C I R P É E

Centre Interuniversitaire sur le Risque, les Politiques Économiques et l'Emploi

Cahier de recherche/Working Paper **08-05**

Practices

Max Blouin
Jean-Marc Bourgeon

Première version/First Version : Mai/May 2008
Version révisée/Revised : Septembre/September 2012

Blouin: Corresponding author. Université du Québec à Montréal and CIRPÉE
blouin.max@uqam.ca
Bourgeon: INRA and Ecole Polytechnique, Paris
bourgeon@agroparistech.fr

We thank participants at the 2008 NSF/NBER/CEME Conference on General Equilibrium and Mathematical Economics (Brown University), as well as Arianna Degan, Claude Fluet, Chris Green and Roberto Serrano, for useful comments. This research was financed by grants from SSHRC (Government of Canada), FQRSC (Government of Quebec) and the Fondation du Risque (Chaire Santé, Risque et Assurance, Allianz).

Abstract:
We examine an economy where professionals provide services to clients and where a professional can sell his practice to another. Professionals vary in quality, and clients in their need (or willingness-to-pay) for high-quality service. Efficiency is measured as the number of matches between high-quality professionals and high-need clients. However, agent types are unobservable a priori. We find that trade in practices can facilitate the transmission of information about agent types. In general full efficiency is achieved, but equilibrium is not always robust to random shocks. A tax on the sale of practices ensures the existence of robust, efficient equilibria.

Keywords: Signalling, professional services, practices, reputation

JEL Classification: C73, D82

1 Introduction

We examine a market for professional (legal, medical, dental, etc.) services. We are interested not only in the interactions between professionals and their clients, but also in the phenomenon of "selling one's practice," which happens when one professional refers his clients to another professional, who pays a sum of money in exchange for this referral.

Such a transaction presents an interesting information problem. When a practice changes hands, the clientele is aware of the change in ownership and is free to take its business somewhere else. Clearly, for a practice to have value (over and above the value of its physical assets), the new owner must be confident that the clientele will remain. But why should it, if the new owner's quality is not observable? Another fact which needs to be explained is that not all young professionals starting out in the business choose to buy practices. Some elect to build up their clientele from scratch. What are the main factors underlying this decision?

The fact that clients choose to remain with the new owner of a practice, rather than take their business elsewhere, could be interpreted as inertia on their part, due to search or switching costs. However we believe that an additional explanation needs to be explored, one having to do with information and uncertainty. Perhaps high-quality professionals are more likely to buy practices than low-quality ones. If this is the case, then the purchase of a practice constitutes a signal of high quality, which would explain why clients choose to stay.

In our model, professionals are either high-quality (H-professionals, or H-pros for short) or low-quality (L-pros). Clients are either type-A (A-clients) or type-B (B-clients), A-clients being the ones with the higher willingness-to-pay for high-quality services. These types are unobservable, but a professional and client do learn each other's types after they have transacted once. In that sense, professional services are an experience good. For simplicity we assume that each professional serves only one client at a time.

An efficient equilibrium maximizes the gains from trade, which in this case means maximizing the number of A-H matches, i.e. matches between A-clients and H-pros. The potential role of practices is to facilitate the creation of such matches.

In this paper, selling a practice means selling the right to negotiate with an A-client.[1] This is not as simple as "selling the client." Here, once a practice changes hands, the client is under no obligation to transact with the new owner: she may, if she wants, take her business elsewhere. This raises a number of questions. First, why would a professional buy a practice if there is no guarantee that the client will stay? Second, why would a client stay in a practice if there is no information about the new owner? Third, how can any of this be efficient?

[1] Practices consisting of B-clients are also possible, and will be discussed at the end of the paper. Including them does not have a qualitative impact on results, though, mainly because A-practices and B-practices are distinguishable on the market and sell for different prices. A professional wishing to be matched with an A-client purchases an A-practice, whether B-practices exist or not.

To put things in perspective, we first study a version of the model without the buying and selling of practices. In this case the market is generally inefficient: young high-quality professionals have no way of signalling their type, so many of them end up with B-clients; similarly, A-clients have no way of screening professionals, so many of them end up with L-pros. Efficiency is only obtained when there is a multitude of A-clients, enough to hire all young professionals and retain the services of the high-quality ones for a second period.

When practices are introduced, there is always an efficient equilibrium. When A-clients are scarce relative to H-pros the equilibrium is separating, in that only H-pros buy and sell practices. The price at which practices are traded is relatively high, so buying one is only profitable to a professional if he intends to keep it for a long time (two periods, in our model). But keeping a practice for a long time means renewing his business relationship with his client even after the client has gotten to know his type. This is where high-quality professionals have an edge. An L-pro can buy a practice and, for a time, pretend that he is high-quality; but eventually (after one period) his true type becomes known to the client, and the game is up. Because of this, L-pros cannot profitably hold on to practices for more than a period, so they do not buy any. Buying a practice is a signal of high quality.

When A-clients are scarcer the efficient equilibrium is semi-pooling: All H-pros buy practices, but some L-pros do so as well. The signal is diluted: when a practice is bought, the client cannot be one hundred percent sure that the new owner is high-quality. But in equilibrium the probability is high enough that she chooses to stay.

We next see if these results are robust to the introduction of a shock. Specifically, we force a fraction of the professional population to leave the economy in mid-career, giving them the opportunity to sell their practices (if they have any) before they go. This is meant to depict a real-life occurrence, namely that people commonly relocate to other areas, for a variety of reasons. We find that some of the semi-pooling efficient equilibria found earlier are robust to this shock, while separating ones are not. In the latter case, we also find that a tax on the sale of practices in mid-career (i.e. a tax from which retiring professionals would be exempt) can be used to restore efficiency.

1.1 Related Literature

The early literature on reputation focuses on long-lived agents and does not consider reputation as an asset that can be traded between firms (see Fudenberg and Tirole, ch. 9, for a presentation of this literature). Kreps (1990) is the first investigation of reputation trading and shows that there exists an equilibrium where reputation is a valuable asset that provides incentives to short-lived firms to exert more effort. However, this reputation equilibrium is one among many and is as likely to happen as less favourable ones.

Mailath and Samuelson (2001) and Tadelis (2002) extend Kreps' setup to investigate the incentive aspect of reputation trading. They consider a heterogeneous set of firms, composed of inept (low-type) and competent (high-type) firms. The latter can choose to exert an effort to increase the probability of providing a high-quality service (a "success;" as opposed to a low-quality service, a "failure"). They show that reputation trading gives incentives for competent firms to exert effort. As in Tadelis (1999, 2003), they also investigate the ability of reputation trading to operate as a screening device between low-type and high-type firms.

In these studies there is a clear distinction between a firm's name and its owner's identity. Also it is assumed that customers cannot observe changes in firm ownership. The name's past performance (or reputation) is publicly known, i.e. customers and entrepreneurs know if the services provided by each name in the past were successes, failures, or any sequence of those events. This name's record determines the price that the owner of the name can charge customers, and thereby, the price of that name on the name market. They show that good reputations cannot serve as sorting devices that separate high-type from low-type entrepreneurs: there is no equilibrium in which only high-type entrepreneurs buy good reputations. Indeed, if good reputations were only bought by high-type entrepreneurs, customers' expectations about the quality of the firm's services would not be violently shaken by the occurrence of a failure, making a good reputation very valuable to low-type firms. As high-type entrepreneurs are more likely than low-type ones to be successful in building their own name, they would value a good name less than their low-type counterparts, and so customers' expectations would not be rational. In two recent studies, Deb (2012) and Wang (2011) obtain similar results with observable changes in name ownership.

As in Deb (2012) and Wang (2011), we assume that a professional's record is not public information: only clients who have benefited from his services in the past know the professional's type. The main difference of our setting compared to this literature is the heterogeneity of clients and the asymmetric information between professionals and clients regarding client types. This makes the purchase of practices more attractive to high-quality professionals than when only reputation effects are at work. Moreover, we introduce a role for government in promoting efficiency through taxation.

Developing an approach somewhat close to ours, Hakenes and Peitz (2007; HP hereafter) find that, for certain parameter values, reputation is tradeable, i.e. that a competitive market allows high-value practices to be sold only to good professionals. This result is obtained by imposing rather strong assumptions regarding what doctors (professionals) can observe and what they can do. In HP's model, doctors selling practices can observe the types of those buying them; and since in equilibrium good doctors only sell their practices to other good doctors, patients can deduce that if their previous doctor was good then their new one must also be good (and similarly for bad doctors). In our model, doctors cannot observe one another's types, and so patients cannot make this kind of inference. Furthermore, in HP's model, doctors can only sell their practices when they retire, not before. Combined with the previous assumption, this means that there is no opportunity in HP for a bad doctor to buy a high-value practice, pass himself off as a good doctor, and resell the practice for a high price before its value has had a chance to diminish, i.e. before the clientele has had a chance to react to the deception. There is, in other words, very little chance of adverse selection. In our model the possibility of such duplicity is one of the main obstacles to efficiency. Good doctors *will* sell practices to and buy practices from bad doctors: the only things that matter in these transactions are the types of clients involved (the real value of the practice) and asking price. And patients will *not* automatically distrust their new doctor if their previous one was bad, since neither doctor's type has any bearing on the other's. Our setup is much more vulnerable to adverse selection; in fact, there are in our model Akerlof-type pooling equilibria in which practices are worthless and not traded, because patients would not trust the doctors who bought them. Compared to HP, then, the conditions for efficiency and for separating equilibrium are more demanding in our setup.

Another difference between the two models is that in HP, prices for services are fixed, and an exogenous switching cost is introduced to prevent clients from seeking new doctors. In our model, prices are fully endogenous and it is these prices which determine whether a patient will stay with his doctor or leave him; there are no switching costs.

A more distantly related paper is Garicano and Santos (2004). In that study, *agents* (akin to our professionals) are either high-skill or low-skill and draw *opportunities* (akin to our clients) of uncertain value. One of the decisions they must make is whether to deal with these opportunities themselves or refer them to other professionals in exchange for some compensation. Their notion of efficiency, the matching of high-skill agents to valuable opportunities, is the same as ours. However, the parallel ends there. The paper studies transactions among professionals, but does not go into the professional-client relationship; in fact, their opportunities are not decision-makers at all. The interdependence of the two kinds of relationships (professional-professional and professional-client) is crucial to our analysis.

To our knowledge, none of the existing literature considers a shock such as the one we investigate. And none considers taxation as a tool for bringing about efficiency, as we do.

2 Benchmark: The Model Without Practices

To give a sense of how the economy functions in our model, we first present a version in which clients hire professionals, but professionals do not buy or sell practices. Much of the reasoning presented here will be used in later sections. For clarity, when a professional and client meet, we will refer to the professional as "him" and to the client as "her."

2.1 Setup

Time elapses discretely without beginning or end. We are interested in steady-state equilibria. The market is for services, which are provided by professionals (or pros) and purchased by clients. We do not call them buyers and sellers, to avoid confusion with the buying and selling of practices, discussed later. All agents are risk-neutral and have a discount factor $\delta \in (0, 1)$.

Professionals enter the economy, work for two periods, then retire, in overlapping-generations fashion. That is, at the start of each period, a measure 1 of young professionals enter the economy; those who were young in the previous period (also a measure 1) now become old; and those who were old in the previous period (again, a measure 1) now retire. Hence there are always a measure 2 of working professionals in the economy, half of them young, half of them old. Retiring professionals do not play a role in this section, but they will later, when we consider the sale of practices. This age structure is meant to mirror reality in a simple way. Professionals do need to begin their careers without having had the chance to establish a reputation; and eventually they do retire, possibly selling their practices, a way of cashing in on accumulated reputation.

Each generation of professionals is composed of a measure q of *high-quality* (type-H) professionals and a measure $1 - q$ of *low-quality* (type-L) ones. These qualities are exogenous and fixed: a professional is born with a certain type, and stays that way. Each professional has one indivisible unit of service for sale in each period; that is to say, he may see only one client per period.

There are of two types of clients, A and B. Type-A clients value high-quality services more than type-B clients do, in a way which will be specified shortly. Clients, unlike professionals, are infinitely-lived.[2]

There is a measure ψ_A of type-A agents and a measure ψ_B of type-B agents present in the economy at all times. We also assume that $\psi_A + \psi_B > 2$, i.e. that clients outnumber working professionals. This seems reasonable, since in reality a large segment of the population does not actually purchase professional services; everyone, however, is a *potential* client. The case where $\psi_A + \psi_B \leq 2$ can also be worked out, with more elaborate proofs and calculations, but with the same general results. Some features of the $\psi_A + \psi_B \leq 2$ case are discussed towards the end of the paper.

Types (A, B, L and H) are private information, as are past histories. Once a pro-

[2]This assumption is made for mathematical convenience. It would be possible to consider the case where each period a client may disappear for exogenous reasons (and be replaced by an identical one who enters the market). As long as this probability of disappearance is relatively low, the properties of the equilibria we derive in the following would not be affected. Alternatively, we may consider that clients have finite lives. In that case, practices (introduced in Section 3) would be assets whose values decline over time. The price of a practice, which (as will be shown) must depend on the selling professional's age, would additionally depend on the client's age, making the model much more cumbersome. We believe that our main insights, regarding the role of practices as costly signals, would carry through regardless.

fessional and client make a transaction, they learn one another's types. They may, on the basis of this acquired information, decide to transact again. A professional's age — i.e. young, old, or retiring — is publicly observable.

The matching mechanism is designed (i) to give professionals who are beginning their second period of work and the clients they served previously an opportunity to renegotiate their business relationship; and (ii) to give all agents access to a Walrasian market for professional services. For this reason each period is composed of a *negotiation phase* and a *market phase*.

First the negotiation phase. As soon as a professional becomes old, i.e. at the beginning of his second period, he negotiates with the client he served when young. Note that at this point the two know each other's types. The professional makes a take-it-or-leave-it offer to the client. The offer is a price for his services in the current period. If the client accepts, the transaction takes place at the offered price, and the two agents are considered matched for the rest of that period, i.e. they do *not* participate in the upcoming market phase. In that case, we say that the match has been renewed. If the client rejects the offer, the match is dissolved: the two part ways and will seek new matches in the market phase, still within the same period.

The negotiation phase is immediately followed by a *market phase*. This is a centralized market for professional services, and involves all unmatched agents. This means (i) all agents whose matches were dissolved during the negotiation phase, and (ii) all those who did not participate in the negotiation phase. The latter group is composed of young professionals (who have just entered the economy) and clients who in the preceding period were served by old professionals (now retired). Market-clearing prices for services are established as if by an auctioneer. Professionals' ages are observable, so there will be two prices in this phase: one for the services of young professionals, which we denote p_Y, and one for those of old professionals, which we denote p_O. In either case, the price is for one period of service.

Per-period utilities are as follows. A professional, whatever his type, has no costs: his utility is simply the price he receives for his unit of service. A type-i client who pays p for a unit of service gets utility $\theta_{iH} - p$ if the professional she deals with is high-quality, and $\theta_L - p$ if low-quality, where

$$0 < \theta_L < \theta_{BH} < \theta_{AH} \quad . \tag{1}$$

Anyone who does not transact gets zero utility for that period.[3] All agents maximize lifetime discounted utility.

An equilibrium consists of:

- a market-clearing price for young professionals on the market, p_Y;
- a market-clearing price for old professionals on the market, p_O;

[3]It may be natural to interpret all this as saying that type-A clients are wealthier than type-B ones. But other interpretations are possible, such as saying that some clients are more prone to illness or legal troubles than others.

- for each possible client-professional pairing (A-H, A-L, B-H and B-L), an optimal price to be offered by the professional when the partnership is up for renewal (i.e. during the negotiation phase), and an optimal acceptance-rejection rule to be implemented by the client.

These values and rules must be the same from period to period.

2.2 Efficiency

Since type-A clients value high-quality service more than type-B ones do (relative to low-quality service and no service), social efficiency can be measured by the number of A-H matches. There are at all times a measure ψ_A of type-A clients in the economy, as well as a measure $2q$ of type-H professionals (q young ones and q old ones). A socially optimal outcome, then, is one where the measure of A-H matches in each period is equal to $\min\{\psi_A, 2q\}$, meaning that either all type-A clients are matched with type-H professionals or vice versa.

2.3 Equilibrium

Let V_i be the value to a type-i client of being on the market, i.e. without a match. And let V_{ij} be the value to a type-i client of being in the negotiation phase with a type-j professional.

Let us look at the negotiation phase first. Consider a client-professional relationship which is up for renewal. By this time the client and professional have learned each other's types. The professional makes a take-it-or-leave-it offer, consisting of a price p for his services for one period. Suppose, to illustrate, that the client's type is A and the professional's is H. If the client accepts the offer, her payoff this period is $\theta_{AH} - p$, and she returns to the market next period (when the professional retires), which makes her total discounted payoff $\theta_{AH} - p + \delta V_A$. If she rejects the offer, she goes on the market this period and gets V_A. The highest price she will consider (let us call it $\bar{p}_{AH}$) is that which makes these two values equal, that is to say $\bar{p}_{AH} = \theta_{AH} - (1-\delta)V_A$. The threshold prices for all possible matches are

$$\bar{p}_{AH} = \theta_{AH} - (1-\delta)V_A \quad ; \tag{2}$$
$$\bar{p}_{BH} = \theta_{BH} - (1-\delta)V_B \quad ; \tag{3}$$
$$\bar{p}_{AL} = \theta_{L} - (1-\delta)V_A \quad ; \tag{4}$$
$$\bar{p}_{BL} = \theta_{L} - (1-\delta)V_B \quad . \tag{5}$$
$$\tag{6}$$

In equilibrium the client will accept any offer $p \leq \bar{p}_{ij}$ and reject any offer $p > \bar{p}_{ij}$.

From the professional's perspective, the situation is quite simple. If his offer of p is accepted, he gets that price; if it is rejected, he goes on the market this period and gets p_O, the market price for old professionals' services. And no matter what happens this period, he will retire next period. Clearly the best thing for him to do if $\bar{p}_{ij} > p_O$ is to offer the price $\bar{p}_{ij}$, which is accepted. If $\bar{p}_{ij} < p_O$, on the other hand, then no agreement is possible: the professional makes an unacceptable offer and both he and the client end up on the market. If $\bar{p}_{ij} = p_O$ he is indifferent between the two outcomes.

We see that the client will never be offered a price below the maximum she is willing to pay. In equilibrium there are two outcomes for her: either she will accept an offer of $\bar{p}_{ij}$, which will leave her no better and no worse off than being on the market; or she will reject an offer of $p > \bar{p}_{ij}$, and actually end up on the market. So both outcomes must have the same value:

$$V_{ij} = V_i \quad . \tag{7}$$

Now let us consider the market phase. On the market, a client can purchase the services of a young professional at price p_Y, those of an old professional at price p_O, or she can purchase nothing. If she hires a young professional, she has a probability q of getting high-quality service. Let q_O denote the probability of receiving high-quality service when hiring an old professional: this is the proportion of high-quality professionals among all old professionals on the market, and it is endogenous. Thus a client gets expected utility $q\theta_{iH} + (1-q)\theta_L - p_Y$ for this period if she hires a young professional, and $q_O\theta_{iH} + (1-q_O)\theta_L - p_O$ if she hires an old one; she gets 0 if she hires no one. In all three cases she expects δV_i from next period onward, whether on the market or in negotiations, since $V_{ij} = V_i$, as we have shown. Thus the client's prospects can be written as

$$\begin{aligned} V_i = \max\{& q\ \theta_{iH} + (1-q)\ \theta_L - p_Y\ + \delta V_i, \\ & q_O\ \theta_{iH} + (1-q_O)\ \theta_L - p_O\ + \delta V_i,\ \delta V_i\} \quad , \end{aligned} \tag{8}$$

or more usefully

$$\begin{aligned} (1-\delta)V_i = \max\{& q\ \theta_{iH} + (1-q)\ \theta_L - p_Y\ , \\ & q_O\ \theta_{iH} + (1-q_O)\ \theta_L - p_O\ ,\ 0\} \quad . \end{aligned} \tag{9}$$

We may now begin to characterize equilibrium. All proofs appear in the appendix.

Lemma 1. *All A-H and B-H matches up for renewal are renewed. Hence* $q_O = 0$.

This establishes that old professionals on the market will not be sought after. The logic is simply that if they were high-quality they would be serving their old clients, not looking for new ones. The following result is what we call a crowding-out equilibrium.

Theorem 1. *If $\psi_A \geq 1 + q$, then in equilibrium all young professionals are hired by A-clients. Equilibrium is efficient.*

When $\psi_A \geq 1 + q$, A-clients are so numerous that they bid the price of young professionals up to $p_Y = q\theta_{AH} + (1-q)\theta_L$, which is too high for B-clients. All young professionals are hired by A-clients, and all A-H matches are renewed. This means that all H-pros are always matched with A-clients, and efficiency is achieved. But it is achieved because A-clients are numerous enough to crowd out B-clients in the market for young professionals. Introducing practices into the economy will not change this result.

Throughout the rest of the paper we will assume that $\psi_A < 1 + q$. This is the more challenging case, as something rather more complex (involving practices) will be required to obtain a socially efficient outcome.

Lemma 2. *Some B-clients hire young pros.*

The last result implies that

$$(1-\delta)V_B = q\theta_{BH} + (1-q)\theta_L - p_Y \quad . \tag{10}$$

To pin down the prices p_Y and p_O, it is necessary to invoke the assumption that clients outnumber professionals. The main implication of this assumption is of course that some clients must do without professional services, and thus get zero utility. Either V_A or V_B must be zero. But looking at (9) we see that $V_A = 0$ implies $V_B = 0$. Sot V_B must be zero in any case. We can then use (10) to find

$$p_Y \quad = \quad q\theta_{BH} + (1-q)\theta_L \quad . \tag{11}$$

There remains to find the market-clearing price for old pros' services. Formally we have the following lemma.

Lemma 3. *In equilibrium $p_O = \theta_L$.*

We can then compute the threshold prices which characterize all negotiations:

$$\bar{p}_{AH} = q\theta_{BH} + (1-q)\theta_{AH} \quad ; \tag{12}$$
$$\bar{p}_{AL} = \theta_L - q(\theta_{AH} - \theta_{BH}) \quad ; \tag{13}$$
$$\bar{p}_{BH} = \theta_{BH} \quad ; \tag{14}$$
$$\bar{p}_{BL} = \theta_L \quad . \tag{15}$$

We see that $\bar{p}_{AL} < p_O$, which means that all A-L matches will be dissolved. As for B-L matches, they can be renewed or dissolved, since L-pros negotiating with B-clients are indifferent between renewing at $\bar{p}_{BL} = \theta_L$ or going on the market to earn $p_O = \theta_L$.

At these prices, A-clients will strictly prefer hiring young pros whenever they are on the market (i.e. when they are not renewing their matches). B-clients, on the other hand, will be indifferent among all three options: hiring young pros, hiring old pros, and hiring no one. And indeed each option must be chosen by *some* B-clients if markets are to clear.

These results give us a full picture of professional-client interactions in the economy. They are illustrated in Figure 1. In this figure, time is measured horizontally. Two generations of professionals are shown: one of them is young at time $t-1$ and old at t; the other is young at t and old at $t+1$. Each horizontal stratum represents a possible history for a professional. First he goes on the market when young, and is matched with either a type-A or type-B client. An arrow indicates that his match is then renewed. If there is no arrow, it means that his match is dissolved; he goes on the market and gets a new client.

The column labelled t gives us an idea of the composition of the economy in steady state, since it comprises two cohorts, one young and one old. Naturally, all clients who obtain services should also be accounted for in this column.

As the figure shows, all A-H and B-H matches are renewed. An L-pro, if he is initially matched with an A-client, will see this match dissolved; in the following period he will go on the market and be hired by a B-client, since only B-clients hire old pros. If the L-pro is initially matched with a B-client, then during negotiations he will be indifferent between renewing at price $\bar{p}_{BL}$ or going on the market and earning p_O, since the two quantities are equal (in the diagram the match is shown to be renewed).

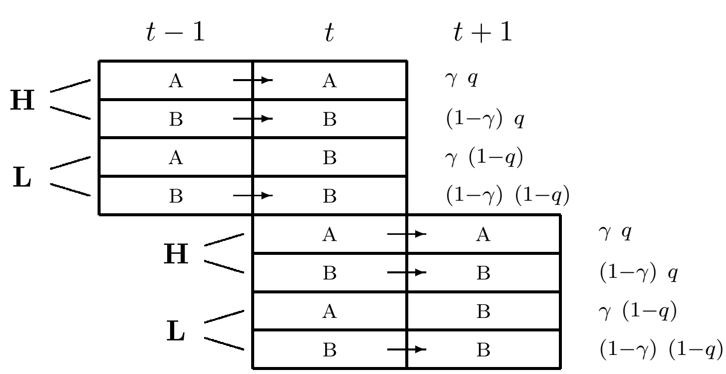

FIGURE 1. Professionals' histories in equilibrium. The letters A and B mean being matched with that type of client. Arrows indicate matches renewed during the negotiation phase.

To the right of each stratum is written the number of pros from each cohort who go through that particular history. The symbol γ denotes the measure of A-clients who hire young professionals in a given period. Since all A-H matches are renewed, there must also be in any given period a measure γq of A-clients served by old H-pros, with whom they have renewed their relationship from the previous period. Since A-L matches are not renewed and type-A clients do not hire old professionals, this accounts for all type-A clients. So the entire population of A-clients, which is ψ_A, must be equal to those who hire young pros (a measure γ) plus those who have renewed with H-pros (a measure γq). Hence $\gamma + \gamma q = \psi_A$, or equivalently

$$\gamma = \frac{\psi_A}{1+q} \quad . \tag{16}$$

The total number of A-H matches in a given period t, including those just formed and those renewed, is $2\gamma q$. This number is less than ψ_A, the population of A-clients, and also less than $2q$, the population of working H-pros. So the number of such matches is less than what it could be. As a result, we have

Theorem 2. *When $\psi_A < 1 + q$, an equilibrium without trade in practices is not socially efficient.*

This inefficiency arises from the fact that A-L and B-H matches are formed on the market. This is inevitable, since both types of clients hire young pros on the market, and these young pros have no way of revealing their types. It is now time to see whether a market for practices can provide such a way.

3 The Model With Practices

We now add a market for practices. Practices in our model are essentially options to negotiate with particular clients. The market for practices is therefore distinct from the market for services. Services are provided by professionals and purchased by clients, whereas practices are traded among professionals only. Professionals who have just reached retirement age play a role in this section, so it is useful to keep in mind that there are *three* ages for professionals: young, old, and retiring. When we say *working* professional, we will mean one who is young or old.

Each period now has three phases: a *practice phase*, a negotiation phase, and a market phase, in that order.

The period opens with a practice phase. This is a centralized market for practices. At this point a professional who served a client in the previous period (i.e. an old or retiring pro) may sell the right to negotiate with this client. This is called selling a practice. Any professional of working age (i.e. young or old) may buy a practice. When a practice is sold, the buyer becomes matched with the client, and the seller becomes unmatched. It is the buyer, then, who moves on to the negotiation phase with the client. We emphasize that we use the term *practice* only in this narrow sense of a clientele that is sold from one professional to another. Matches formed during the market phase are not called practices.

Note that if an old professional wants to buy a practice, he must first break off his previous match. This can be done by simply abandoning his client (who will seek a new match during the market phase), or by selling his old practice. In the latter case, the professional is simultaneously a buyer and seller of practices.

For simplicity we will assume that only matches with A-clients are sold as practices. Including B-practices in our model, while possible, is not informative enough to warrant the added complications.[4]

Obviously, a pro who buys a practice will want proof that the client he is acquiring is indeed an A-client. We assume that sellers can provide such proof. We know that a pro who has served a client for one period has learned the client's type. We are assuming that he can pass on this information to another professional in a verifiable manner. Later we will consider other possible assumptions regarding the revelation of client types.

When a practice is sold, the client is aware of the transaction (she sees that she is matched with someone new) but does not participate in it. However, she *will* have the choice whether to accept this new match or take her business elsewhere in the ensuing negotiation phase.

Once the practice phase is over, retiring professionals formally retire and have no further role in the economy.

The negotiation phase comes next. As before, all professionals who are matched with clients make take-it-or-leave-it offers to those clients. This time the professionals involved are: (i) all those (young or old) who have just bought practices, and (ii) all old professionals who have not sold practices. When a client accepts, the transaction proceeds at the proposed price and the two parties are considered matched for the remainder of the period. When she refuses, both parties go on the market.

The one difference between negotiations here and negotiations in the previous section is that here information is not always perfect. If the professional has just acquired the practice, he and the client have not had an opportunity to learn each other's types. Beliefs, described below, will come into play.

Then comes the market phase. As before, it is a centralized market for professional services, in which all unmatched agents participate.

Figure 2 summarizes the timing.

3.1 Beliefs and practice prices

In the benchmark model, the negotiation phase necessarily involved agents who knew each other's types. Here this is not always the case. If a professional has just bought a practice, then he and the client have just met. We have assumed that the

[4]The value of a B-practice lies not in the nature of the client herself, but in the opportunity for a professional to reveal his type to that client. Some results and examples involving B-practices are available from the authors.

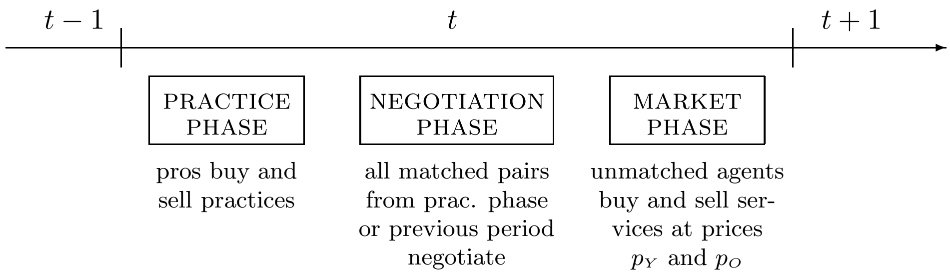

Figure 2. Timing.

professional learns the client's type from the seller of the practice. But the client must form a belief as to the professional's type, based on the information at hand.

Let us say that she ascribes a probability μ to the professional being type-H and $1 - \mu$ to his being type-L. She forms this belief based on what she knows or can observe: the age and type of the seller (previous owner) and the age of the buyer (new owner). The age of the new owner is clearly relevant, since this may tell her something about his type, which she cares about. But what about the characteristics of the *previous* owner, someone with whom she will have no more dealings?

The seller's age is relevant because it is also observed by the buyer, and so it can act as a coordination device between the client and the buyer. For example, we could imagine an equilibrium in which only young H-pros buy practices from retiring sellers, but equal numbers of young H-pros and young L-pros buy them from old sellers. Client beliefs must reflect this reality: when the practice is sold by a retiree to a young pro, the client should be certain that the new owner is an H-pro; but in the case of an old seller and a young buyer, she should think it equally likely that the new owner is one type or the other.

The previous owner's *type*, since it is not observable by the new owner, cannot be such a coordination device. It will therefore not figure in the formation of beliefs.

There are, then, four beliefs to specify: μ_{YO} when the practice is bought by a young professional from an old one, μ_{YR} when it is bought by a young pro from a retiring one, μ_{OO} when both buyer and seller are old, and μ_{OR} when the buyer is old and the seller is retiring. Beliefs are common knowledge in equilibrium.

These beliefs in turn affect how much a client is willing to pay for the services offered by the practice's new owner. In this way they affect the *value* of a practice. It follows that practices supplied by old and retiring professional are different things. As such they can sell for different prices. Let us denote by k_O the price of a practice sold by an old pro, and by k_R the price of a practice sold by a retiring one.

3.2 Negotiation strategies

As before, the client's strategy during negotiations is to have a threshold price: she accepts any price equal to or less than this threshold and rejects any price above it. This threshold is based on her knowledge of (or belief about) the professional's type and on her reservation payoff.

Let V_i be the value to a type-i client of being on the open market, just as in Section 2. This is her reservation payoff, since the open market is where she goes if she rejects an offer. The value to her of being in negotiations with *any* professional must be equal to V_i in equilibrium. The reasoning is basically the same as before. During negotiations, the professional knows (or is sure of) his client's type. He knows, then, the client's threshold price. His optimal strategy is to offer this price (if he wants the offer to be accepted) or a higher one (if he wants the offer to be rejected). In either case, the client gets a payoff equal to V_i.

When the professional and client know each other from before, the maximum price the client is willing to accept is found in the same way as in Section 2. We again call this price $\bar{p}_{ij}$, where i is the client's type and j the professional's. The four possibilities are given by equations (2) through (5).

When the professional and client do not know each other from before, i.e. when the professional has just bought the practice, then the threshold price depends on μ, the client's belief that the professional is type-H. If the client accepts an offer of p, her expected payoff for this period is $\mu\theta_{AH} + (1-\mu)\theta_L - p$. Her payoff from the next period onward is V_A (whether on the market or via negotiations), making the total $\mu\theta_{AH} + (1-\mu)\theta_L - p + \delta V_A$. If she rejects the offer, she goes on the market this period and gets V_A. The highest price she will consider is that which makes these two values equal:

$$\bar{p}_A(\mu) = \mu\theta_{AH} + (1-\mu)\theta_L - (1-\delta)V_A \quad . \tag{17}$$

As explained previously, the value of μ depends on the ages of the two professionals involved in the sale of the practice. Note that $\bar{p}_A(1) = \bar{p}_{AH}$ and $\bar{p}_A(0) = \bar{p}_{AL}$.

As in Section 2, the professional has to decide whether to offer the threshold price, which the client will accept, or a higher one, which the client will reject. But here he may have a few more things to consider, such as the revenue from selling the practice in a later period. Also, the professional may be young, in which case a renewed match can mean more negotiations next period. The various possibilities will be explored shortly.

3.3 Market values

When young professionals buy practices, this affects the proportion of type-H pros among young pros left on the market. We can no longer count on this proportion being q as in the benchmark model; it is now endogenous and we will call it q_Y.

For old professionals, we define q_O as before. Clients on the market face the same choices (hire a young pro, hire an old pro, hire no one), yielding this time

$$(1-\delta)V_i = \max\{q_Y\ \theta_{iH} + (1-q_Y)\ \theta_L - p_Y\ ,\\ q_O\ \theta_{iH} + (1-q_O)\ \theta_L - p_O\ ,\ 0\} \quad . \tag{18}$$

3.4 Professional strategies

There are various life histories possible for a professional. We must account for the buying and selling of practices, the renewal or dissolution of matches and possibly the uncertainty as to which type of client he is matched with on the market.

There is one thing we can rule out: the dissolution of a match involving the new owner of a practice. When a pro buys a practice, he is certain that the client is type-A, and so he knows what price he must offer her to keep her as a client. If this price does not suit him, then it was a waste of money to buy the practice in the first place. We may be assured, then, that new practice owners will serve their clients for at least one period.

Let us consider the ways that an old pro can get a new client. He can buy a practice from a retiring pro: this will cost him k_R and allow him to charge $\bar{p}_A(\mu_{OR})$ this period, then he can sell the practice for k_R next period. He can buy a practice from an old pro instead. Or he can buy no practice at all, dissolve his match during the negotiation phase, and go on the market, where he will get p_O. We define V_O as the highest of the three payoffs:

$$V_O = \max\{-k_R + \bar{p}_A(\mu_{OR}) + \delta k_R\ ,\ -k_O + \bar{p}_A(\mu_{OO}) + \delta k_R\ ,\ p_O\} \quad . \tag{19}$$

Note that we do not distinguish between H-pros and L-pros in this case, because this value is necessarily the same for both. An old pro is set to retire the following period. So by the time the client finds out the pro's type, it will be too late for her to do anything about it.

Now we turn to the case of a young H-pro who has just entered the economy. If he buys a practice from a retiring pro and keeps it until he retires, he will get

$$V_H^R = -k_R + \bar{p}_A(\mu_{YR}) + \delta \max\{\ \bar{p}_{AH} + \delta k_R\ ,\ k_O + V_O\ \} \quad . \tag{20}$$

He buys the practice for an amount k_R. His client has belief μ_{YR}, so during the first period he obtains $\bar{p}_A(\mu_{YR})$ for his services. By the second period, however, his type is known. He can renew his match at price $\bar{p}_{AH}$ and sell the practice when he retires. Or he can sell the practice now and find a new client (in one of the three ways listed above).

The same scenarios are possible with a practice bought from an old (rather than a retiring) professional. The payoff in that case is

$$V_H^O = -k_O + \bar{p}_A(\mu_{YO}) + \delta \max\{ \bar{p}_{AH} + \delta k_R \ , \ k_O + V_O \ \} \quad . \tag{21}$$

Finally, the young H-pro might choose not to buy a practice. He will get p_Y for his services when young. His full expected payoff is

$$V_H^M = p_Y + \alpha\delta \max\{ \bar{p}_{AH} + \delta k_R \ , \ k_O + V_O \ \} + (1-\alpha)\delta \max\{\bar{p}_{BH}, V_O\} \quad . \tag{22}$$

Here α denotes the probability that his first client is type-A. If he is indeed matched with an A-client, then he will be able to sell that match as a practice. Thus a new practice will have been created. This obviously has steady-state implications, which will be discussed shortly.

Now we turn to young L-pros. If a young L-pro buys a practice from a retiring pro, he gets $\bar{p}_A(\mu_{YR})$ in the first period, just as an H-pro would. In the following period, however, the client will know his type. So he can only renew the match at price $\bar{p}_{AL}$. His payoff, therefore, is

$$V_L^R = -k_R + \bar{p}_A(\mu_{YR}) + \delta \max\{ \bar{p}_{AL} + \delta k_R \ , \ k_O + V_O \ \} \quad . \tag{23}$$

If the practice had been bought from an old pro instead, the payoff would be

$$V_L^O = -k_O + \bar{p}_A(\mu_{YO}) + \delta \max\{ \bar{p}_{AH} + \delta k_R \ , \ k_O + V_O \ \} \quad . \tag{24}$$

And if he does not buy a practice when young his career payoff is

$$V_L^M = p_Y + \alpha\delta \max\{ \bar{p}_{AL} + \delta k_R \ , \ k_O + V_O \ \} + (1-\alpha)\delta \max\{\bar{p}_{BL}, V_O\} \quad . \tag{25}$$

Professionals must choose among these strategies. They do so taking practice prices (k_R and k_O) and market prices (p_Y and p_O) as given, and correctly anticipating clients' beliefs and threshold prices. It is professionals' choices among these strategies that determine q_Y and q_O. An equilibrium is a situation where all these choices are optimal and consistent with clients' beliefs.

3.5 Pooling equilibrium

A pooling equilibrium is one in which practices are not traded. Clients ascribe a low probability μ to new owners being type-H (whatever the ages of the buyers and sellers). Therefore whoever buys a practice cannot hope to get a higher price from his client than what he could get on the market. As a result, practices are worthless and no one buys them.

All other prices, proportions, payoffs and behaviour are exactly as in the benchmark model. And so, like the equilibrium without practices, the pooling equilibrium is inefficient.

A pooling equilibrium always exists.

3.6 Efficient equilibrium

Our main line of inquiry, however, is whether efficiency can be obtained when there is a market for practices. For this reason we will now focus on cases where practices have positive value. Because practices have positive value, everyone who is matched with an A-client will wish to sell this match as a practice. A steady-state equilibrium can only be achieved if all A-clients are in practices, and none of them can be found on the market. Thus we will necessarily have $\alpha = 0$ in equations (22) and (25).

Our first main result is that a market for practices does indeed provide a mechanism for achieving efficiency.

Theorem 3. *An efficient equilibrium always exists.*

The proof is in the appendix. Much depends on whether or not A-clients outnumber H-pros. When $\psi_A \leq 2q$, it is possible to have an equilibrium in which only H-pros buy and sell practices. We call this a separating equilibrium.

When $\psi_A > 2q$, however, this is no longer possible: even if all H-pros buy practices, there are still some A-clients left over, and these are traded as practices by L-pros. An equilibrium of this kind can nevertheless be efficient: if all H-pros have practices, the number of A-H matches is maximized. We refer to an equilibrium in which both types of pros buy practices as a semi-pooling equilibrium.

Even when both types of professionals buy practices, there is a difference in their behaviour. H-pros, because they can renew their matches at a high price ($\bar{p}_{AH}$), are more likely to keep their practices for two periods. L-pros, because they can only renew their matches at a low price ($\bar{p}_{AL}$), have a greater incentive to get rid of them after a single period. It is this difference which we will exploit when discussing sales taxes on practices in Section 4.3.

We offer two numerical examples, both based on the proof of Theorem 3. In the first, H-pros outnumber A-clients; in the second, we have the reverse.

Example 1. Let parameter values be $\theta_L = 1$, $\theta_{BH} = 2$, $\theta_{AH} = 3$, $q = 0.5$, $\psi_A = 0.8$, $\psi_B = 2$, and $\delta = 2/3$. Then the following is an equilibrium, illustrated by Figure 3. Market prices are $p_Y = 7/6$ and $p_O = 1$. Market proportions are $q_Y = 1/6$ and $q_O = \alpha = 0$. Threshold prices are $\bar{p}_{AH} = 17/6$, $\bar{p}_{BH} = 2$, $\bar{p}_{AL} = 5/6$, and $\bar{p}_{BL} = 1$. Market values are $V_A = 0.5$ and $V_B = 0$. Beliefs are $\mu_{YR} = 1$ and $\mu_{YO} = \mu_{OR} = \mu_{OO} = 0$. Practice prices are $k_R = 4$ and $k_O = 3$. Lifetime utility is $V_H^{R2} = 47/18$ for type-H pros and $V_L^M = 11/6$ for type-L pros. Since $p_O = \bar{p}_{BL}$, B-L matches may be renewed or not.

Example 2. Let parameter values be $\theta_L = 1$, $\theta_{BH} = 2$, $\theta_{AH} = 3$, $q = 0.4$, $\psi_A = 1$, $\psi_B = 2$, and $\delta = 0.5$. Then the following is an equilibrium, illustrated by Figure 4. Market prices are $p_Y = p_O = 1$. Market proportions are $q_Y = q_O = \alpha = 0$. Threshold prices are $\bar{p}_{AH} = 3$, $\bar{p}_{BH} = 2$, and

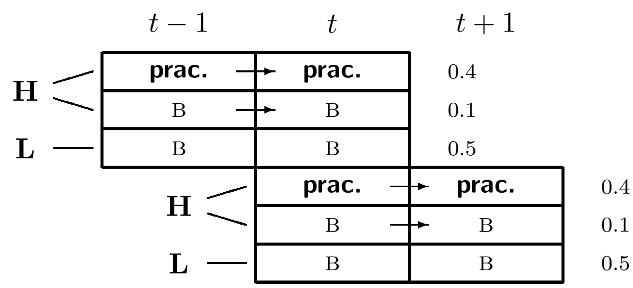

FIGURE 3. Professionals' histories in Example 1.

$\bar{p}_{AL} = \bar{p}_{BL} = 1$. Market values are $V_A = V_B = 0$. Each period a measure 0.32 of young H-pros buy practices from retiring pros; and another 0.08 buy them from old pros. Also, a measure 0.08 of young L-pros buy practices from retiring pros, and another 0.12 buy them from old pros. H-pros keep their practices for two periods; L-pros keep theirs for only one. Beliefs are $\mu_{YR} = 0.8$, $\mu_{YO} = 0.4$ and $\mu_{OR} = \mu_{OO} = 0$. Practice prices are $k_R = 2.4$ and $k_O = 1.6$. Lifetime utility is $V_H^R = 2.3$ for type-H pros and $V_L^M = 1.5$ for type-L pros. Since $p_O = \bar{p}_{BL}$, B-L matches may be renewed or not.

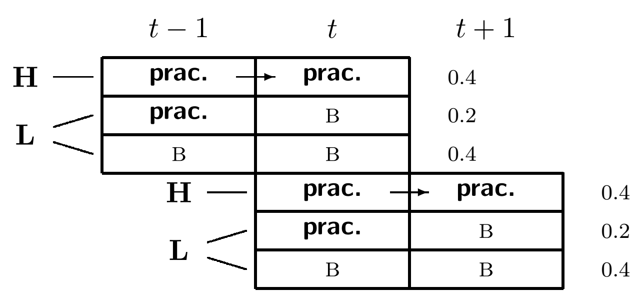

FIGURE 4. Professionals' histories in Example 2.

4 Robustness

In this part of the paper we want to add a realistic shock to the model, and see which, if any, of the model's efficient equilibria are robust to it. We also want to see if a taxation scheme on the part of the government can remedy any lack of robustness we might find.

The shock is simple: each period a positive fraction ϵ of all old professionals must leave the economy. Those who have to leave are informed of this *before* the practice phase, but their departure takes place immediately *after* the practice phase. This gives them a chance to sell their practices, if they have any, before they leave. Agents who leave in this way are not replaced. The remaining fraction $1-\epsilon$ are said to "survive the shock," i.e. they remain in the economy until they retire.

The relevance is easy to see. In the real world, people sometimes have to relocate. Or maybe they need money right away and cannot borrow. The fact that a professional sells his practice before the end of his career is not necessarily due to his quality. Granted, we could have modelled relocation as a choice. But if conditions (prices, etc.) are the same across locations, it is hard to see why an old H-pro would choose to relocate and give up the trust he has built up with his clientele. So we prefer to think of relocation as something beyond the agent's control, i.e. a shock.

When he enters the economy, a young pro knows there is a probability ϵ that he will have to leave a period later. He needs to factor that into his expected payoffs (V_H^{R2} and so on). If he buys a practice, and then it turns out that he *is* chosen to leave, then obviously he will sell that practice before leaving, as long as $k_O > 0$.

All pros are equally likely to be forced to leave: H-pros, L-pros, those who bought practices, those who did not, those who are matched with A-clients, those who are matched with B-clients. With the addition of this shock, Equations (22) to (24) become

$$\begin{aligned} V_H^R &= -k_R + \bar{p}_A(\mu_{YR}) + \delta\epsilon k_O \\ &\qquad + \delta(1-\epsilon)\max\{\ \bar{p}_{AH} + \delta k_R\ ,\ k_O + V_O\ \} \quad ; \end{aligned} \tag{26}$$

$$\begin{aligned} V_H^O &= -k_O + \bar{p}_A(\mu_{YO}) + \delta\epsilon k_O \\ &\qquad + \delta(1-\epsilon)\max\{\ \bar{p}_{AH} + \delta k_R\ ,\ k_O + V_O\ \} \quad ; \end{aligned} \tag{27}$$

$$V_H^M = p_Y + \delta(1-\epsilon)\max\{\bar{p}_{BH}, V_O\} \quad ; \tag{28}$$

$$\begin{aligned} V_L^R &= -k_R + \bar{p}_A(\mu_{YR}) + \delta\epsilon k_O \\ &\qquad + \delta(1-\epsilon)\max\{\ \bar{p}_{AL} + \delta k_R\ ,\ k_O + V_O\ \} \quad ; \end{aligned} \tag{29}$$

$$\begin{aligned} V_L^O &= -k_O + \bar{p}_A(\mu_{YO}) + \delta\epsilon k_O \\ &\qquad + \delta(1-\epsilon)\max\{\ \bar{p}_{AL} + \delta k_R\ ,\ k_O + V_O\ \} \quad ; \end{aligned} \tag{30}$$

$$V_L^M = p_Y + \delta(1-\epsilon)\max\{\bar{p}_{BL}, V_O\} \quad . \tag{31}$$

We have in mind a small value for ϵ. In fact, we are interested in results that hold true as ϵ tends to zero. So the ϵ-shock will not have an appreciable impact on the expected payoffs themselves. Its major impact will come from the fact that now there will necessarily be old pros selling their practices, even when $\psi_A \leq 2q$. The separating equilibria discussed in Section 3.6, in which only retiring H-pros sold practices, may no longer exist. This will explained in detail in Section 4.1 below.

We consider an equilibrium to be much more realistic if it is robust to this shock than if it is not. We proceed now to find conditions under which an equilibrium which is both efficient and robust exists.

4.1 Non-robustness of efficient separating equilibria

The separating equilibria described in the first part of the proof of Theorem 3 (which require $\psi_A \leq 2q$) are vulnerable to the shock we just discussed. In those equilibria only retiring pros sold practices. Because $\mu_{YO} = \mu_{OO} = 0$, anyone who bought a practice from an old pro would be presumed low-quality. If we add the ϵ-shock to these equilibria, we create a situation where some of the pros who bought practices when young sell them at price k_O when old. For markets to clear, other pros have to buy these practices from them: buying a practice from an old pro now has to be optimal. This is only possible if either μ_{YO} or μ_{OO} is sufficiently high. With this change of beliefs, some of the incentives which made the equilibrium work no longer hold. So these particular equilibria are not robust. The following theorem and its proof work out the details thoroughly.

Theorem 4. *If $\psi_A \leq 2q$, equilibrium cannot be both efficient and robust.*

This is significant. Theorem 3, showing the existence of an efficient separating equilibrium, gave us reason to believe that the market for practices could serve as a strong signalling device, perfectly separating H-pros from L-pros. But such an equilibrium is not robust to the small, realistic shock to which we subjected it.

Semi-pooling equilibria can also be efficient, as shown in the second part of the proof of Theorem 3. We must determine if they can also be robust.

4.2 Robustness of efficient semi-pooling equilibria

Intuitively, semi-pooling equilibria such as that in Example 2 have better chances of being robust. They involve the purchase of practices by young L-pros, who sell them after a single period. Someone, then, does buy practices from old pros. The belief μ_{YO} refers to purchase scenarios that actually take place in equilibrium; it is not an off-equilibrium belief. Therefore the ϵ-shock should not affect things too much. This intuition is correct, as the following theorem shows.

Theorem 5. *If $\psi_A > 2q$, a robust, efficient equilibrium exists.*

The caveat that A-clients must outnumber H-pros ($\psi_A > 2q$) is a holdover from Theorem 3. Semi-pooling equilibria do exist when $\psi_A \leq 2q$, and they are robust. They are just not efficient.

In the next section, we show that an efficient, robust separating equilibrium can always be induced through taxation. In equilibrium the tax is only levied against people leaving the economy, hence does not raise much revenue. Its purpose is of course to provide the incentives which lead to efficiency and robustness.

4.3 Taxation

We will now consider a tax on the sale of practices as a possible way of ensuring efficiency and robustness when the practice market alone fails to do so, i.e. when $\psi_A \leq 2q$. A tax effectively drives a wedge between the price paid by the buyer and that received by the seller, and can thus give us more leeway in our effort to satisfy incentive constraints.

Essentially we wish to make the purchase of practices less attractive to L-pros without making them less attractive to H-pros. We know that, on the whole, H-pros have no trouble holding their practices for two periods, whereas L-pros, if they do buy practices, are likely to sell them quickly. Types are unobservable, even to the government. But a professional's *age* is fully observable. Levying a tax on the sale of a practice, contingent on the seller's age, might be the answer to the efficiency/robustness problem.

We imagine a proportional tax τ on the sale of practices, to be paid by the seller. Retiring pros are exempt from this tax. Therefore only old pros selling their practices must pay it. The idea is that if the tax is set at a suitable level, only agents who are compelled to do so by the ϵ-shock will sell their practices when old. This may make it possible for all the other equilibrium, efficiency and robustness conditions to be satisfied.

With the tax in place, V_H^M and V_L^M are still given by (28) and (31), but the other payoffs are now

$$V_H^R = -k_R + \bar{p}_A(\mu_{YR}) + \delta\epsilon k_O(1-\tau) + \delta(1-\epsilon)\max\{\ \bar{p}_{AH} + \delta k_R\ ,\ k_O(1-\tau) + V_O\ \} \quad ; \quad (32)$$

$$V_H^O = -k_O + \bar{p}_A(\mu_{YO}) + \delta\epsilon k_O(1-\tau) + \delta(1-\epsilon)\max\{\ \bar{p}_{AH} + \delta k_R\ ,\ k_O(1-\tau) + V_O\ \} \quad ; \quad (33)$$

$$V_L^R = -k_R + \bar{p}_A(\mu_{YR}) + \delta\epsilon k_O(1-\tau) + \delta(1-\epsilon)\max\{\ \bar{p}_{AL} + \delta k_R\ ,\ k_O(1-\tau) + V_O\ \} \quad ; \quad (34)$$

$$V_L^O = -k_O + \bar{p}_A(\mu_{YO}) + \delta\epsilon k_O(1-\tau) + \delta(1-\epsilon)\max\{\ \bar{p}_{AL} + \delta k_R\ ,\ k_O(1-\tau) + V_O\ \} \quad . \quad (35)$$

The taxation instrument does achieve the goal, as the next theorem will show.

Theorem 6. *Suppose that $\psi_A \leq 2q$. If the tax τ satisfies*

$$\tau \quad \geq \quad \left[\frac{(1-\delta)(\theta_{BH}-\theta_L)}{(\theta_{AH}-\theta_L)+\delta(\theta_{AH}-\theta_{BH})}\right] \quad , \tag{36}$$

then there exists an efficient, robust separating equilibrium.

5 Discussion

This section discusses various possible extensions alluded to earlier in the paper.

5.1 Revelation of client types

At the beginning of Section 3 we assumed that the seller of a practice can prove his client's type to prospective buyers. The main consequence of this is that anyone with an A-client can sell this match as a practice. In any steady-state equilibrium in which practices have value, new practices will not be created, since old ones will net be discarded. Thus matches with A-clients cannot be formed on the market. But other assumptions are possible with regards to the transfer of information about a client's type.

At one extreme, suppose client types were perfectly observable. Then professionals would seek out A-clients on the market, applying price discrimination. The role of practices would be lessened, but not eliminated. High-quality professionals would still have an incentive to buy practices, as a way of signalling their type. The assumption does not seem quite reasonable: a client's willingness-to-pay or level of need is seldom an outwardly apparent characteristic.

At the other extreme, if sellers of practices had no way of revealing their clients' types, then buyers would have no way of obtaining this information. All pros, including those with B-clients, could claim that they were matched with A-clients and try to sell their match as an A-practice. Buyers would be in the dark as to whether the client was truly type-A or not. A-practices and B-practices would be sold for the same price, or not at all.

A more reasonable assumption would be the following. When a professional offers his practice for sale, he makes known to a prospective buyer the last price paid by the client. If that price is high enough, then the buyer will be satisfied that only an A-client would have paid it. In actual practice markets, a professional buying a practice may have access to "the books," or accounting records, which include records of past transactions between the seller of the practice and his clientele.

In our model, this would add the following requirements: when a retiring pro sells a practice, the last price paid must be greater than what B-clients pay old pros in equilibrium ($\bar{p}_{BH}$, $\bar{p}_{BL}$ or p_O); similarly, when an old pro sells a practice, the last price paid must be greater than what B-clients pay young pros in equilibrium (p_Y). For H-pros who buy practices when young and want to sell them when they retire, this is no problem: the last price they receive from their clients is $\bar{p}_{AH}$. But L-pros cannot hold practices for two periods; and depending on μ_{OO} and μ_{OR}, old pros may not be able to buy practices and sell them for k_R a period later. Being with an A-client no longer guarantees one has a practice to sell.

These added restrictions would make the separation of H-pros and L-pros easier to achieve. There would in fact be efficient separating equilibria for $\psi_A > 2q$. In these equilibria, not all clients are in practices: some of them hire pros during the market phase, and hence α can be positive.[5] However, they would not be robust to the ϵ-shock. The equilibria in our current model would continue to be equilibria with this new assumption, and they would still have the same efficiency and robustness properties. All our theorems would continue to hold.

5.2 Clients on the short side

We have assumed throughout that $\psi_A + \psi_B > 2$, i.e. that clients outnumber professionals. Here we consider some implications of assuming that $\psi_A + \psi_B < 2$, i.e. that it was professionals who outnumbered clients. We nonetheless assume $\psi_A + \psi_B > 2q$, to prevent H-pros from servicing the entire population.

Because professionals outnumber clients, competition forces some of them to offer their services for free. Moreover, some professionals must remain unemployed, at least for one period. Indeed there is no guarantee that a professional who works will do so for two periods. In Example 3, which follows, some pros work when young and old, others work only when young, and the rest do not work at all. Specifically, each period a measure 0.2 of young H-pros buy practices and the remaining young H-pros go on the market (where there are only B-clients). All H-pros' matches are renewed, hence they work for two periods. A measure 0.2 of young L-pros go on the market, see their matches dissolved, and do not return to the market — "no" means they do not offer their services. Finally, a measure 0.4 of L-pros do not work in either period.

Example 3. Let parameter values be $\theta_L = 1$, $\theta_{BH} = 2$, $\theta_{AH} = 3$, $q = 0.4$, $\psi_A = 0.4$, $\psi_B = 0.6$, and $\delta = 2/3$. Then the following is an equilibrium, illustrated by Figure 5. Market prices are $p_Y = 0$ and $p_O = 0$. Market proportions are $q_Y = 0.5$ and $q_O = 0$. Threshold prices are $\bar{p}_{AH} = 1$, $\bar{p}_{BH} = 0.5$, $\bar{p}_{AL} = -1$ and $\bar{p}_{BL} = -0.5$. Market values are $V_A = 6$ and $V_B = 4.5$. Beliefs are $\mu_{YR} = 1$ and $\mu_{YO} = \mu_{OR} = \mu_{OO} = 0$. Practice prices are $k_R = 2.4$. Lifetime utility is $1/3$ for type-H pros and 0 for type-L pros.

Very little else changes, however. All of the theorems in the paper continue to hold. The equilibria constructed in the proofs of those theorems continue to be equilibria, if we set $p_Y = p_O = 0$ and adjust the threshold prices ($\bar{p}_{AH}$ and so on) accordingly. Therefore the main results do not depend on who is on the short side of the market.

[5] Examples are available upon request.

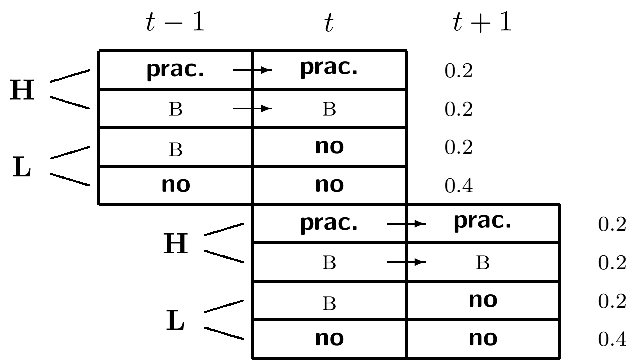

FIGURE 5. Professionals' histories in Example 3. Here "no" means the professional does not supply his services and is unemployed.

5.3 B-practices

Throughout the paper we have assumed that a practice is a match with an A-client. This was made for convenience, to keep things as simple as possible. The model can certainly accommodate trade in B-practices as well. Because a professional selling a practice is able to reveal (verifiably) his client's type to prospective buyers, A-practices and B-practices would be distinct commodities on the market — and of course one would be still have to be make the distinction between practices sold by retiring pros and those sold by old ones, making four different prices for practices.

The following shows an example. Here we call k_{BO} the price of a B-practice sold by an old professional, and k_{BR} the price of one sold by a retiring one.

Example 4. Let parameter values be $\theta_L = 1$, $\theta_{BH} = 2$, $\theta_{AH} = 3$, $q = 0.3$, $\psi_A = 0.4$, $\psi_B = 2$, and $\delta = 2/3$. Then the following is an equilibrium, illustrated by Figure 6. Market prices are $p_Y = p_O = 1$. Market proportions are $q_Y = q_O = \alpha = 0$. Threshold prices are $\bar{p}_{AH} = 3$, $\bar{p}_{BH} = 2$, and $\bar{p}_{AL} = \bar{p}_{BL} = 1$. Market values are $V_A = V_B = 0$. Beliefs are $\mu_{YR} = 1$ and $\mu_{YO} = \mu_{OR} = \mu_{OO} = 0$ (for both types of practices). Practice prices are $k_R = 4$, $k_O = 2.5$, $k_{BR} = 1$ and $k_{BO} = 0$. Lifetime utility is 25/9 for type-H pros and 5/3 for type-L pros.

One may ask why a professional would buy a B-practice, since it means acquiring a relatively low-paying client. The reason is that it gives the professional a way to signal his type. This is apparent in the example. H-pros who buy B-practices are able to charge their clients $\bar{p}_{BH} = 2$. L-pros, who are also servicing B-clients, can only charge them $p_Y = p_O = 1$.

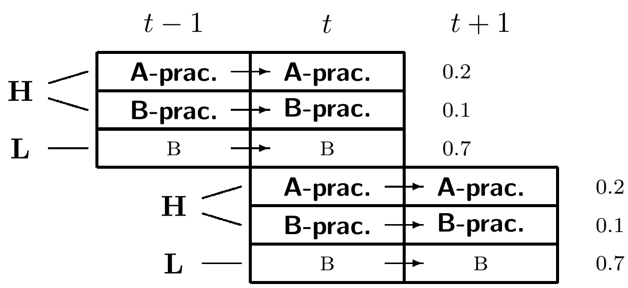

FIGURE 6. Professionals' histories in Example 4.

6 Conclusion

In this paper we have presented a model in which professionals provide services to clients as well as buy and sell practices from each other. We began by laying out a mathematical structure which allows the study of these concurrent markets. This structure is characterized by (i) complete endogeneity of prices for both services and practices; (ii) very little knowledge by agents regarding other agents' types; (iii) the absence of any exogenous frictions such as switching costs.

It was shown that without trade in practices, the market for services is generally inefficient. The introduction of practices provides high-quality professionals with the opportunity to signal their types by the purchase of a practice, and so yields a socially efficient outcome. Our next step was the introduction of a simple shock: each period, an exogenously determined portion of all professionals in mid-career are forced to leave the economy, but are allowed to sell practices before they go (as commonly happens to real professionals). We found that for some parameter values, the socially efficient equilibria obtained previously were no longer tenable when the shock was added. In these cases, however, a tax levied on mid-career sales of practices could ensure the existence of a robust efficient equilibrium.

Appendix

Proof of Lemma 1. Recall that matches between i-clients and j-pros are renewed if $\bar{p}_{ij} > p_O$ and dissolved if $\bar{p}_{ij} < p_O$. From equations (2) through (5) we can see that $\bar{p}_{AL} < \bar{p}_{AH}$ and $\bar{p}_{BL} < \bar{p}_{BH}$. This means that L-pros are at least as likely to see their matches dissolved as H-pros are. Hence the proportion of old H-pros among all old pros on the market cannot be higher than q. So we have, as a preliminary result, $q_O \leq q$.

If any A-clients hire young pros, it means that $(1-\delta)V_A = q\theta_{AH}+(1-q)\theta_L-p_Y$. Some of these will be young pros will be type-H. If any of these A-H matches are dissolved, it means that $p_O \geq \bar{p}_{AH}$. Combining these two results with (2) leads to

$p_O > p_Y$. But with $p_O > p_Y$ and $q_O \leq q$, there will be no demand on the market for these old pros whose matches were just dissolved [see (9)]. That would mean excess supply, which cannot happen in equilibrium. We conclude that all A-H matches are renewed. Following a similar reasoning, all B-H matches are renewed as well. Therefore no H-pros end up on the market when old, and so $q_O = 0$ in equilibrium.[6]

Proof of Theorem 1. Suppose $\psi_A \geq 1 + q$. Suppose that some young pros are hired by B-clients. This means that for an unmatched B-client, hiring a young pro is weakly preferred to the other two options (hiring an old pro, hiring no one). If that is the case, then it will be *strictly* preferred for an unmatched A-client — we can see this by inspecting (9) and recalling from Lemma 1 that $q_O = 0$. We can use these facts, together with (4), to find that $p_O > \bar{p}_{AL}$, so that all A-L matches will be dissolved. A-H matches, however, will be renewed (see Lemma 1). Putting all this together: in any period there will be A-clients hiring young pros, but that will be a measure less than 1 since some young pros are hired by B-clients; there will be A-clients renewing matches with H-pros, but that will be a measure less than q; the rest will be unmatched. But unmatched A-clients only want to hire young pros: hence we have excess demand. We conclude that B-clients will not hire young pros if $\psi_A \geq 1 + q$.

Further work shows that when $\psi_A \geq 1+q$ markets clear at $p_Y = q\theta_{AH} + (1-q)\theta_L$ and $p_O = \theta_L$. In equilibrium all young pros will be hired by A-clients. The H-pros among these will renew their matches. Therefore all H-pros will be matched with A-clients, and so equilibrium is efficient.

Proof of Lemma 2. If B-clients do not hire any young pros, then all young pros must be hired by A-clients. We know from Lemma 1 that all A-H matches are renewed. So A-clients are required to keep all young pros (a measure 1) and all old H-pros (a measure q) employed. However, this situation has been ruled out by our assumption that $\psi_A < 1 + q$.

Proof of Lemma 3. We cannot have $p_O < \theta_L$, as that would mean $V_B > 0$, which has already been ruled out. We know from Lemma 2 that B-clients hire young pros; hence B-L matches will be formed. If $p_O > \theta_L$, we have $p_O > \bar{p}_{BL}$ and these matches will be dissolved. Consequently some old pros will end up supplying their services on the market. But with $p_O > \theta_L$ (and the fact that $q_O = 0$) no client will want to hire an old pro; hence there will be excess supply.

[6]If there are no old pros at all on the market, then technically speaking q_O is not well defined. This situation cannot arise in the benchmark model, but is possible in the model with practices. If this is to happen, then q_O and p_O must be such that old pros' services are neither supplied nor demanded. One can show that this is only possible with $q_O = 0$ and $p_O = \theta_L$.

Proof of Theorem 3. We first address the case where H-pros outnumber A-clients, i.e. where $\psi_A \leq 2q$, then consider the complementary case $\psi_A > 2q$.

Part I. Suppose $\psi_A \leq 2q$. An equilibrium exists, described as follows. Each period a measure $\psi_A/2$ of young H-pros buy practices; each one holds it for two periods and sells it when he retires. No one else buys practices. This means there will be $q - (\psi_A/2)$ young H-pros on the market in each period. There will also be $1 - q$ young L-pros, for a total of $1 - (\psi_A/2)$ young pros. Therefore

$$q_Y = \frac{q - \psi_A/2}{1 - \psi_A/2} = \frac{2q - \psi_A}{2 - \psi_A} \quad . \tag{37}$$

Let market prices be $p_Y = q_Y\theta_{BH} + (1 - q_Y)\theta_L$ and $p_O = \theta_L$. Let threshold prices be $\bar{p}_{AH} = q_Y\theta_{BH} + (1-q_Y)\theta_{AH}$, $\bar{p}_{BH} = \theta_{BH}$, $\bar{p}_{AL} = \theta_L - q_Y(\theta_{AH} - \theta_{BH})$ and $\bar{p}_{BL} = \theta_L$. All A-H and B-H matches are renewed, all A-L matches are dissolved, and B-L matches are either renewed or dissolved. There are no old H-pros on the market, so $q_O = 0$. All A-clients are in practices, so $\alpha = 0$.

For client beliefs, we have $\mu_{YR} = 1$, which is consistent with the above behaviour, and $\mu_{YO} = \mu_{OO} = \mu_{OR} = 0$, which are off the equilibrium path. Let practice prices be the following:

$$k_R = (1 - q_Y)\left[\frac{(\theta_{AH} - \theta_L) + \delta(\theta_{AH} - \theta_{BH})}{1 - \delta^2}\right] \quad ; \tag{38}$$

$$k_O = \delta(1 - q_Y)\left[\frac{(\theta_{AH} - \theta_L) + (\theta_{AH} - \theta_{BH})}{1 - \delta^2}\right] \quad ; \tag{39}$$

We can verify that no professional would want to buy a practice when old, hence $V_O = p_O = \theta_L$. Also, we can check that $\bar{p}_{AH} + \delta k_R > k_O + V_O$, meaning that H-pros prefer keeping their practices for two periods rather than only one. Substituting our results into the career-payoff equations, we can verify that

$$V_H^R = V_H^M > V_H^O \quad ; \tag{40}$$

$$V_L^M > \max\{V_L^R, V_L^O\} \quad . \tag{41}$$

These show that professionals' behaviour is optimal as stated. Note that $\bar{p}_{AH} > \theta_{BH}$, so a retiring pro selling a practice can prove that his client is type-A.

All equilibrium requirements are met. All A-clients are in practices owned by H-pros, so the number of A-H matches is at its maximum, and the equilibrium is efficient.

Part II. If $\psi_A > 2q$ there exists an equilibrium, described as follows. Each period n_H^R young H-pros and n_L^R young L-pros buy practices from retiring pros, while n_H^O young H-pros and n_L^O young L-pros buy them from old pros, where

$$n_H^R = q^2 \left[\frac{1 + \delta - (\psi_A - 2q)R}{\delta \psi_A + (1-\delta) q} \right] \quad ; \tag{42}$$
$$n_L^R = n_H^O = q - n_H^R \quad ; \tag{43}$$
$$n_L^O = \psi_A - 3q + n_H^R \quad ; \tag{44}$$

where $R \equiv (\theta_{AH} - \theta_{BH})/(\theta_{AH} - \theta_L)$. The four quantities n_H^R, n_L^R, n_H^O and n_L^O are all strictly between 0 and 1. Old pros never buy practices. H-pros keep their practices for two periods, whereas L-pros only keep them for one. Note that $n_H^R + n_H^O = q$ (meaning that all H-pros buy practices) and that $n_L^R + n_L^O = \psi_A - 2q$.

At the close of the practice phase, $n_H^R + n_H^O$ young H-pros and $n_L^R + n_L^O$ young L-pros own practices; also, $n_H^R + n_H^O$ old H-pros still own theirs, purchased the period before. The total number of practices is therefore $2(n_H^R + n_H^O) + (n_L^R + n_L^O)$, which is equal to ψ_A. That is, all A-clients are in practices. Each period the number of buyers who buy practices from retiring pros is $n_H^R + n_L^R = q$. The number of H-pros who retire and sell theirs is $n_H^R + n_H^O = q$. The number of buyers who buy practices from old pros is $n_H^O + n_L^O = \psi_A - 2q$. The number of old L-pros who sell theirs is $n_L^R + n_L^O = \psi_A - 2q$. Hence practice markets clear. For client beliefs, we have

$$\mu_{YR} = \frac{n_H^R}{q} \quad ; \quad \mu_{YO} = \frac{q - n_H^R}{\psi_A - 2q} \quad , \tag{45}$$

which are consistent with the behaviour described above; along with $\mu_{OR} = \mu_{OO} = 0$, which are off the equilibrium path.

All H-pros and A-clients are tied up in practices, so $q_Y = q_O = \alpha = 0$. Let market prices be $p_Y = p_O = \theta_L$. Threshold prices are $\bar{p}_{AH} = \theta_{AH}$, $\bar{p}_{BH} = \theta_{BH}$, and $\bar{p}_{AL} = \bar{p}_{BL} = \theta_L$. All matches with H-pros are renewed; A-L matches are not; B-L matches may or may not be. Let practice prices be

$$k_R = \left[\frac{\delta \mu_{YO} + (1-\delta) \mu_{YR}}{1-\delta} \right] (\theta_{AH} - \theta_L) \quad ; \tag{46}$$

$$k_O = \left[\frac{\mu_{YO}}{1-\delta} \right] (\theta_{AH} - \theta_L) \quad . \tag{47}$$

It is straightforward to show that old pros will not want to buy practices, that H-pros will choose to keep practices for two periods, while L-pros who buy practices will only want to keep them for one. Payoffs satisfy

$$V_H^R = V_H^O > V_H^M \quad ; \tag{48}$$
$$V_L^M = V_L^R = V_L^O \quad . \tag{49}$$

The behaviour with which we began the proof is optimal. All equilibrium requirements are met. Moreover, since all H-pros own practices (and so are matched with A-clients), this equilibrium is efficient.

Proof of Theorem 4. Suppose $\psi_A \leq 2q$. Suppose there is an efficient, robust equilibrium. If a positive measure of L-pros are matched with A-clients, there will be fewer than $2q$ A-clients left and some H-pros will have to be matched with B-clients. Hence equilibrium will be inefficient. So L-pros cannot buy practices, nor can they be hired by A-clients on the market. This has a number of implications.

FIRST RESULT. All young L-pros go on the market, since they do not buy practices. Hence $q_Y \leq q$. They must be hired by B-clients, which implies $p_Y = q_Y\theta_{BH} + (1 - q_Y)\theta_L$, since we must have $V_B = 0$. This yields the result $p_Y < \theta_{BH}$, which we will use towards the end of the proof.

SECOND RESULT. When L-pros become old (and survive the shock) they cannot buy practices: their matches must be renewed at $\bar{p}_{BL}$ or they must be hired by B-clients on the market at p_O. If $p_O > q_O\theta_{BH} + (1 - q_O)\theta_L$ neither of these things can happen: matches will be dissolved because $p_O > \bar{p}_{BL}$, and B-clients will not hire old pros because doing so would give them a negative payoff. We know that $p_O \geq q_O\theta_{BH} + (1-q_O)\theta_L$, since we must have $V_B = 0$. Therefore $p_O = q_O\theta_{BH} + (1-q_O)\theta_L$. Any positive value for q_O would mean $p_O > \bar{p}_{BL}$, so that all old L-pros would end up on the market; so regardless of whether q_O is positive or zero, we will certainly have $q_O \leq q$. We conclude that $p_O < \theta_{BH}$, our second result.

THIRD RESULT. Because L-pros do not buy practices when old, we must have $V_O = p_O$. Substituting this into (28) and (31), keeping in mind that $\theta_L \leq p_O < \theta_{BH}$, we see that $V_H^M > V_L^M$. Because L-pros do not buy practices when young, V_L^M must be the maximum among all three L-pro payoffs. So we have $V_H^M > V_L^R$ and $V_H^M > V_L^O$.

FOURTH RESULT. Suppose some young H-pros buy practices from retiring ones. This implies $V_H^R \geq V_H^M$, hence $V_H^R > V_L^R$ (see third result). Looking at (26) and (29), we see that this is only possible if $\bar{p}_{AH} + \delta k_R > k_O + V_O$. This means that those who survive the shock will renew their matches. Now suppose that some H-pros buy practices from old pros. This implies $V_H^O \geq V_H^M$, hence $V_H^O > V_L^O$ Again, this can only happen if $\bar{p}_{AH} + \delta k_R > k_O + V_O$, which means that those who survive the shock renew their matches. And if some young pros go on the market, those who survive the shock will also renew their matches, since $\bar{p}_{BH} = \theta_{BH} > p_O = V_O$. We conclude that no H-pros ever buy practices when old.

FIFTH RESULT. Since someone *must* buy practices (otherwise we would be in a pooling equilibrium), it must be young H-pros. In fact young H-pros must buy practices from both old and retiring pros. If they only bought from retiring pros, some of the buyers would be affected by the shock and wish to sell their practices when old. But there would be no one willing to buy them. If they only bought from old pros, then those buyers who survived the shock would renew their matches (see fourth result) and sell their practices upon retiring. But there would be one willing to buy them. Hence young H-pros will buy from both old and retiring professionals. Consistency of beliefs requires therefore $\mu_{YR} = \mu_{YO} = 1$, which gives us $\bar{p}_A(\mu_{YR}) = \bar{p}_A(\mu_{YO}) = \bar{p}_{AH}$. $V_H^R = V_H^O$ is required for optimality, and since $\bar{p}_A(\mu_{YR}) = \bar{p}_A(\mu_{YO})$ we must have $k_R = k_O$.

SIXTH RESULT. Combining the results we have so far, we can rewrite V_H^R and V_H^M as follows:

$$V_H^R = -k_R + \bar{p}_{AH} + \delta\epsilon k_R + \delta(1-\epsilon)(\bar{p}_{AH} + \delta k_R) \quad ; \tag{50}$$
$$V_H^M = p_Y + \delta(1-\epsilon)\bar{p}_{BH} \quad ; \tag{51}$$

By definition $V_O \geq -k_O + \bar{p}_A(\mu_{OO}) + \delta k_R$. Together with the fact that $\bar{p}_A(\mu_{OO}) \geq \bar{p}_{AL}$, this means that $k_O + V_O \geq \bar{p}_{AL} + \delta k_R$. Also, note that $\bar{p}_{BL} = \theta_L \leq p_O$. We can now rewrite V_L^R and V_L^M as

$$V_L^R = -k_R + \bar{p}_{AH} + \delta\epsilon k_R + \delta(1-\epsilon)(k_R + V_O) \quad ; \tag{52}$$
$$V_L^M = p_Y + \delta(1-\epsilon)V_O \quad . \tag{53}$$

Optimality requires $V_H^R \geq V_H^M$. Using (50) and (51), this can be rewritten as

$$(1-\delta)k_R \leq \bar{p}_{AH} - \left[\frac{p_Y + \delta(1-\epsilon)\bar{p}_{BH}}{1+\delta(1-\epsilon)}\right] \quad . \tag{54}$$

Since $\bar{p}_{BH} = \theta_{BH} > p_Y$ by the first result, (54) leads to $(1-\delta)k_R < \bar{p}_{AH} - p_Y$. For L-pros not to buy practices we need $V_L^M \geq V_L^R$. Using (52) and (53), this can be rewritten as $(1-\delta)k_R \geq \bar{p}_{AH} - p_Y$. Thus we have a contradiction. This establishes that we cannot have an efficient equilibrium that is robust to the ϵ-shock.

Proof of Theorem 5. We will show that the equilibrium described in Part II of the proof of Theorem 3 is robust, by slightly modifying some of its quantities to account for the ϵ-shock. Note that as ϵ tends to zero, all the quantities in this proof tend to their counterparts in the proof of Theorem 3. Each period n_H^R young H-pros and n_L^R young L-pros buy practices from retiring pros, while n_H^O young H-pros and n_L^O young L-pros buy practices from old pros, where n_H^R is given by (42), and

$$n_L^R = q(1-\epsilon) - n_H^R \quad ; \tag{55}$$
$$n_H^O = q - n_H^R \quad ; \tag{56}$$
$$n_L^O = \psi_A - 3q + 2\epsilon q + n_H^R \quad . \tag{57}$$

These quantities are all positive (for ϵ small enough). Old pros never buy practices. H-pros keep their practices for two periods (unless they are forced to leave), and L-pros who buy practices sell them when they are old. Each period, the number of old pros who sell their practices is $(n_L^R + n_L^O) + \epsilon(n_H^R + n_H^O)$; this is equal to $n_H^O + n_L^O$, the number who buy practices from old pros. The number of retiring pros who sell practices is $(1-\epsilon)(n_H^R + n_H^O)$; this is equal to $n_H^R + n_L^R$, the number who buy practices from retiring pros. The total number of practices is $(2-\epsilon)(n_H^R + n_H^O) + (n_L^R + n_L^O) = \psi_A$.

Client beliefs are now

$$\mu_{YR} = \frac{n_H^R}{(1-\epsilon)q} \quad ; \quad \mu_{YO} = \frac{q - n_H^R}{\psi_A - 2q + 2\epsilon q} \quad , \tag{58}$$

and $\mu_{OR} = \mu_{OO} = 0$. Market prices and proportions are the same as before. Practice prices are again given by (46) and (47), but with the new beliefs. Optimality conditions (48) and (49) are satisfied, for ϵ small enough. All equilibrium conditions are met; and in particular there are buyers for the practices sold by shock-affected professionals, therefore the equilibrium is robust. Since all q young H-pros and all $(1-\epsilon)q$ surviving old H-pros own practices, the equilibrium is efficient.

Proof of Theorem 6. Suppose $\psi_A \leq 2q$. Then an equilibrium exists, as follows. Each period n_H^R young H-pros buy practices from retiring pros, and n_H^O buy them from old pros, where

$$n_H^R = \left(\frac{1-\epsilon}{2-\epsilon}\right)\psi_A \quad \text{and} \quad n_H^O = \left(\frac{\epsilon}{2-\epsilon}\right)\psi_A \quad . \tag{59}$$

No one else buys practices. Those who buy practices keep them for two periods, except when affected by the shock. At the end of each practice phase, $n_H^R + n_H^O$ young pros and $(1-\epsilon)(n_H^R + n_H^O)$ old pros own practices, making the total ψ_A. That is, all A-clients are in practices. A measure $(1-\epsilon)(n_H^R + n_H^O)$ are sold by retiring pros; this is equal to n_H^R. A measure $\epsilon(n_H^R + n_H^O)$ are sold by old pros, i.e. those practice-owners who are forced to leave; this is equal to n_H^O. So practice markets clear.

Client beliefs are $\mu_{YR} = \mu_{YO} = 1$ and $\mu_{OR} = \mu_{OO} = 0$. Any young H-pros not buying practices go on the market. They form of a proportion q_Y of young pros on the market, with

$$q_Y = \frac{q - n_H^R - n_H^O}{1 - n_H^R - n_H^O} = \frac{(2-\epsilon)q - \psi_A}{2 - \epsilon - \psi_A} \quad . \tag{60}$$

For ϵ small enough, this is well-defined and positive, since $\psi_A < 2q$. As for old pros, we have $q_O = 0$. Market prices are $p_Y = q_Y\theta_{BH} + (1-q_Y)\theta_L$ and $p_O = \theta_L$. Threshold prices are $\bar{p}_{AH} = q_Y\theta_{BH} + (1-q_Y)\theta_{AH}$, $\bar{p}_{BH} = \theta_{BH}$, $\bar{p}_{AL} = \theta_L - q_Y(\theta_{AH} - \theta_{BH})$ and $\bar{p}_{BL} = \theta_L$.

Let practice prices be

$$k_R = k_O = (1-q_Y)\left[\frac{(\theta_{AH} - \theta_L) + \delta(1-\epsilon)(\theta_{AH} - \theta_{BH})}{1 - \delta^2(1-\epsilon) - \delta\epsilon(1-\tau)}\right] \tag{61}$$

Then it is straightforward to verify that

$$V_H^M = V_H^R = V_H^O \quad ; \tag{62}$$

$$V_L^M \geq V_L^R = V_L^O \quad ; \tag{63}$$

the latter with strict inequality if (36) is satisfied strictly. We can also verify that old pros have no incentive to buy or sell practices (unless forced to leave the economy). Thus the behaviour described above is optimal. The equilibrium is efficient, since all A-clients are in practices owned by H-pros. It is robust, since it takes the shock into account.

References

Deb, Joyee. 2012. "Observability and Sorting in a Market for Names." *Journal of Economics and Management Strategy* 21(2): 301-338.

Fudenberg, Drew and Jean Tirole. 1991. *Game Theory.* Cambridge, MA: MIT Press.

Garicano, Luis and Tano Santos. 2004. "Referrals." *American Economic Review*, 94(3): 499-525.

Hakenes, Hendrik and Martin Peitz. 2007. "Observable Reputation Trading." *International Economic Review*, 48(2): 693-730.

Kreps, David M. 1990. "Corporate Culture and Economic Theory." In *Perspectives on Positive Political Economy*, ed. James E. Alt and Kenneth A. Shepsle, 90-143. Cambridge, UK: Cambridge University Press.

Mailath, George J. and Larry Samuelson. 2001. "Who Wants a Good Reputation?" *Review of Economic Studies*, 68(2): 415-441.

Tadelis, Steven. 1999. "What's in a Name? Reputation as a Tradeable Asset." *American Economic Review*, 89(3): 548-563.

Tadelis, Steven. 2002. "The Market for Reputations as an Incentive Mechanism." *Journal of Political Economy*, 110(4): 854-882.

Tadelis, Steven. 2003. "Firm Reputation with Hidden Information." *Economic Theory*, 21(2-3): 635-651.

Wang, Tianxi. 2011. "The Dynamics of Names: A Model of Reputations." *International Economic Review*, 52(4): 1039-1058.