2$ and $\mathcal{A}_3$, price discrimination yields an increase in buyer H's level of quality and a decrease in buyer L's level of quality, while the overall quality (i.e., the sum of the two levels of quality) increases. The increase in buyer H's level of quality leads to an increase in his gross benefits (i.e., $u_H - t_H$). The increase in overall quality leads to an increase as well in the congestion cost. In area $\mathcal{A}_2$, buyer H's gross benefit increases more than the congestion

[13] Recall that $u_L^{**} - t_L^{**} = \hat{u}_L - \hat{t}_L = 0$.

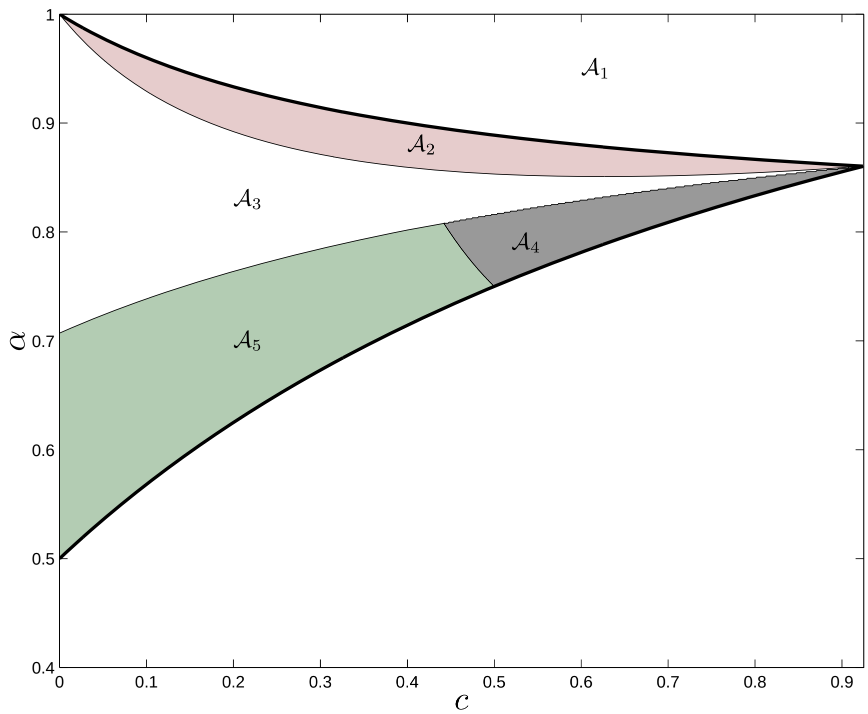

Figure 5: Effect of Price Discrimination on Consumer Surplus. For $i = 1, 2, 3, 5$, area $\mathcal{A}_i$ regroups the set of pairs $\{\alpha, c\}$ for different cases for the effect of congestion on consumer surplus. Area $\mathcal{A}_1$: Buyer L is never excluded, $q_H^{**} = q_L^{**} = \hat{q}$, and $CS^{**} = \widehat{CS}$. Area $\mathcal{A}_2$: Buyer L is never excluded, $q_H^{**} > \hat{q} > q_L^{**}$, and $CS^{**} > \widehat{CS}$. Area $\mathcal{A}_3$: Buyer L is never excluded, $q_H^{**} > \hat{q} > q_L^{**}$, and $CS^{**} < \widehat{CS}$. Areas $\mathcal{A}_4$ and $\mathcal{A}_5$: Buyer L is excluded in the case of no price discrimination, $q_H^{**} < \hat{q}, q_L^{**} > 0$, and $CS^{**} > \widehat{CS}$.

cost, and, thus, consumer surplus increases. In area $\mathcal{A}_3$, buyer H's gross benefit increases less than the congestion cost, and, thus, consumer surplus decreases.

Remark 3.4. *From Figure 5, when both buyers are serviced under no price discrimination and the seller offers different packages under second-degree price discrimination, then price discrimination*

1. *increases consumer surplus in area* $\mathcal{A}_2$*, and*
2. *decreases consumer surplus in area* $\mathcal{A}_3$*.*

Remark 3.5 states that the possibility to price discriminate may have no effect on consumer surplus. Since with congestion, the seller may decide to offer the same package to both buyers under second-degree price discrimination (Recall Figure 2), and, thus, there is no difference with the case of no price discrimination.

Remark 3.5. *From Figure 5, when both buyers are serviced under no price discrimination, second-degree price discrimination has no effect on consumer surplus in area* $\mathcal{A}_1$*.*

Remark 3.6 recalls the standard positive effect of price discrimination on consumer surplus when the seller excludes buyer L under no price discrimination. This effect is present regardless of congestion.

Remark 3.6. *From Figure 5, when buyer L is excluded under no price discrimination, second-degree price discrimination increases consumer surplus in areas* $\mathcal{A}_4$ *and* $\mathcal{A}_5$*.*

In area $\mathcal{A}_4$, the increase is due to an increase of the gross benefit and, if there is congestion, a decrease of the congestion cost. Here, buyer L is offered a higher level of quality while buyer H's level of quality is reduced under second-degree price discrimination (compared to no price discrimination). Since the congestion cost borne by buyer H is more affected by his own level of quality than buyer L's level of quality, congestion cost decreases for buyer H. The fee charged to buyer H is also reduced so that gross benefit increases. In area $\mathcal{A}_4$, and in no price discrimination, buyer L is excluded, and the seller extracts most of buyer H's surplus. Under second-degree price discrimination, buyer L is not excluded, and the seller captures less of buyer H's utility. The overall result in area $\mathcal{A}_4$ is an increase in the consumer surplus. In area $\mathcal{A}_5$, both gross benefit and congestion cost increase with price discrimination. In comparison with area $\mathcal{A}_4$, α is relatively low. Therefore, the variation of q_L^{**} and q_H^{**} observed in area $\mathcal{A}_4$ is attenuated in area $\mathcal{A}_5$.[14] As a consequence, the congestion cost increases. However, the increase in the gross benefit overcomes the increase in the congestion cost.

4 Final Remarks

In this paper, we provide an analysis of second-degree price discrimination with congestion. We show that the seller does not always provide distinct contracts (i.e., it is not always optimal to price discriminate). We also show that congestion makes it impossible to obtain efficient allocations for any buyers. Finally, with congestion and for values of the parameters for which all types are serviced, consumer surplus under second-degree price discrimination may be greater than consumer surplus under no price discrimination.

The existence of a region of the parameter space for which second-degree price discrimination can improve welfare suggests there is a need for empirical work studying markets for services where the benefit that consumers derive from the service can be limited by congestion. The market for residential broadband service is an example. Often these localized markets are characterized by a monopolist practicing second-degree price discrimination where packages are vertically differentiated (e.g., connection speeds or usage allowances). This market has also been characterized by rapid growth in the intensity with which consumers utilize the service, which makes congestion a bigger concern. The applicability of our results suggesting that consumers may benefit from price discrimination, along with the many important policy debates (e.g., net-neutrality) and recent consolidation (e.g., Comcast-Time

[14] Remember that q_L^{**} is increasing in α and q_H^{**} is decreasing in α.

Warner), make this an exciting market for future analysis. However, to date very little is known about the efficiency properties of second-degree price discrimination in this important market.[15]

As a first step in studying screening with congestion, we have abstracted from the buyer's usage decision once usage allowance is chosen and ignored a richer pricing scheme with a variable component linked to usage. Future research should consider a three-stage game in which decisions on usage allowance and usage are split. In the first stage, the seller offers several packages. In the second stage, the buyers choose one of the packages. In the third stage, the consumers interact strategically by choosing their usage levels. This would further our understanding on price discrimination when buyers choose both a type of service and a level of consumption when there is congestion.

[15] There are a few notable exceptions (Varian, 2001; Goolsbee and Klenow, 2006; Lambrecht et al., 2007).

A Second-Degree Price Discrimination

In this appendix, we provide a proof of Proposition 2.3. Using (4) and (5), it follows that

$$\begin{aligned} &\alpha q_L - q_L^2 - c q_L(q_L + q_H) - t_L \\ &+ q_H - q_H^2 - c q_H(q_L + q_H) - t_H \\ &\geq \alpha q_H - q_H^2 - c q_H(q_H + q_H) - t_H \\ &+ q_L - q_L^2 - c q_L(q_L + q_L) - t_L, \end{aligned} \tag{15}$$

so that

$$(q_H - q_L)(1 - \alpha) + c(q_H - q_L)^2 \geq 0. \tag{16}$$

If $c = 0$, then, from (16), $q_H \geq q_L$. If $c > 0$, then we need to consider three cases. 1) If $q_H > q_L$, then (16) implies that

$$q_H > q_L - \frac{1 - \alpha}{c}, \tag{17}$$

which is always true when $q_H \geq q_L$. 2) If $q_H = q_L$, then (16) holds. 3) If $q_H < q_L$, then (16) implies that the condition

$$q_H + \frac{1 - \alpha}{c} < q_L \tag{18}$$

must hold as well.

1. Suppose first that $q_H > q_L$.

   (a) At the optimum, (6) is active. To see this, using (5), $\alpha \in (0, 1)$, and $q_H > q_L$, it follows that

$$q_H - q_H^2 - c q_H(q_L + q_H) - t_H \geq q_L - q_L^2 - c q_L(q_L + q_L) - t_L, \tag{19}$$

$$\geq \alpha q_L - q_L^2 - c q_L(q_L + q_H) - t_L. \tag{20}$$

Suppose to the contrary that (6) is inactive, i.e., $\alpha q_L - q_L^2 - cq_L(q_L + q_H) - t_L > 0$, then so is (7). This cannot be an optimum since t_L and t_H can be increased without any effect on incentive compatibility and individual rationality. Hence, (6) is active, i.e.,

$$t_L = \alpha q_L - q_L^2 - cq_L(q_L + q_H), \tag{21}$$
$$= \alpha q_L - (1 + c)q_L^2 - cq_L q_H \tag{22}$$

at the optimum.

(b) At the optimum, (5) is active. To see this, using (19) and (22),

$$q_H - q_H^2 - cq_H(q_L + q_H) - t_H \geq q_L - q_L^2 - cq_L(q_L + q_L) - t_L, \tag{23}$$
$$\geq \alpha q_L - q_L^2 - cq_L(q_L + q_H) - t_L = 0. \tag{24}$$

Suppose to the contrary that (5) is inactive, i.e., $q_H - q_H^2 - cq_H(q_L + q_H) - t_H > q_L - q_L^2 - cq_L(q_L + q_L) - t_L$, then t_H can be increased without any effect on incentive compatibility or individual rationality. Hence, from (5),

$$q_H - q_H^2 - cq_H(q_L + q_H) - t_H = q_L - q_L^2 - cq_L(q_L + q_L) - t_L, \tag{25}$$

so that

$$t_H = q_H - (1 + c)q_H^2 - (1 - \alpha)q_L + cq_L^2 - 2cq_H q_L \tag{26}$$

at the optimum.

(c) Expressions (4) and (7) can be neglected.

(d) Plugging (22) and (26) into (3) yields

$$\Pi^{**}(\alpha, c) = \max_{q_L, q_H \geq 0} \alpha q_L - (1+c)q_L^2 - cq_Lq_H + q_H - (1+c)q_H^2 - (1-\alpha)q_L + cq_L^2 - 2cq_Hq_L, \tag{27}$$

$$= \max_{q_L, q_H \geq 0} (2\alpha - 1)q_L - q_L^2 + q_H - (1+c)q_H^2 - 3cq_Hq_L. \tag{28}$$

We consider two subcases.

- Suppose first that $\alpha \in \left(\frac{2+5c}{4(1+c)}, \frac{2(1+2c)}{2+5c}\right)$. Then, the first-order conditions are[16]

$$\frac{\partial}{\partial q_L} : 2\alpha - 1 - 2q_L - 3cq_H = 0, \tag{30}$$

$$\frac{\partial}{\partial q_H} : 1 - 2(1+c)q_H - 3cq_L = 0. \tag{31}$$

  Solving (30) and (31) yields (9) and (10) such that $q_H^{**} > q_L^{**} > 0$ when $\alpha \in \left(\frac{2+5c}{4(1+c)}, \frac{2(1+2c)}{2+5c}\right)$.[17]

- Suppose next that $\alpha \in \left(0, \frac{2+5c}{4(1+c)}\right]$. Then, $q_L^{**} = 0$, and, from (28) evaluated at $q_L = 0$, the first-order condition is

$$\frac{\partial}{\partial q_H} : 1 - 2(1+c)q_H = 0, \tag{32}$$

  which yields $q_H^{**} = \frac{1}{2(1+c)}$, so that, using (26), $t_H^{**} = \frac{1}{4(1+c)}$, as in (8)

2. Before considering the case $\alpha \in \left[\frac{2(1+2c)}{2+5c}, 1\right)$, we show that $q_H < q_L$ is not possible in equilibrium. Suppose to the contrary that $q_H < q_L$.

[16] Given Assumption 2.2, the Hessian matrix

$$\mathcal{H} = \begin{bmatrix} -2 & -3c \\ -3c & -2(1+c) \end{bmatrix} \tag{29}$$

is negative definite.

[17] Here, $\alpha > \frac{2+5c}{4(1+c)}$ implies that $q_L^{**} > 0$, while $\alpha < \frac{2(1+2c)}{2+5c}$ implies that $q_H^{**} > q_L^{**}$.

(a) At the optimum, (7) is active. To see this, using (4) and the fact that $\alpha \in (0,1)$ and that (18) must hold in this case, it follows that

$$\alpha q_L - q_L^2 - cq_L(q_L + q_H) - t_L \geq \alpha q_H - q_H^2 - cq_H(q_H + q_H) - t_H, \tag{33}$$

$$> q_H - q_H^2 - cq_H(q_H + q_L) - t_H. \tag{34}$$

Suppose to the contrary that (7) is inactive, i.e., $q_H - q_H^2 - cq_H(q_H + q_L) - t_H > 0$, then so is (6). This cannot be an optimum since t_L and t_H can be increased without any effect on incentive compatibility and individual rationality. Hence, (7) is active, i.e.,

$$t_H = q_H - q_H^2 - cq_H(q_H + q_L), \tag{35}$$

$$= q_H - (1+c)q_H^2 - cq_L q_H \tag{36}$$

at the optimum.

(b) At the optimum, (4) is active. To see this, using (34) and (36),

$$\alpha q_L - q_L^2 - cq_L(q_L + q_H) - t_L \geq \alpha q_H - q_H^2 - cq_H(q_H + q_H) - t_H, \tag{37}$$

$$\geq q_H - q_H^2 - cq_H(q_H + q_L) - t_H = 0. \tag{38}$$

Suppose to the contrary that (4) is inactive, i.e., $\alpha q_L - q_L^2 - cq_L(q_L + q_H) - t_L > \alpha q_H - q_H^2 - cq_H(q_H + q_H) - t_H$, then t_L can be increased without any effect on incentive compatibility or individual rationality. Hence,

$$\alpha q_L - q_L^2 - cq_L(q_L + q_H) - t_L = \alpha q_H - q_H^2 - cq_H(q_H + q_H) - t_H, \tag{39}$$

so that

$$t_L = \alpha q_L - (1+c)q_L^2 + (1-\alpha)q_H + cq_H^2 - 2cq_L q_H. \tag{40}$$

(c) Plugging (36) and (40) into (3) yields

$$\Pi^{**}(\alpha, c) = \max_{q_L, q_H} \alpha q_L - (1+c)q_L^2 + (1-\alpha)q_H + cq_H^2 - 2cq_Lq_H$$
$$+ q_H - (1+c)q_H^2 - cq_Lq_H, \tag{41}$$
$$= \max_{q_L, q_H} \alpha q_L - (1+c)q_L^2 + (2-\alpha)q_H - q_H^2 - 3cq_Lq_H. \tag{42}$$

We consider two subcases.

- Suppose first that $q_L^{**} > q_H^{**} > 0$. Then, the first-order conditions are

$$\frac{\partial}{\partial q_L} : \alpha - 2(1+c)q_L - 3cq_H = 0, \tag{43}$$
$$\frac{\partial}{\partial q_H} : 2 - \alpha - 2q_H - 3cq_L = 0, \tag{44}$$

so that

$$q_L^{**} = \frac{2\alpha - 3(2-\alpha)c}{4(1+c) - 9c^2}, \tag{45}$$
$$q_H^{**} = \frac{2(2-\alpha) + (4-5\alpha)c}{4(1+c) - 9c^2}. \tag{46}$$

Given (18), we need

$$q_H^{**} + \frac{1-\alpha}{c} < q_L^{**} \tag{47}$$

or $8(1-\alpha)c + (1+\alpha)c^2 + 4(1-\alpha) < 0$, which is impossible.

- Suppose next that $q_L^{**} > q_H^{**} = 0$. Then, the first-order condition is

$$\frac{\partial}{\partial q_L} : \alpha - 2(1+c)q_L = 0, \tag{48}$$

so that

$$q_L^{**} = \frac{\alpha}{2(1+c)}, \tag{49}$$

and thus profit is

$$\Pi^{**}(\alpha, c) = \frac{\alpha^2}{4(1+c)}. \tag{50}$$

However, this cannot be an optimum since the solution $q_H^{**} = \frac{1}{2(1+c)}$, $q_L^{**} = 0$, $t_L^{**} = 0$ and $t_H^{**} = \frac{1}{4(1+c)}$ yields profit $\frac{1}{4(1+c)}$ that is strictly greater than (50).

3. Suppose finally that $\alpha \in \left[\frac{2(1+2c)}{2+5c}, 1\right)$. Since $q_H^{**} > q_L^{**}$ cannot hold, and $q_H^{**} < q_L^{**}$ is not possible, it must be that $q_H^{**} = q_L^{**}$. Hence, (3) is rewritten as

$$\Pi^{**}(\alpha, c) = \max_q 2(\alpha q - (1+2c)q^2), \tag{51}$$

which yields (11).

B First-Degree Price Discrimination

In this appendix, we state and prove the optimal contract under first-degree (perfect) price discrimination. Suppose that the seller can perfectly price discriminate. Hence, he solves the following program

$$\Pi^*(\alpha, c) = \max_{q_L, q_H, t_L, t_H \geq 0} t_L + t_H, \tag{52}$$

subject to the IR (individual rationality) constraints

$$(\underline{IR}) : \alpha q_L - q_L^2 - c(q_L + q_H)q_L \geq t_L, \tag{53}$$
$$(\overline{IR}) : q_H - q_H^2 - c(q_L + q_H)q_H \geq t_H. \tag{54}$$

Proposition B.1 provides the optimal packages under perfect price discrimination.

Proposition B.1. *Under perfect price discrimination,* $q_H^* > q_L^* \geq 0$ *and* $t_H^* > t_L^* \geq 0$*, such that*

1. *For* $\alpha \in \left(0, \frac{c}{1+c}\right]$, $\{q_L^*, t_L^*\} = \{0, 0\}$ *and*

$$\{q_H^*, t_H^*\} = \left\{\frac{1}{2(1+c)}, \frac{1}{4(1+c)}\right\}, \tag{55}$$

and

2. *For* $\alpha \in \left(\frac{c}{1+c}, 1\right)$,

$$\{q_L^*, t_L^*\} = \left\{\frac{\alpha - (1-\alpha)c}{2(1+2c)}, \frac{\alpha^2 - (1-\alpha)\alpha c}{4(1+2c)}\right\}, \tag{56}$$
$$\{q_H^*, t_H^*\} = \left\{\frac{1 + (1-\alpha)c}{2(1+2c)}, \frac{1 + (1-\alpha)c}{4(1+2c)}\right\}. \tag{57}$$

Proof. Since both (53) and (54) are active, (52) is rewritten as

$$\Pi^*(\alpha, c) = \max_{q_L, q_H \geq 0} \alpha q_L - q_L^2 + q_H - q_H^2 - c(q_L + q_H)^2. \tag{58}$$

1. Suppose first that $\alpha \in \left(0, \frac{c}{1+c}\right]$. Then, $q_L^* = 0$, and, thus $t_L^* = 0$. From (58) evaluated at $q_L = 0$, the first-order condition is

$$\frac{\partial}{\partial q_H} : 1 - 2q_H - 2cq_H = 0, \tag{59}$$

which yields $q_H = \frac{1+(1-\alpha)c}{2(1+2c)}$ and $t_H^* = \frac{1+(1-\alpha)c}{4(1+2c)}$, as in (55).

2. Suppose next that $\alpha \in \left(\frac{c}{1+c}, 1\right)$. Then, the unique interior solution is characterized by the first-order conditions[18]

$$\frac{\partial}{\partial q_L} : \alpha - 2q_L - 2c(q_L + q_H) = 0, \tag{60}$$

$$\frac{\partial}{\partial q_H} : 1 - 2q_H - 2c(q_L + q_H) = 0. \tag{61}$$

Solving (60) and (61) yields q_L^* and q_H^* as in (56) and (57) such that $q_L^* > 0$ when $\alpha \in \left(\frac{c}{1+c}, 1\right)$. Plugging back q_L^* and q_H^* into (53) and (54) yields t_L^* and t_H^* as in (56) and (57).

□

[18]The Hessian matrix $\mathcal{H} = -2 \begin{bmatrix} 1+c & c \\ c & 1+c \end{bmatrix}$ is negative definite.

C No Price Discrimination

Suppose that the seller cannot price discriminate among the two buyers. Then, he proposes the same package $\{q, t\}$ to both buyers, and he solves the following program:

$$\hat{\Pi} = \max_{q,t \geq 0} t1_{[u_L(q,q) \geq t]} + t1_{[u_H(q,q) \geq t]} \tag{62}$$

Here, $1_{[\cdot]}$ is an indicator function equal to one if the statement in $[\cdot]$ is true, and zero otherwise.

Proposition C.1 provides the optimal solutions for the case in which the seller cannot price discriminate. For low values of α, the low-valuation buyer is excluded. Because the low-valuation buyer does not value the good (α is low), he will not be disposed to pay a high flat fee. As a consequence, selling to the low-valuation buyer will not be profitable for the seller. For high values of α, both buyers accept to consume the good. The presence of a congestion cost renders first-degree price discrimination more difficult. Specifically, as heterogeneity between the two buyers increases (i.e., a decrease in α), the seller finds it more costly to offer a non-zero contract to the low-valuation buyer. The reason is that offering an extra unit of quality to the low-valuation buyer yields extra revenue that is dominated by the extra congestion cost. An increase in the cost of congestion makes it more likely for the low-valuation buyer to be excluded as well.

Proposition C.1. *Suppose the seller cannot price discriminate.*

1. *For $\alpha \in \left(0, \sqrt{\frac{1+2c}{2(1+c)}}\right]$, the seller excludes the low-valuation buyer and service the high-valuation buyer by offering quality*

$$\hat{q} = \frac{1}{2(1+c)} \tag{63}$$

for a fee

$$\hat{t} = \frac{1}{4(1+c)}. \tag{64}$$

The seller's payoff (or social welfare) is

$$\hat{\Pi} = \hat{W} = \frac{1}{4(1+c)}. \tag{65}$$

2. *For* $\alpha \in \left(\sqrt{\frac{1+2c}{2(1+c)}}, 1\right)$, *the seller services both buyers by offering quality*

$$\hat{q} = \frac{\alpha}{2(1+2c)} \tag{66}$$

for a fee

$$\hat{t} = \frac{\alpha^2}{4(1+2c)}. \tag{67}$$

The seller's payoff is

$$\hat{\Pi} = \frac{\alpha^2}{2(1+2c)}, \tag{68}$$

while welfare is

$$\hat{W} = \frac{\alpha}{2(1+2c)}. \tag{69}$$

Proof. We solve the problem piecewise. The seller will not propose $\{q, t\}$ such that $u_L(q,q) < t$ and $u_H(q,q) < t$. We then distinguish two cases: $u_L(q,q) < t \leq u_H(q,q)$ and $t \leq u_L(q,q) < u_H(q,q)$.

1. *Case 1:* $u_L(q,q) < t \leq u_H(q,q)$. The low-valuation buyer does not accept the offer, so that the congestion cost is simply cq^2, then the utility of the high-valuation buyer is $u_H(q,q) = q - q^2 - cq^2 \geq t$. At the optimum, $u_H(q,q) = t$ because if $u_H(q,q) > t$ the seller can increase t without affecting the constraints. The seller's program is rewritten as

$$\hat{\Pi} = \max_{q \geq 0} q - q^2 - cq^2, \tag{70}$$

so that the first-order condition is $\frac{\partial}{\partial q} : 1 - 2q - 2cq = 0$, which yields

$$\hat{q} = \frac{1}{2(1+c)} \tag{71}$$

The buyers' utilities are $u_L(\hat{q}, \hat{q}) = 0$ and $u_H(\hat{q}, \hat{q}) = \hat{t} = \frac{1}{4(1+c)}$. The seller's profit $\hat{\Pi}$ and the social welfare $\hat{W}$ are

$$\hat{\Pi} = \hat{W} = \frac{1}{4(1+c)}. \tag{72}$$

2. *Case 2:* $t \leq u_L(q,q) < u_H(q,q)$. Both buyers accept the package $\{q,t\}$. $u_L(q,q) = \alpha q - q^2 - c(q+q)q$, and $u_H(q,q) = q - q^2 - c(q+q)q$. At the optimum, $u_L(q,q) = t$ because if $u_L(q,q) > t$ the seller can increase t without affecting the constraints. The seller's program is rewritten as

$$\hat{\Pi} = \max_{q \geq 0} 2(\alpha q - q^2 - cq(q+q)), \tag{73}$$

so that the first-order condition is $\frac{\partial}{\partial q} : \alpha - 2(1+2c)q = 0$, which yields

$$\hat{q} = \frac{\alpha}{2(1+2c)}, \tag{74}$$

Hence,

$$\hat{\Pi} = \frac{\alpha^2}{2(1+2c)}. \tag{75}$$

and

$$\hat{W} = \hat{u}_H + \hat{u}_L, \tag{76}$$

$$= \frac{\alpha}{2(1+2c)}. \tag{77}$$

3. It follows that the low-valuation buyer is excluded if and only if (72) is greater than (75), i.e., $\alpha \in \left(0, \sqrt{\frac{1+2c}{2(1+c)}}\right]$.

□

D Figures

Figures 6 and 7 provide information about the partial effects of α and c, respectively, on optimal quality under second-degree price discrimination, holding constant the other parameter.

Figures 8 and 9 provide a general three-dimensional view of the difference in levels of quality under perfect price discrimination and second-degree price discrimination.

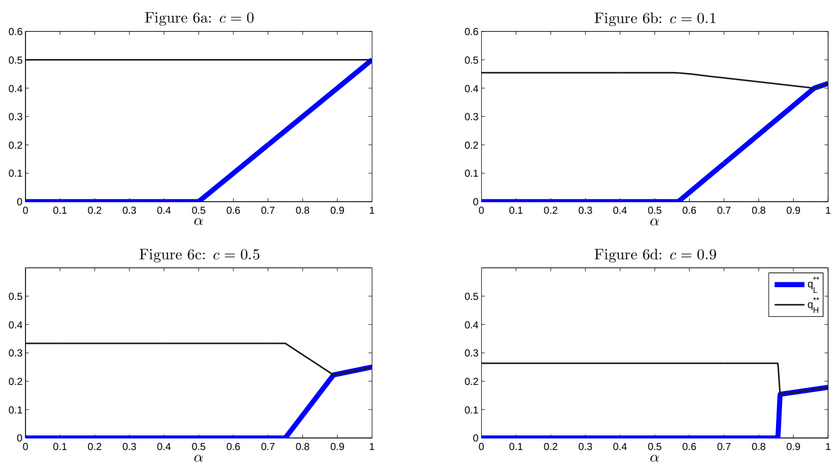

Figure 6: Effect of Buyer L's Valuation on Levels of Quality. The optimal levels of quality q_L^{**} and q_H^{**} under second-degree price discrimination are plotted as functions of $\alpha \in [0, 1]$ for different values of $c \in \{0, 0.1, 0.5, 0.9\}$. An increase in α reduces the difference in levels of quality between the two types of buyers.

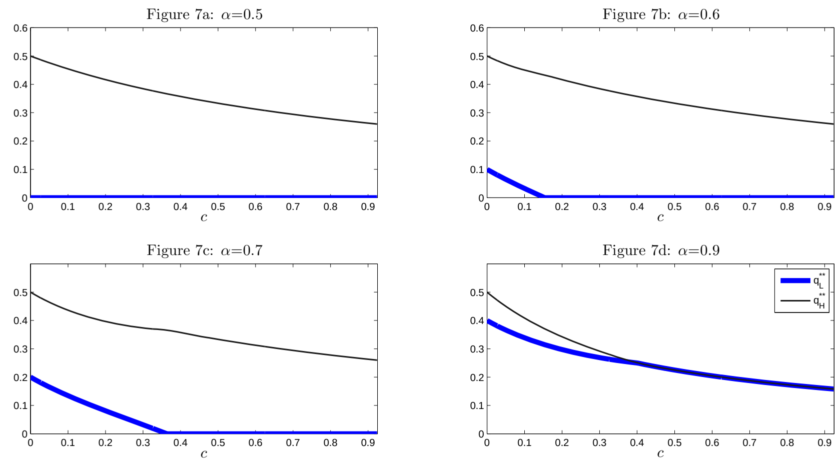

Figure 7: Effect of Congestion Cost on Levels of Quality. The optimal levels of quality q_L^{**} and q_H^{**} under second-degree price discrimination are plotted as functions of $c \in [0, \bar{c}]$ for different values of $\alpha \in \{0.5, 0.6, 0.7, 0.9\}$. An increase in c reduces the difference in levels of quality between the two types of buyers.

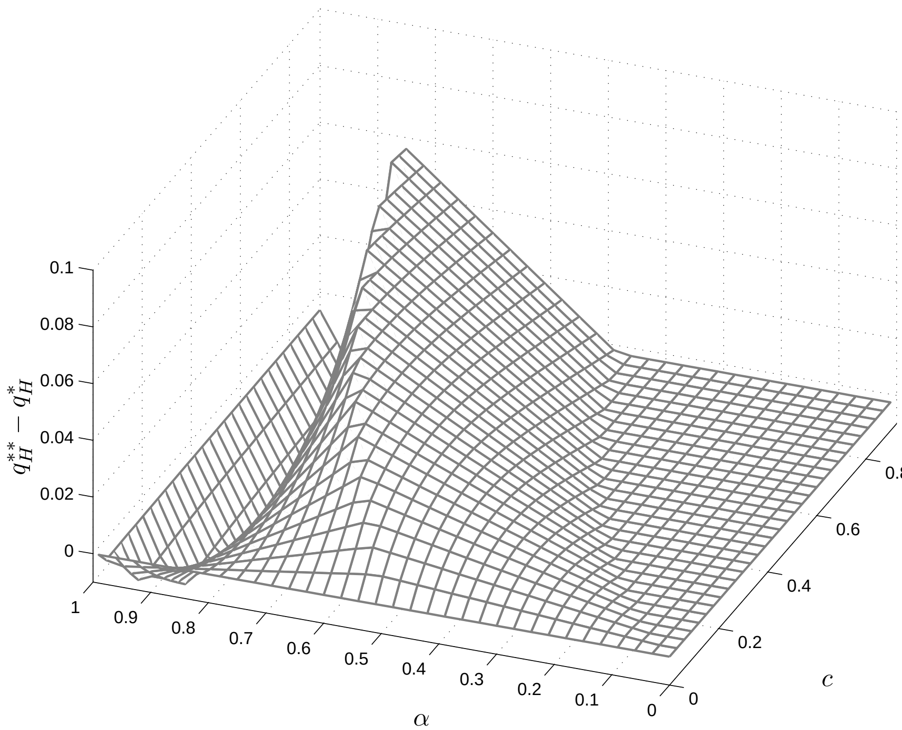

Figure 8: Efficiency for Buyer H's Level of Quality, 3D. A three-dimensional view of the difference in buyer H's levels of quality between second-degree and perfect price discrimination is provided. That is, $q_H^{**} - q_H^*$ is plotted for all values of α and c.

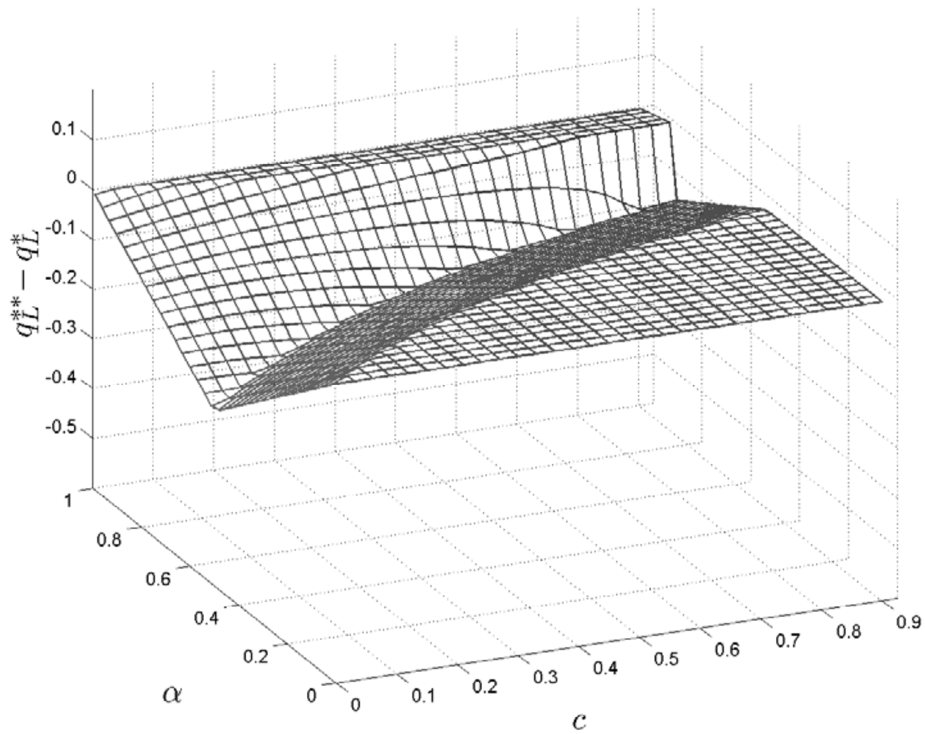

Figure 9: Efficiency for Buyer L's Level of Quality, 3D. A three-dimensional view of the difference in buyer L's levels of quality between second-degree and perfect price discrimination is provided. That is, $q_L^{**} - q_L^*$ is plotted for all values of α and c.

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