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Comparative Ross Risk Aversion in the Presence of Mean Dependent Risks

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Abstract:
This paper studies comparative risk aversion between risk averse agents in the presence of a background risk. Although the literature covers this question extensively, our contribution differs from most of the literature in two respects. First, background risk does not need to be additive or multiplicative. Second, the two risks are not necessary mean independent, and may be conditional expectation increasing or decreasing. We show that our order of cross Ross risk aversion is equivalent to the order of partial risk premium, while our index of decreasing cross Ross risk aversion is equivalent to decreasing partial risk premium. These results generalize the comparative risk aversion model developed by Ross (1981) for mean independent risks. Finally, we show that decreasing cross Ross risk aversion gives rise to the utility function family belonging to the class of *n*-switch utility functions.

Keywords: Comparative cross Ross risk aversion, Dependent background risk, Partial risk premium, Decreasing cross Ross risk aversion, *n*-switch utility function

JEL Classification: D81

1 Introduction

Arrow (1965) and Pratt (1964) propose an important theorem stating that risk aversion comparisons using risk premia and measures of risk aversion always give the same result. Ross (1981) shows that when an agent faces more than one risky variable, Arrow-Pratt measures are not strong enough to support the plausible association between absolute risk aversion and the size of the risk premium. He proposes a stronger ordering called Ross risk aversion. Several studies extend Ross' results. Most papers generalize them to higher-orders of risk aversion for univariate utility functions (see Modica and Scarsini, 2005; Jindapon and Neilson, 2007; Li, 2009; Denuit and Eeckhoudt, 2010a). This paper provides another direction to this line of research.

There is growing concern on risk attitudes of bivariate utility function in the literature (see Bleichrodt et al., 2003; Eeckhoudt et al., 2007; Courbage and Rey, 2007; Menegatti, 2009 a,b; Denuit and Eeckhoudt, 2010b; Li, 2011; Denuit et al., 2011a). To our knowledge, these studies do not analyze comparative risk aversion. The first paper that looks at preservation of "more risk averse" with general multivariate preferences and background risk is Nachman (1982). However, in his setting, the background risk is independent. Pratt (1988) also considers the comparison of risk aversion both with and without the presence of an independent background risk using a two-argument utility function.

We generalize the model of comparative risk aversion developed by Ross (1981). We introduce the notion of cross Ross risk aversion and show that more cross Ross risk aversion is associated with a higher partial risk premium in the presence of a conditional expectation increasing (or decreasing) background risk. Hence, we demonstrate that the index of cross Ross risk aversion is equivalent to the order of partial risk premium. We also propose the concept of decreasing cross Ross risk aversion and derive necessary and sufficient conditions for obtaining an equivalence between decreasing cross Ross risk aversion and decreasing partial risk premium in the presence of a conditional expectation increasing (or decreasing) background risk. We apply this result to examine the effects of changes in wealth and financial background risk on the intensity of risk aversion. Finally, we show that specific assumptions about the behavior of the decreasing cross Ross risk aversion gives rise to the utility function form that belongs to the class of n-switch utility functions (Abbas and Bell, 2011).

Our paper is organized as follows. Sections 2 and 3 offer the necessary and sufficient conditions for comparing two agents' attitudes towards risk with different utility functions and the same agent's attitude at different wealth levels under a conditional expectation increasing (or decreasing) background risk. Section 4 applies our results to financial background risks. Section 5 relates decreasing cross Ross risk aversion to n-switch independence property. Section 6 concludes the paper.

2 Comparative cross risk attitudes

We consider an economic agent whose preference for wealth, $\tilde{w}$ and a random variable, $\tilde{y}$, can be represented by a bivariate model of expected utility. We let $u(w, y)$ denote the utility function, and let $u_1(w, y)$ denote $\frac{\partial u}{\partial w}$ and $u_2(w, y)$ denote $\frac{\partial u}{\partial y}$. We follow the same subscript convention for the derivatives $u_{11}(w, y)$, $u_{12}(w, y)$ and so on, and assume that the partial derivatives required for any definition all exist and are continuous.

Pratt (1990) and Chalfant and Finkelshtain (1993) introduce the following definition of partial risk premia into the economic literature.

Definition 2.1 For u and v, the partial risk premia π_u and π_v for risk $\tilde{x}$ in the presence of risk $\tilde{y}$,are defined as

$$Eu(w + \tilde{x}, \tilde{y}) = Eu(w + E\tilde{x} - \pi_u, \tilde{y}) \tag{1}$$

and

$$Ev(w + \tilde{x}, \tilde{y}) = Ev(w + E\tilde{x} - \pi_v, \tilde{y}). \tag{2}$$

The partial risk premia π_u and π_v are the maximal monetary amounts individuals u and v are willing to pay for removing one risk in the presence of a second risk. We derive necessary and sufficient conditions for comparative partial risk premia in the presence of a conditional expectation increasing background risk. Extension of the analysis to conditional decreasing background risk is discussed later. Let us introduce two definitions of comparative risk aversion motivated by Ross (1981). The following definition uses $-\frac{u_{12}(w,y)}{u_1(w,y)}$ and $-\frac{v_{12}(w,y)}{v_1(w,y)}$ as local measures of correlation aversion.

Definition 2.2 u is more cross Ross risk averse than v if and only if there exists $\lambda_1, \lambda_2 > 0$ such that for all w, y and y'

$$\frac{u_{12}(w, y)}{v_{12}(w, y)} \geq \lambda_1 \geq \frac{u_1(w, y')}{v_1(w, y')} \tag{3}$$

and

$$\frac{u_{11}(w, y)}{v_{11}(w, y)} \geq \lambda_2 \geq \frac{u_1(w, y')}{v_1(w, y')}. \tag{4}$$

The interpretation of the sign of the second mixed derivative goes back to De Finetti (1952) and has been studied and extended by Epstein and Tanny (1980); Richard (1975); Scarsini (1988) and Eeckhoudt et al. (2007). For example, Eeckhoudt et al. (2007) show that $u_{12} \leq 0$ is necessary and sufficient for defining "correlation aversion", meaning that a higher level of the second argument mitigates the detrimental effect of a reduction in the first argument. An agent is correlation averse if she always prefers a 50-50 gamble of a loss in wealth or a loss in the second argument over another 50-50 gamble offering a loss in both arguments.

When $u(w, y) = U(w + y)$ in (3) and (4), we obtain the definition of comparative Ross risk aversion for mean independent risks. However, we are interested in comparisons when the agents face two dependent risks which is more general than mean independence. We consider the notion of conditional background risk. Two random variables are conditional risk dependent when they are not mean independent.

Definition 2.3 $\tilde{y}$ is a conditional background risk for $\tilde{x}$ if $E[\tilde{x}|\tilde{y} = y] \neq E[x]$.

The following proposition provides an equivalent comparison between risk aversion and partial risk premium in the presence of conditional increasing background risks.

Proposition 2.1 *For u, v with $u_1 > 0$, $v_1 > 0$, $v_{11} < 0$, $u_{11} < 0$, $u_{12} < 0$ and $v_{12} < 0$, the following three conditions are equivalent:*

(i) u is more cross Ross risk averse than v.

(ii) There exists $\phi : R \times R \to R$ with $\phi_1 \leq 0$, $\phi_{12} \leq 0$ and $\phi_{11} \leq 0$, and $\lambda > 0$ such that $u = \lambda v + \phi$.

(iii) $\pi_u \geq \pi_v$ for $\forall$ w and $(\tilde{x}, \tilde{y})$ such that $E[\tilde{x}|\tilde{y} = y]$ is non-decreasing in y.

Proof See the Appendix.

When an agent faces a conditional expectation increasing background risk, the cross Ross risk aversion relationship establishes an unambiguous equivalence between more risk aversion and a larger partial risk premium.

3 Decreasing cross Ross risk aversion with respect to wealth

In this section, we examine how the partial risk premium for a given risk $\tilde{x}$ is affected by a change in initial wealth w, in the presence of a dependent background risk. Fully differentiating equation (1) with respect to w yields[1]

$$Eu_1(w+\tilde{x},\tilde{y}) = Eu_1(w+E\tilde{x}-\pi_u,\tilde{y}) - \pi'(w)Eu_1(w+E\tilde{x}-\pi_u,\tilde{y}), \tag{5}$$

hence,

$$\pi'(w) = \frac{Eu_1(w+E\tilde{x}-\pi_u,\tilde{y}) - Eu_1(w+\tilde{x},\tilde{y})}{Eu_1(w+E\tilde{x}-\pi_u,\tilde{y})}. \tag{6}$$

Thus, the partial risk premium is decreasing in wealth if and only if

$$Eh(w+E\tilde{x}-\pi_u,\tilde{y}) \geq Eh(w+\tilde{x},\tilde{y}), \tag{7}$$

where $h \equiv -u_1$ is defined as minus the partial derivative of function u. Since $h_1 = -u_{11} \geq 0$, condition (7) then just states that the partial risk premium of agent h is larger than the partial risk premium of agent u. From Proposition 2.1, this is true if and only if h is more cross Ross risk averse than u. That is, $\exists \lambda_1, \lambda_2 > 0$, for all w,y and y', such that

$$\frac{h_{12}(w,y)}{u_{12}(w,y)} \geq \lambda_1 \geq \frac{h_1(w,y')}{u_1(w,y')} \tag{8}$$

and

$$\frac{h_{11}(w,y)}{u_{11}(w,y)} \geq \lambda_2 \geq \frac{h_1(w,y')}{u_1(w,y')}, \tag{9}$$

or, equivalently,

$$-\frac{u_{112}(w,y)}{u_{12}(w,y)} \geq \lambda_1 \geq -\frac{u_{11}(w,y')}{u_1(w,y')} \tag{10}$$

and

$$-\frac{u_{111}(w,y)}{u_{11}(w,y)} \geq \lambda_2 \geq -\frac{u_{11}(w,y')}{u_1(w,y')}. \tag{11}$$

Proposition 3.1 introduces $-\frac{u_{112}(w,y)}{u_{11}(w,y)}$ and $-\frac{u_{111}(w,y)}{u_{11}(w,y)}$ as local measurements of cross-prudence and prudence. These local measures of prudence are essentially identical to the measure proposed by Kimball (1990). It is well known that, for the single-risk case, DARA is equivalent to the utility function $-u'(.)$ being more concave than $u(.)$ (see for example, Gollier, 2001). Proposition 3.1 is an extension of this result to bivariate risks under a conditional expectation increasing background risk.

We obtain the following proposition:

Proposition 3.1 *For u with $u_1 > 0$, $u_{11} < 0$, $u_{12} < 0$, $u_{111} \geq 0$ and $u_{112} \geq 0$, the following three conditions are equivalent:*

[1] Equation (5) has a univariate counterpart in Eeckhoudt and Kimball (1992).

(i) the partial risk premium π_u, associated with any $(\tilde{x}, \tilde{y})$ such that $E[\tilde{x}|\tilde{y} = y]$ is non-decreasing in y, is decreasing in wealth;

(ii) There exists $\phi : R \times R \to R$ with $\phi_1 \leq 0$, $\phi_{12} \leq 0$ and $\phi_{11} \leq 0$, and $\lambda > 0$ such that $-u_1 = \lambda u + \phi$;

(iii) $\exists \lambda_1, \lambda_2 > 0$, for all w, y and y', such that

$$-\frac{u_{112}(w,y)}{u_{12}(w,y)} \geq \lambda_1 \geq -\frac{u_{11}(w,y')}{u_1(w,y')} \tag{12}$$

and

$$-\frac{u_{111}(w,y)}{u_{11}(w,y)} \geq \lambda_2 \geq -\frac{u_{11}(w,y')}{u_1(w,y')}. \tag{13}$$

The proof of Proposition 3.1 is obtained by (5) to (11).

An interpretation of the sign of u_{112} is provided by Eeckhoudt et al. (2007), who showed that $u_{112} > 0$ is a necessary and sufficient condition for "cross-prudence in its second argument", meaning that a higher level of second argument mitigates the detrimental effect of the monetary risk.

There are economic applications where negative dependence is more convenient. If $E[\tilde{x}|\tilde{y} = y]$ is non-increasing in y, then $E[-\tilde{x}|\tilde{y} = y]$ is non-decreasing in y. We can define $\bar{u}(x, y) = u(-x, y)$ and $\bar{v}(x, y) = v(-x, y)$. Then Propositions 2.1 and 3.1 can be extended to $\bar{u}(x, y)$ and $\bar{v}(x, y)$ directly. More specifically, we can propose the following results.

Proposition 3.2 *For $\bar{u}$, $\bar{v}$ with $\bar{u}_1 > 0$, $\bar{v}_1 > 0$, $\bar{u}_{11} < 0$, $\bar{v}_{11} < 0$, $\bar{u}_{12} < 0$ and $\bar{v}_{12} < 0$, the following three conditions are equivalent:*

(i) $\bar{u}$ is more cross Ross risk averse than $\bar{v}$.

(ii) There exists $\phi : R \times R \to R$ with $\phi_1 \leq 0$, $\phi_{12} \leq 0$ and $\phi_{11} \leq 0$, and $\lambda > 0$ such that $u = \lambda v + \phi$.

(iii) $\pi_{\bar{u}} \geq \pi_{\bar{v}}$ for $\forall$ w and $(\tilde{x}, \tilde{y})$ such that $E[\tilde{x}|\tilde{y} = y]$ is non-increasing in y.

and

Proposition 3.3 *For $\bar{u}$ with $\bar{u}_1 > 0$, $\bar{u}_{11} < 0$, $\bar{u}_{12} < 0$, $\bar{u}_{111} \geq 0$ and $\bar{u}_{112} \geq 0$, the following three conditions are equivalent:*

(i) the partial risk premium $\pi_{\bar{u}}$ associated with any $(\tilde{x}, \tilde{y})$ such that $E[\tilde{x}|\tilde{y} = y]$ is non-increasing in y, is decreasing in wealth;

(ii) There exists $\phi : R \times R \to R$ *with* $\phi_1 \leq 0$, $\phi_{12} \leq 0$ *and* $\phi_{11} \leq 0$, *and* $\lambda > 0$ *such that* $-u_1 = \lambda u + \phi$;

(iii) $\exists \lambda_1, \lambda_2 > 0$, *for all* w, y *and* y', *such that*

$$-\frac{\bar{u}_{112}(w,y)}{\bar{u}_{12}(w,y)} \geq \lambda_1 \geq -\frac{\bar{u}_{11}(w,y')}{\bar{u}_1(w,y')} \tag{14}$$

and

$$-\frac{\bar{u}_{111}(w,y)}{\bar{u}_{11}(w,y)} \geq \lambda_2 \geq -\frac{\bar{u}_{11}(w,y')}{\bar{u}_1(w,y')}. \tag{15}$$

4 Comparative risk aversion in the presence of a financial background risk

In the economic literature, the financial background risk has received much attention. For additive financial background risk, we refer to Doherty and Schlesinger (1983a,b, 1986), Kischka (1988), Eeckhoudt and Kimball (1992), Eeckhoudt and Gollier, (2000), Schlesinger (2000), Gollier (2001), Eeckhoudt et al. (2007) and Franke et al. (2011). For multiplicative financial background risk, see Franke et al. (2006, 2011). In this section, we consider some examples to illustrate the use of Propositions 2.1 and 3.1 in the framework of additive or multiplicative background risks.

4.1 Additive background risk

First, we show that Proposition 2.1 allows us to extend the results of Ross (1981) for an additive background risk. Note that, for an additive background risk $\tilde{y}$, we have

$$u(w,y) = U(w+y) \tag{16}$$

and

$$v(w,y) = V(w+y). \tag{17}$$

Here w can be interpreted as the random wealth of an agent and y as a random increment to wealth, i.e., random income or financial portfolio.

Since,

$$u_1 = U' \quad , \quad u_{11} = u_{12} = U'' \;\; and \;\; u_{111} = u_{112} = U''' \tag{18}$$

and

$$v_1 = V' \quad , \quad v_{11} = v_{12} = V'' \;\; and \;\; v_{111} = v_{112} = V'''. \tag{19}$$

Ross (1981) proposed the following results

Proposition 4.1 *(Ross (1981, Theorem 3)) For $u(w,y) = U(w+y)$, $v(w,y) = V(w+y)$ with $U' > 0$, $V' > 0$, $U'' < 0$ and $V'' < 0$, the following two conditions are equivalent:*

(i) $\exists \lambda > 0$
$$\frac{U''(w+y)}{V''(w+y)} \geq \lambda \geq \frac{U'(w+y')}{V'(w+y')} \quad for \;\; all \;\; w \;, y \; and \; y'. \tag{20}$$

(ii) $\pi_u \geq \pi_v$ *for* $\forall$ w, *any zero-mean risk* $\tilde{x}$ *and* $\tilde{y}$ *with* $E[\tilde{x}|\tilde{y} = y] = E\tilde{x} = 0$.

Proposition 4.2 *(Ross (1981, Theorem 4)) For $u(w,y) = U(w+y)$, with $U' > 0$, $U'' < 0$ and $U''' > 0$, the partial risk premium associated to any zero-mean risk $\tilde{x}$ with $E[\tilde{x}|\tilde{y} = y] = 0$ is decreasing in wealth if and only if, $\exists \lambda > 0$, for all w, y and y',*

$$-\frac{U'''(w+y)}{U''(w+y)} \geq \lambda \geq -\frac{U''(w+y')}{U'(w+y')} \tag{21}$$

We now show that corollaries 4.2 and 4.3 generalize Ross' conditions.

Conditions (3) and (4) imply

$$\frac{U''(w+y)}{V''(w+y)} \geq \lambda \geq \frac{U'(w+y')}{V'(w+y')} \quad for \;\; all \;\; w \;, y \; and \; y'. \tag{22}$$

Then, Proposition 2.1, (18), (19) and (22) immediately entail the following result.

Corollary 4.3 *For $u(w,y) = U(w+y)$, $v(w,y) = V(w+y)$ with $U' > 0$, $V' > 0$, $U'' < 0$ and $V'' < 0$, the following two conditions are equivalent:*

(i) $\exists \lambda > 0$
$$\frac{U''(w+y)}{V''(w+y)} \geq \lambda \geq \frac{U'(w+y')}{V'(w+y')} \quad for \;\; all \;\; w \;, y \; and \; y'. \tag{23}$$

(ii) $\pi_u \geq \pi_v$ *for* $\forall$ w *and* $(\tilde{x}, \tilde{y})$ *such that* $E[\tilde{x}|\tilde{y} = y]$ *is non-decreasing in* y.

Conditions (14) and (15) imply, for all w, y and y',

$$-\frac{U'''(w+y)}{U''(w+y)} \geq \lambda \geq -\frac{U''(w+y')}{U'(w+y')} \tag{24}$$

From Proposition 3.1, (18), (19) and (22), we obtain the following corollary:

Corollary 4.4 *For $u(w,y) = U(w+y)$, with $U' > 0$, $U'' < 0$ and $U''' > 0$, the following two conditions are equivalent:*

(i) the partial risk premium associated with any $(\tilde{x}, \tilde{y})$ such that $E[\tilde{x}|\tilde{y} = y]$ is non-decreasing in y, is decreasing in wealth.

(ii) $\exists \lambda > 0$, for all w, y and y',

$$-\frac{U'''(w+y)}{U''(w+y)} \geq \lambda \geq -\frac{U''(w+y')}{U'(w+y')} \tag{25}$$

In Corollary 4.4, the condition for decreasing risk premia under conditional expectation increasing risks is equivalent to that for a first-order stochastic dominance (FSD) improvement in an independent background risk to decrease the risk premium, as shown by Eeckhoudt et al. (1996).

4.2 Multiplicative background risk

For a multiplicative background risk $\tilde{y}$, we have

$$u(w, y) = U(wy) \tag{26}$$

and

$$v(w, y) = V(wy). \tag{27}$$

Here w may represent the random wealth invested in a risky asset and y may represent a multiplicative random shock on random wealth.

Since,

$$u_1 = yU', \quad u_{11} = y^2U'', \quad u_{12} = U' + wyU'', \quad u_{111} = y^3U''' \;\; \textit{and} \;\; u_{112} = 2yU''^2U''' \tag{28}$$

and

$$v_1 = yV', \quad v_{11} = y^2V'', \quad v_{12} = V' + wyV'', \quad v_{111} = y^3V''' \;\; \textit{and} \;\; v_{112} = 2yV''^2V'''. \tag{29}$$

Conditions (3) and (4) imply, $\exists \lambda_1, \lambda_2 > 0$, for all w, y and y',

$$\frac{U'(wy) + wyU''(wy)}{V'(wy) + wyV''(wy)} \geq \lambda_1 \geq \frac{U'(wy')}{V'(wy')} \tag{30}$$

and

$$\frac{U''(wy)}{V''(wy)} \geq \lambda_2 \geq \frac{U'(wy')}{V'(wy')}. \tag{31}$$

Then, from Proposition 2.1, (28), (29), (30) and (31), we obtain

Corollary 4.5 *For $u(w,y) = U(wy)$, $v(w,y) = V(wy)$ with $U' > 0$, $V' > 0$, $U'' < 0$ and $V'' < 0$, the following two conditions are equivalent:*

(i) $\exists \lambda_1, \lambda_2 > 0$, for all w,y and y',

$$\frac{U'(wy) + wyU''(wy)}{V'(wy) + wyV''(wy)} \geq \lambda_1 \geq \frac{U'(wy')}{V'(wy')} \tag{32}$$

and

$$\frac{U''(wy)}{V''(wy)} \geq \lambda_2 \geq \frac{U'(wy')}{V'(wy')}. \tag{33}$$

(ii) $\pi_u \geq \pi_v$ for $\forall$ w and $(\tilde{x}, \tilde{y})$ such that $E[\tilde{x}|\tilde{y} = y]$ is non-decreasing in y.

Since

$$\begin{aligned} & \frac{U'(wy) + wyU''(wy)}{V'(wy) + wyV''(wy)} \\ = & \frac{U''(wy)(\frac{U'(wy)}{U''(wy)} + wy)}{V''(wy)(\frac{V'(wy)}{V''(wy)} + wy)} \\ = & \frac{U''(wy)(wy - \frac{1}{RA_U(wy)})}{V''(wy)(wy - \frac{1}{RA_V(wy)})}, \end{aligned} \tag{34}$$

where $RA_U(wy) = -\frac{U''(wy)}{U'(wy)}$ and $RA_V(wy) = -\frac{V''(wy)}{V'(wy)}$ are indices of absolute risk aversion. We can obtain a more short cut sufficient condition from Corollary 4.5.

Corollary 4.6 *For $u(w,y) = U(wy)$, $v(w,y) = V(wy)$ with $w > 0$, $\tilde{y} > 0$ almost surely, $U' > 0$, $V' > 0$, $U'' < 0$ and $V'' < 0$, If $\exists \lambda > 0$, for all w,y and y',*

$$\frac{U''(wy)}{V''(wy)} \geq \lambda \geq \frac{U'(wy')}{V'(wy')}, \tag{35}$$

then $\pi_u \geq \pi_v$ for $\forall$ w and $(\tilde{x}, \tilde{y})$ such that $E[\tilde{x}|\tilde{y} = y]$ is non-decreasing in y.

Proof From the above argument, we know that for all w,y and y',

$$\frac{U''(wy)}{V''(wy)} \geq \lambda \geq \frac{U'(wy')}{V'(wy')}. \tag{36}$$

$RA_U(wy) \geq RA_V(wy)$ implies that $\pi_u \geq \pi_v$ for $\forall$ w and $(\tilde{x}, \tilde{y})$ such that $E[\tilde{x}|\tilde{y} = y]$ is non-decreasing in y. Using the fact that "U is more Ross risk averse than $V \Rightarrow RA_U(wy) \geq RA_V(wy)$", we obtain the result. Q.E.D.

Corollary 4.5 states that "more Ross risk aversion" is a sufficient condition to order partial risk premium in the presence of conditional expectation increasing multiplicative background risk.

From Proposition 3.1, we obtain

Corollary 4.7 *For $u(w,y) = U(wy)$, with $U' > 0$, $U'' < 0$ and $U''' > 0$, the partial risk premiums associated with any $(\tilde{x}, \tilde{y})$ such that $E[\tilde{x}|\tilde{y} = y]$ is non-decreasing in y, is decreasing in wealth if and only if, $\exists\lambda_1, \lambda_2 > 0$, for all w,y and y',*

$$-\frac{2yU''^2U'''(wy)}{U'(wy) + wyU''(wy)} \geq \lambda_1 \geq -\frac{y'U''(wy')}{U'(wy')} \tag{37}$$

and

$$-\frac{yU'''(wy)}{U''(wy)} \geq \lambda_2 \geq -\frac{y'U''(wy')}{U'(wy')}. \tag{38}$$

Since

$$\begin{aligned} & -\frac{2yU''^2U'''(wy)}{U'(wy) + wyU''(wy)} \\ = & -\frac{yU'''(wy)(2\frac{U''(wy)}{U'''(wy)} + wy)}{U''(wy)(\frac{U'(wy)}{U''(wy)} + wy)} \\ = & -\frac{yU'''(wy)(wy - 2\frac{1}{P_U(wy)})}{U''(wy)(wy - \frac{1}{RA_U(wy)})}, \end{aligned} \tag{39}$$

where $P_U(wy) = -\frac{U'''(wy)}{U''(wy)}$ is the index of absolute prudence. We can obtain a shorter sufficient condition from Corollary 4.7.

Corollary 4.8 *For $u(w,y) = U(wy)$, with $w > 0$, $\tilde{y} > 0$ almost surely, $U' > 0$, $U'' < 0$ and $U''' > 0$, The partial risk premium associated with any risk $(\tilde{x}, \tilde{y})$ such that $E[\tilde{x}|\tilde{y} = y]$ is non-decreasing in y, is decreasing in wealth if , $\exists\lambda > 0$, for all w, y and y',*

$$-\frac{yU'''(wy)}{U''(wy)} \geq \lambda \geq -\frac{y'U''(wy')}{U'(wy')} \tag{40}$$

and $P_U(wy) \geq 2RA_U(wy)$.

Moreover, (40) can be multiplied by w on both sides to obtain the results in terms of measures of relative risk aversion and relative prudence:

$$-\frac{wyU'''(wy)}{U''(wy)} \geq \lambda \geq -\frac{wy'U''(wy')}{U'(wy')}, \tag{41}$$

which implies "min relative prudence $\geq$ max relative risk aversion". Whereas in the literature, $P_U \geq 2RA_U$ is an important condition for risk vulnerability (see Gollier 2001, p129), Corollary 4.8 shows that $\min P_U \geq \max RA_U$ is an important condition for comparative risk aversion in the presence of a conditional expectation increasing multiplicative background risk.

5 Decreasing cross Ross risk aversion and n-switch independence property

Because the conditions derived in Ross (1981) are fairly restrictive upon preference, some readers may regard Ross' results as negative, because no standard utility functions (logarithmic, power, mixture of exponentials) satisfy these conditions. Pratt (1990) suggests that probability distributions restrictions stronger than mean independence may provide more satisfactory comparative statics. On a very different ground, Bell (1988) proposes that agents are likely to be characterized by a utility function satisfying the one-switch rule: there exists at most one unique critical wealth level at which the decision-maker switches from preferring one alternative to the other. He shows that the linex function (linear plus exponential) is the only relevant utility function family if one adds to the one-switch rule some very reasonable requirements. Such utility function has been studied by Bell and Fishburn (2001), Sandvik and Thorlund-Petersen (2010) and Abbas and Bell (2011). In a recent paper, Denuit et al. (2011b) show that Ross' stronger measure of risk aversion gives rise to the linex utility function and therefore they provide not only a utility function family but also some intuitive and convenient properties for Ross' measure.

Abbas and Bell (2011) extend the one-switch independence property to two-attribute utility functions and propose a new independence assumption based on the one-switch property: n-switch independence.

Definition 5.1 (Abbas and Bell 2011) For utility function $u(x, y)$, X is n-switch independent of Y if two gamblers $\tilde{x}_1$ and $\tilde{x}_2$ can switch in preference at most n times as Y progresses from its lowest to its highest value.

They provide the following propositions:

Proposition 5.1 *(Abbas and Bell 2011) X is one-switch independent of Y if and only if*

$$u(x, y) = g_0(y) + f_1(x)g_1(y) + f_1(x)g_2(y), \tag{42}$$

where $g_1(y)$ has constant sign, and $g_2(y) = g_1(y)\phi(y)$ for some monotonic function ϕ.

Proposition 5.2 *(Abbas and Bell 2011) If X is n-switch independent of Y, then there exist some functions f_i, g_i such that*

$$u(x, y) = g_0(y) + \sum_{i=1}^{n+1} f_i(x)g_i(y). \tag{43}$$

We now show that the one-switch property of Proposition 5.1 is a consequence of Proposition 3.1. We also argue that (43) is a utility function that satisfies the decreasing cross Ross risk aversion condition proposed in Section 3.

From Proposition 3.1 we know that the partial risk premium π_u, associated with any $(\tilde{x}, \tilde{y})$ such that $E[\tilde{x}|\tilde{y} = y]$ is non-decreasing in y, is decreasing in wealth, if and only if there exists $\phi : R \times R \to R$ with $\phi_1 \leq 0$, $\phi_{12} \leq 0$ and $\phi_{11} \leq 0$, and $\lambda > 0$ such that

$$-u_1(x, y) = \lambda u(x, y) + \phi(x, y). \tag{44}$$

Solving the above differential equation implies that u is of the form

$$u(x, y) = -\int_{-\infty}^{x} e^{\lambda t}\phi(t, y)dte^{-\lambda x}. \tag{45}$$

If we take $\phi(x, y) = -H(x)J(y)$ such that $J(y)$ has a constant sign, then we get

$$\begin{aligned} u(x, y) &= \int_{-\infty}^{x} e^{\lambda t}H(t)dte^{-\lambda x}J(y) \\ &= [\frac{1}{\lambda}e^{\lambda x}H(x) - \frac{1}{\lambda}\int_{-\infty}^{x} e^{\lambda t}H'(t)dt]e^{-\lambda x}J(y) \\ &= \frac{1}{\lambda}H(x)J(y) - \frac{1}{\lambda}\int_{-\infty}^{x} e^{\lambda t}H'(t)dte^{-\lambda x}J(y). \end{aligned} \tag{46}$$

Defining $g_1(y) = g_2(y) = \frac{1}{\lambda}J(y)$, $f_1(x) = H(x)$ and $f_2(x) = -\int_{-\infty}^{x} e^{\lambda t}H'(t)dte^{-\lambda x}$, then we recognize the functional form in Proposition 5.1.

Integrating the integral term of (46) by parts again and again, we obtain

$$\begin{aligned} u(x, y) &= \sum_{i=1}^{n} e^{\lambda x}\frac{(-1)^{i-1}H^{(i-1)}(x)}{\lambda^i} + \frac{1}{\lambda^n}\int_{-\infty}^{x} e^{\lambda t}(-1)^n H^{(n)}(t)dt]e^{-\lambda x}J(y) \\ &= \sum_{i=1}^{n} J(y)\frac{(-1)^{i-1}H^{(i-1)}(x)}{\lambda^i} + \frac{1}{\lambda^n}\int_{-\infty}^{x} e^{\lambda t}(-1)^n H^{(n)}(t)dte^{-\lambda x}J(y) \\ &= \sum_{i=1}^{n+1} f_i(x)g_i(y), \end{aligned} \tag{47}$$

where $f_i(x) = (-1)^{(i-1)}H^{(i-1)}(x)$ for $i = 1, ..., n$, $f_{n+1}(x) = \int_{-\infty}^{x} e^{\lambda t}(-1)^n H^{(n)}(t)dte^{-\lambda x}$, $g_i(y) = \frac{1}{\lambda^i}J(y)$ for $i = 1, .., n$ and $g_{n+1}(y) = \frac{1}{\lambda^n}J(y)$. Therefore we obtain the functional form in Proposition 5.2 from decreasing cross Ross risk aversion. Although coming from very different approaches, decreasing cross Ross risk aversion and n-switch independence reach the same functional form. Our result thus provides a connection between decreasing cross Ross risk aversion and n-switch independence.

6 Conclusion

In this paper we consider expected-utility preferences in a bivariate setting. The analysis focuses on random variables that satisfy the conditional expectation dependence. The main focus is on the risk premium for removing one of the risks in the presence of a second risk. To this end, we extend Ross' (1981) contribution by defining the concept of "cross Ross risk aversion." We derive several equivalence theorems relating measures of risk premia with measures of risk aversion. We then consider additive risks and multiplicative risks as two special cases. We also show that decreasing cross Ross risk aversion assumption about behavior gives rise to the utility function family that belongs to the class of n-switch utility functions. The analysis and the index of risk aversion in this paper may be instrumental in obtaining comparative static predictions in various applications.

7 Appendix: Proof of Proposition 2.1

Proof (i) implies (ii): We note that

$$\frac{u_{12}(w,y)}{v_{12}(w,y)} \geq \lambda_1 \geq \frac{u_1(w,y')}{v_1(w,y')} \Leftrightarrow \frac{-u_{12}(w,y)}{-v_{12}(w,y)} \geq \lambda_1 \geq \frac{u_1(w,y')}{v_1(w,y')}. \tag{48}$$

$$\frac{u_{11}(w,y)}{v_{11}(w,y)} \geq \lambda_2 \geq \frac{u_1(w,y')}{v_1(w,y')} \Leftrightarrow \frac{-u_{11}(w,y)}{-v_{11}(w,y)} \geq \lambda_2 \geq \frac{u_1(w,y')}{v_1(w,y')}. \tag{49}$$

Defining $\phi = u - \lambda v$, where $\lambda = \min\{\lambda_1, \lambda_2\}$, and differentiating, one obtains $\phi_1 = u_1 - \lambda v_1$, $\phi_{12} = u_{12} - \lambda v_{12}$ and $\phi_{11} = u_{11} - \lambda v_{11}$, then (48) and (49) imply that $\phi_1 \leq 0$, $\phi_{12} \leq 0$ and $\phi_{11} \leq 0$.

(ii) implies (iii): From Theorem 2 of Finkelshtain et al. (1999), we know that,

(a) $\phi_{11} \leq 0$ and $\phi_{12} \leq 0 \Leftrightarrow E\phi(w+\tilde{x},\tilde{y}) \leq E\phi(w+E\tilde{x},\tilde{y})$ for any risk $(\tilde{x},\tilde{y})$ such that $E[\tilde{x}|\tilde{y}=y]$ is non-decreasing in y;

(b) when $v_1 \geq 0$, $v_{11} \leq 0$ and $v_{12} \leq 0$ if and only if $\pi_v \geq 0$ for any risk $(\tilde{x},\tilde{y})$ such that $E[\tilde{x}|\tilde{y}=y]$ is non-decreasing in y.

Since $\pi_v \geq 0$, we have $\phi_1 \leq 0 \Rightarrow \phi(w,y) \leq \phi(w-\pi_v,y)$.

The following proof is as in Ross (1981):

$$\begin{aligned} Eu(w+E\tilde{x}-\pi_u,\tilde{y}) &= Eu(w+\tilde{x},\tilde{y}) \qquad (50)\\ &= E[\lambda v(w+\tilde{x},\tilde{y}) + \phi(w+\tilde{x},\tilde{y})] \end{aligned}$$

$$
\begin{aligned}
&= \lambda Ev(w+E\tilde{x}-\pi_v,\tilde{y})+E\phi(w+\tilde{x},\tilde{y})\\
&\le \lambda Ev(w+E\tilde{x}-\pi_v,\tilde{y})+E\phi(w+E\tilde{x},\tilde{y})\\
&\le \lambda Ev(w+E\tilde{x}-\pi_v,\tilde{y})+E\phi(w+E\tilde{x}-\pi_v,\tilde{y})\\
&= Eu(w+E\tilde{x}-\pi_v,\tilde{y}).
\end{aligned}
$$

Since $u_1>0$, $\pi_u \ge \pi_v$.

(iii) implies (i): We prove this claim by contradiction. Suppose that there exists some w, y and y' such that $\frac{u_{12}(w,y)}{v_{12}(w,y)} < \frac{u_1(w,y')}{v_1(w,y')}$. Because u_1, v_1, u_{12} and v_{12} are continuous, we have

$$\frac{u_{12}(w,y)}{v_{12}(w,y)} < \frac{u_1(w,y')}{v_1(w,y')} \quad for \quad (w,y),(w,y') \in [m_1,m_2]\times[n_1,n_2], \tag{51}$$

which implies

$$\frac{-u_{12}(w,y)}{-v_{12}(w,y)} < \frac{u_1(w,y')}{v_1(w,y')} \quad for \quad (w,y),(w,y') \in [m_1,m_2]\times[n_1,n_2], \tag{52}$$

and then

$$\frac{v_1(w,y')}{-v_{12}(w,y)} < \frac{u_1(w,y')}{-u_{12}(w,y)} \quad for \quad (w,y),(w,y') \in [m_1,m_2]\times[n_1,n_2]. \tag{53}$$

If $G(x,y)$ is a distribution function and $G_Y(y)$ is the marginal distribution function of $\tilde{y}$. such that $G_Y(y)$ has positive support on interval $[n_1,n_2]$ then we have

$$\frac{Ev_1(w,\tilde{y})}{-v_{12}(w,y)} < \frac{Eu_1(w,\tilde{y})}{-u_{12}(w,y)} \quad for \quad (w,y) \in [m_1,m_2]\times[n_1,n_2], \tag{54}$$

which can be written as

$$\frac{u_{12}(w,y)}{Eu_1(w,\tilde{y})} > \frac{v_{12}(w,y)}{Ev_1(w,\tilde{y})} \quad for \quad (w,y) \in [m_1,m_2]\times[n_1,n_2]. \tag{55}$$

Let us consider $w_0 \in [m_1,m_2]$ and $\tilde{x}=k\tilde{z}$ with $k>0$, where $\tilde{z}$ is a zero-mean risk with a distribution function $G(z,y)$ such that $G_Z(\tilde{z}\le z|\tilde{y}=y)$ is non-increasing in y. We notice that

(a) $G_Z(\tilde{z}\le z|\tilde{y}=y)$ is non-increasing in $y \Rightarrow E[\tilde{z}|\tilde{y}=y]$, is non-decreasing in y;

(b) $G_Z(\tilde{z}\le z|\tilde{y}=y)$ is non-increasing in $y \Rightarrow G(\tilde{y}\le y,\tilde{z}\le z) \ge G_Y(\tilde{y}\le y)G_Z(\tilde{z}\le z)$ (see Lehmann 1966, Lemma 4).

Let $\pi_u(k)$ denote its associated partial risk premium, which is

$$Eu(w_0+k\tilde{z},\tilde{y}) = Eu(w_0-\pi_u(k),\tilde{y}). \tag{56}$$

Differentiating the above equality with respect to k yields

$$E\tilde{z}u_1(w_0+k\tilde{z},\tilde{y}) = -\pi_u'(k)Eu_1(w_0-\pi_u(k),\tilde{y}). \tag{57}$$

Observing that $\pi_u(0) = 0$, we get

$$\begin{aligned}
\pi_u'(0) &= -\frac{E\tilde{z}u_1(w_0,\tilde{y})}{Eu_1(w_0,\tilde{y})} \qquad (58)\\
&= -\frac{E\tilde{z}Eu_1(w_0,\tilde{y}) + Cov(\tilde{z}, u_1(w_0,\tilde{y}))}{Eu_1(w_0,\tilde{y})}\\
&= -\frac{Cov(\tilde{z}, u_1(w_0,\tilde{y}))}{Eu_1(w_0,\tilde{y})}\\
&= -\frac{\int\int[G(z,y) - G_Z(z)G_Y(y)]dzd_y u_1(w_0,y)}{Eu_1(w_0,\tilde{y})} \quad (by\ Cuadras\ 2002,\ Theorem\ 1)\\
&= -\int\int[G(z,y) - G_Z(z)G_Y(y)]\frac{u_{12}(w_0,y)}{Eu_1(w_0,\tilde{y})}dzdy
\end{aligned}$$

Similarly, for v we have

$$\pi_v'(0) = -\int\int[G(z,y) - G_Z(z)G_Y(y)]\frac{v_{12}(w_0,y)}{Ev_1(w_0,\tilde{y})}dzdy. \qquad (59)$$

Now π_u and π_v can be written as the forms of Taylor expansion around $k = 0$:

$$\pi_u(k) = -k\int\int[G(z,y) - G_Z(z)G_Y(y)]\frac{u_{12}(w_0,y)}{Eu_1(w_0,\tilde{y})}dzdy + o(k) \qquad (60)$$

and

$$\pi_v(k) = -k\int\int[G(z,y) - G_Z(z)G_Y(y)]\frac{v_{12}(w_0,y)}{Ev_1(w_0,\tilde{y})}dzdy + o(k). \qquad (61)$$

Then, from (55), we know that, when $k \to 0$, we get $\pi_u < \pi_v$ for $G(z,y)$ such that $G_Y(y)$ has positive support on interval $[n_1, n_2]$ and $G(z,y) - G_Z(z)G_Y(y)$ is positive on domain $[m_1, m_2] \times [n_1, n_2]$. This is a contradiction.

Now let us turn to the other condition. Suppose that there exists some w, y and y' such that $\frac{u_{11}(w,y)}{v_{11}(w,y)} < \frac{u_1(w,y')}{v_1(w,y')}$. Because u_1, v_1, u_{11} and v_{11} are continuous, we have

$$\frac{u_{11}(w,y)}{v_{11}(w,y)} < \frac{u_1(w,y')}{v_1(w,y')} \quad for \quad (w,y),(w,y') \in [m_1', m_2'] \times [n_1', n_2'], \qquad (62)$$

which implies

$$\frac{-u_{11}(w,y)}{-v_{11}(w,y)} < \frac{u_1(w,y')}{v_1(w,y')} \quad for \quad (w,y),(w,y') \in [m_1', m_2'] \times [n_1', n_2'], \qquad (63)$$

and then

$$\frac{-u_{11}(w,y)}{u_1(w,y')} < \frac{-v_{11}(w,y)}{v_1(w,y')} \quad for \quad (w,y),(w,y') \in [m_1', m_2'] \times [n_1', n_2']. \qquad (64)$$

If $G(x,y)$ is a distribution function such that $G_Y(y)$ has positive support on interval $[n_1', n_2']$, then we have

$$\frac{-Eu_{11}(w,\tilde{y})}{u_1(w,y')} < \frac{-Ev_{11}(w,\tilde{y})}{v_1(w,y')} \quad for \quad (w,y') \in [m_1', m_2'] \times [n_1', n_2'] \qquad (65)$$

and

$$\frac{-Eu_{11}(w,\tilde{y})}{Eu_1(w,\tilde{y})} < \frac{-Ev_{11}(w,\tilde{y})}{Ev_1(w,\tilde{y})}. \tag{66}$$

Let us consider $w_0 \in [m_1', m_2']$ and $\tilde{x} = k\tilde{z}$, where $\tilde{z}$ is a zero-mean risk and $\tilde{z}$ and $\tilde{y}$ are independent. Let $\pi_u(k)$ denote its associated partial risk premium, which is

$$Eu(w_0 + k\tilde{z}, \tilde{y}) = Eu(w_0 - \pi_u(k), \tilde{y}). \tag{67}$$

Differentiating the equality above with respect to k yields

$$E\tilde{z}u_1(w_0 + k\tilde{z}, \tilde{y}) = -\pi_u'(k)Eu_1(w_0 - \pi_u(k), \tilde{y}), \tag{68}$$

and so $\pi_u'(0) = 0$ since $E\tilde{z} = 0$. Differentiating once again with respect to k yields

$$E\tilde{z}^2u_{11}(w_0 + k\tilde{z}, \tilde{y}) = [\pi_u'^2 Eu_{11}(w_0 - \pi_u(k), \tilde{y}) - \pi_u''(k)Eu_1(w_0 - \pi_u(k), \tilde{y}). \tag{69}$$

This implies that

$$\pi_u''(0) = -\frac{Eu_{11}(w_0,\tilde{y})}{Eu_1(w_0,\tilde{y})}E\tilde{z}2. \tag{70}$$

Similarly, for v we have

$$\pi_v''(0) = -\frac{Ev_{11}(w_0,\tilde{y})}{Ev_1(w_0,\tilde{y})}E\tilde{z}2. \tag{71}$$

Now π_u and π_v can be written as the forms of Taylor expansion around $k = 0$:

$$\pi_u(k) = -\frac{Eu_{11}(w_0,\tilde{y})}{Eu_1(w_0,\tilde{y})}E\tilde{z}^2k2 + o(k2) \tag{72}$$

and

$$\pi_v(k) = -\frac{Ev_{11}(w_0,\tilde{y})}{Ev_1(w_0,\tilde{y})}E\tilde{z}^2k2 + o(k2). \tag{73}$$

From (66) we know that, when $k \to 0$, we get $\pi_u < \pi_v$ for $G(x,y)$ such that $G_Y(y)$ has positive support on interval $[n_1', n_2']$. This is a contradiction. Q.E.D.

8 References

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