The Optimal Amount of Falsified Testimony

Winand Emons
Claude Fluet

Juin/June 2005

We thank Bruno Deffains, Oliver Fabel, Marie-Cécile Fagart, Christian Ghiglino, Gerd Mühlheusser, and Harris Schlesinger for helpful comments. The usual disclaimer applies.
Abstract:
An arbiter can decide a case on the basis of his priors or he can ask for further evidence from the two parties to the conflict. The parties may misrepresent evidence in their favor at a cost. The arbiter is concerned about accuracy and low procedural costs. When both parties testify, each of them distorts the evidence less than when they testify alone. When the fixed cost of testifying is low, the arbiter hears both, for intermediate values one, and for high values no party to all. The ability to commit to an adjudication scheme makes it more likely that the arbiter requires evidence.

Keywords: Evidence production, procedure, costly state falsification, adversarial, inquisitorial

JEL Classification: D82, K41, K42
1 Introduction

How much testimony should an arbiter require when he knows that the parties to the conflict spend considerable resources to misrepresent evidence in their favor? When he hears no witnesses, no resources are wasted on fabricating evidence, yet the judge’s adjudication will be erroneous, leading to a social loss from inaccurate decisions. If parties testify, the decision will be more accurate, yet resources are wasted on fabricating evidence. Requiring, e.g., testimony from two rather than one party will lead to a duplication of the costs to produce misleading information. The purpose of this paper is to analyze this trade-off between procedural costs and the benefits of truth-finding.

An arbiter has to decide on an issue which we take to be a real number; for example, the adjudicated value are the damages that one party owes to the other. The defendant wants the damages to be small whereas the plaintiff wants them to be large. The parties thus have conflicting interests. The arbiter can decide the case solely on the basis of his priors. Alternatively, he can ask for further evidence from the two parties to the conflict.

Both parties know the actual realization of the damages. Presenting evidence involves a fixed cost. Moreover, they can boost the evidence in either direction. Distorting the evidence is, however, costly. The further a party moves away from the truth, the higher the cost; for example, expert witnesses charge more the more they distort the truth.

The arbiter first announces whether he wants to hear no, one, or both parties. Then he announces an adjudication rule which maps the parties’ reports into an adjudicated value. For simplicity, we take this adjudication function to be linear. The arbiter minimizes the sum from the loss of inaccurate adjudication plus the weighted parties’ submission costs. He thus trades-off the social benefits of truth-finding against the cost of obtaining evidence.

We first look at the case where the arbiter can commit to an adjudication
function. When he hears no party, he adjudicates the mean. When he decides to hear the parties, the adjudicated value is the weighted average of the true value and the mean. The more weight he gives to the parties’ submission costs, the more the adjudicated value is biased towards the mean. The arbiter commits ex ante not to give too much weight to the evidence presented by the parties, thereby inducing them not to boost the evidence excessively.

When the arbiter hears only one party, this party lies more than his extent of lying when both parties submit. When only one party presents evidence, the arbiter gives more weight to the party’s submission, thereby inducing him to falsify more. Accordingly, confronting the parties in adversarial hearings induces either of them to distort the evidence less than when only one party is heard. Yet when both parties are heard, both are involved in boosting the evidence.

The optimal number of parties to submit evidence depends on the fixed submission cost and the weight given to the cost of obtaining evidence. When the fixed cost of presenting evidence and the weight given to submission costs are small, the arbiter hears both parties. For intermediate values he goes for one party, and for large values he hears no party at all. Accordingly, even when the cost of obtaining evidence is accounted for, it may still be optimal to hear both parties: the duplication of the fixed submission costs is more than compensated by the lower cost of boosted evidence.

The parties’ submissions are monotone in the true value at stake. Therefore, the arbiter can ex post infer the true value. By the choice of the adjudication function he can, however, commit not to adjudicate the true value. This implies a loss from inaccurate adjudication. The arbiter is thus tempted to renege ex post on the adjudication function to minimize the loss from inaccurate adjudication.

In a second step we look, therefore, at the scenario where the adjudicator can no longer ex ante commit to adjudication schemes. If the parties’ statements reveal the truth, the arbiter will ex post adjudicate the true value.
More weight is therefore now given to the parties’ presentations than in the commitment case. This implies that the parties will boost the evidence more. Roughly speaking, in the no commitment scenario the arbiter can give either “full weight” to the parties’ evidence or no weight at all by not listening to them in the first place. This in turn implies that it becomes more likely that the arbiter doesn’t require any evidence at all from the two parties.

We thus develop a simple framework which allows us to determine when an arbiter should hear two, one, or no party at all. The lower the fixed costs of making a presentation and the less the arbiter weighs the submission costs, the more parties should be heard in the proceedings. The ability to commit to an adjudication scheme turns out to be crucial. The inability to commit makes it more likely that the arbiter hears no party at all and adjudicates solely on the basis of his priors. If it is optimal to receive evidence, the inability to commit also makes it more likely that the two parties will be heard rather than a single one.

It is standard in the literature to view accuracy in adjudication and procedural economy as the objectives at which legal procedures should aim. Adversarial systems of discovery clearly motivate parties to provide evidence. Nevertheless, they are often criticized for yielding excessive “influence” expenditures in the sense of Milgrom (1988): they lead to unnecessary duplication and costly overproduction of misleading information. Tullock (1975, 1980) provides a well-known statement of this opinion. Our contribution is to tackle the cost/accuracy trade-off on the basis of the so-called “costly state falsification” approach; see Lacker and Weinberg (1989), Maggi and Rodriguez-Clare (1995), and Crocker and Morgan (1998).\footnote{For example, Crocker and Morgan (1998) analyze the falsification of insurance claims. The agent is privately informed about the true value of the loss and is able to misrepresent this quantity at a cost.}

One approach to court decision making views the trial outcome as an exogenous function of the litigants’ levels of effort or expenditure. See Cooter and Rubinfeld (1989) for a review of the earlier literature; other examples...
include Bernardo, Talley, and Welch (2000), Farmer and Pecorino (1999), Katz (1988), and Parisi (2002). In these papers adjudication is a zero-one variable where a party either wins or loses. Parties engage in a rent-seeking game, leading to excessive expenditures. Our approach differs in that the arbiter’s adjudication function is not specified exogenously; we derive the optimal adjudication function. In our model, the outcome is based on the evidence provided by the parties, rather than on unobservable effort. Moreover, in our set-up the arbiter understands the parties’ incentives to “falsify” the submitted evidence.

Our approach also differs from other expenditure-based models which consider guilty or innocent defendants; see, e.g., Rubinfeld and Sappington (1987). A defendant’s type is private information. The defendant’s level of effort determines the probability that he will be found innocent. This probability function is exogenously given and differs between types. The arbiter minimizes the sum of the losses from type 1 and type 2 errors plus the defendant’s expected effort cost with respect to the standard of proof and the penalty for conviction. When effort is not observable, both types of defendant provide effort, yet the innocent defendant more than the guilty one. The major difference to our set-up is that the judge faces just one defendant who can be of two types. Rubinfeld and Sappington do not address the question of how many witnesses should be heard.

We also differ from another well-known strand of literature in which parties cannot falsify the verifiable evidence as such, but are able to misrepresent it by disclosing only what they see fit; see Sobel (1985), Milgrom and Roberts (1986), and Shin (1998). Finally, our paper is related to the literature comparing adversarial with inquisitorial procedures of truth-finding; see Shin (1998), Dewatripont and Tirole (1999), and Palumbo (2001). In the inquisitorial system a neutral investigator searches for evidence, in the adversarial system the parties to the conflict present the evidence. The last two papers compare the two procedures in terms of the costs to motivate
agents to gather and produce verifiable information. By contrast, we look at the question how much testimony from interested parties should be used. Our judge or arbiter is therefore an active agent since he directs how the procedure will evolve.\(^2\)

The paper is organized as follows. In the next section we describe our basic set-up. The following two sections derive the optimal procedures for the case where the judge can commit to an adjudication scheme. In the subsequent section we look at the case where the judge cannot commit. Section 6 concludes.

2 The Model

The issue to be settled is the value of \(x \in \mathbb{R}\). The adjudicator—regulatory commission, court, etc.—has prior beliefs represented by the density \(f(x)\) with full support over the real line and mean \(\mu\). Units are normalized so that the variance equals unity.\(^3\) The arbiter’s initial beliefs may be taken as being shaped from information publicly available at the beginning of the proceedings.

The arbiter can adjudicate solely on the basis of his priors. Alternatively, he can require further evidence to be submitted from perfectly informed but self-interested actors denoted \(A\) and \(B\). Party \(A\) would like the adjudicated value of \(x\) to be large while party \(B\) would like it to be small. For example, the adjudicated value may be the damages that should be paid to the plaintiff

\(^2\)It is of course possible to interpret our cases where the judge hears no or one agent as inquisitorial and the case where he hears both parties as adversarial. Nevertheless, note that our judge has full control over whom he wants to hear, a feature typically associated with inquisitorial systems; see Posner (1973, 1999). Yet another approach can be found in Froeb and Kobayashi (1996, 2001) and Daughety and Reinganum (2000) who model the adversarial provision of evidence as a game in which two parties engage in strategic sequential search.

\(^3\)We assume full support over the real line in order to avoid boundary conditions. The probability of extreme values of \(x\) can be made, however, arbitrarily small.
A by the defendant $B$; in a divorce case it may be the amount of support $A$ should get from $B$; in regulatory hearings about the rental charge for a local loop the incumbent wants the charge to be high whereas the entrant wants it to be low.

Submissions by the parties are costly. A submission is of the form “the value of the quantity at issue is $x_i$, $i = A, B$. It should be thought of as a story or argument rendering $x_i$ plausible, together with the supporting documents, witnesses, etc. The cost of a presentation is

$$c_i(x_i, x) = \gamma + \frac{1}{2} \gamma_i (x_i - x)^2, \quad i = A, B,$$

where $\gamma \geq 0$ and $\gamma_i > 0$. $x$ is the actual value, which is observed by the party, and $x_i$ is the testimony or the statement submitted.

A distorting presentation is more costly than simply reporting the naked truth as it involves more fabrication. We take a quadratic function to capture the idea that the cost of misrepresenting the evidence increases at an increasing rate the further one moves away from the truth: it becomes more difficult to produce the corresponding documents or experts charge more the more they distort the truth.\footnote{Using quadratic falsification costs is standard in the literature. Maggi and Rodríguez-Clare (1995) interpret $\gamma_i$ as capturing the publicness of information. If $\gamma_i = 0$, falsification is costless, therefore, information is purely private. As $\gamma_i$ increases, it becomes more costly to falsify information and for an arbitrarily large $\gamma_i$ the public-information model obtains.}

The parties’ capacity to falsify—their “credibility”—is common knowledge. Falsification costs may differ between the parties, but for simplicity the cost of reporting the true state of the world, $\gamma$, is the same. Total submission cost is $C = 0$ if no evidence is required from the parties. It is $C = c_i$ if only party $i$, $i = A, B$, submits. Otherwise it is $C = c_A + c_B$.

The arbiter is concerned about the loss from inaccuracy in adjudication and the parties’ submission costs. Accordingly, there is a potential trade-off between procedural costs and the social benefits of correct adjudication.
From the arbiter’s perspective, the total social loss is

\[ L = l + \theta C \]

where \( l \) is the loss from inaccurate adjudication or “error costs”, \( C \) is total submission costs, and \( \theta \geq 0 \) is the rate at which the arbiter is willing to trade-off submission costs against accuracy. If, e.g., \( \theta = 0 \), accuracy is his only concern; for \( \theta > 0 \), the arbiter is willing to sacrifice some accuracy to save on submission costs.

Let \( \hat{x} \) denote the arbiter’s decision. The loss from inaccurate adjudication is

\[ l(\hat{x}, x) = \frac{1}{2} (\hat{x} - x)^2. \]

If the true value is adjudicated, error costs are zero. The more the decision errs in either direction, the higher the losses from inaccurate adjudication. Losses increase at an increasing rate the further one moves away from the truth.

At the start of the proceedings, the arbiter determines a procedure. This consists of two elements: first, the procedure specifies which party, if any, is required to submit evidence; secondly, it specifies how the arbiter’s decision will depend on the evidence submitted. We denote the first part by the decision \( d \in \{N, S, J\} \) where \( N \) stands for no party being heard, \( S \) for only a single party being heard (this would specify which one), and \( J \) for joint submissions.

The second element of the procedure is an adjudication rule \( \hat{x}(\cdot) \) determining \( \hat{x} \) as a function of the evidence submitted. For simplicity, we focus on linear adjudication functions. When both parties are heard, i.e., \( d = J \), the adjudicated \( \hat{x} \) is given by

\[ \hat{x} = ax_A + bx_B + c. \]

Here the arbiter chooses \( a, b, \) and \( c \) so as to minimize the expected value of the social loss \( L \). If the arbiter only hears, say, party \( B \), i.e., in procedure
\( S, \hat{x} = bx_B + c. \) For \( d = N, \hat{x} = \mu. \) The linear scheme allows us to easily compare the different scenarios of listening to both, to only one, and to no party.\(^5\)

The arbiter’s decision implies a gain of \( \hat{x} \) for party A and a loss of \( \hat{x} \) for party B. The parties’ payoff as a function of the true state of the world \( x \), of their presentations \( x_A \) and \( x_B \), if any, and of the adjudicated \( \hat{x} \) are therefore

\[
\begin{align*}
\pi_A(\hat{x}, x_A, x) &= \hat{x} - c_A(x_A, x) \quad \text{and} \\
\pi_B(\hat{x}, x_B, x) &= -\hat{x} - c_B(x_B, x).
\end{align*}
\]

To summarize, the sequencing is as follows: (i) the arbiter announces a procedure; (ii) parties submit a presentation if asked to do so; (iii) the quantity at issue is adjudicated according to the announced adjudication function. In making their presentation, the parties seek to maximize their payoffs \( \pi_A \) and \( \pi_B \). In choosing the procedure, the arbiter seeks to minimizes the expected loss \( E(L) := \bar{L} \). We first solve for the best adjudication functions under each procedure.

### 3 Adjudication Rules

**Both parties submit**

With both parties submitting evidence, i.e., \( d = J \), the expected loss is

\[
\bar{L}_J = E \{ l(\hat{x}(x_A, x_B), x) + \theta [c_A(x_A, x) + c_B(x_B, x)] \}.
\]

The arbiter chooses the adjudication rule \( \hat{x}(\cdot, \cdot) \) so as to minimize \( \bar{L}_J \) subject to the incentive constraints

\[
\begin{align*}
x_A(x) &= \arg \max_{x_A} \hat{x}(x_A, x_B(x)) - c_A(x_A, x) \quad \text{and} \\
x_B(x) &= \arg \max_{x_B} \hat{x}(x_A, x_B(x)) - c_B(x_B, x).
\end{align*}
\]

\(^5\)In the conclusion we comment on the implications of allowing for more general adjudication functions.
These constraints describe the Nash equilibrium of the parties’ submission game. Since the agents’ payoff functions are concave in their testimony, the equilibrium reports are given by the first-order conditions

\[ x_A = x + \frac{a}{\gamma_A} \quad \text{and} \quad x_B = x - \frac{b}{\gamma_B}. \]  

(1)

Substitution yields

\[ L_J = E \left\{ \frac{1}{2} \left[ a \left( x + \frac{a}{\gamma_A} \right) + b \left( x - \frac{b}{\gamma_B} \right) + c - x \right]^2 \right\} + \theta \left[ 2\gamma + \frac{a^2}{2\gamma_A} + \frac{b^2}{2\gamma_B} \right]. \]

The first term in the expression is the expected error cost \( \bar{L} \). Minimizing this term with respect to \( c \) yields

\[ c = \frac{b^2}{\gamma_B} - \frac{a^2}{\gamma_A} - \mu(a + b - 1). \]

Substituting in \( \bar{L} \) and recalling the unit variance assumption, we get

\[ \bar{L} = \frac{1}{2} (a + b - 1)^2 + \theta \left[ 2\gamma + \frac{a^2}{2\gamma_A} + \frac{b^2}{2\gamma_B} \right]. \]

Minimizing with respect to \( a \) and \( b \), we finally obtain

\[ a = \frac{\gamma_A}{\gamma_A + \gamma_B + \theta} \quad \text{and} \quad b = \frac{\gamma_B}{\gamma_A + \gamma_B + \theta}. \]  

(2)

(3)

The parameters describe the weight given to the parties’ reports. The weights depend on the parties’ relative credibility and on the arbiter’s concern for submission costs. If, say, party \( A \) has a lower cost of lying than party \( B \), i.e., \( \gamma_A < \gamma_B \), the arbiter gives less importance to \( A \)’s rather than \( B \)’s report. Moreover, the weights sum to less than unity and the more the arbiter cares about submission costs, the less weight he gives to both parties’ reports. The weights are such that in equilibrium the extent of lying is the same for both parties, i.e., \( |x_i - x| = 1/(\gamma_A + \gamma_B + \theta) \), \( i = A, B \).
The adjudicated value is
\[ \hat{x} = ax_A + bx_B + c = \left( \frac{\theta}{\gamma_A + \gamma_B + \theta} \right) \mu + \left( \frac{\gamma_A + \gamma_B}{\gamma_A + \gamma_B + \theta} \right) x. \] (4)

This is the weighted average of the mean \( \mu \) and the true value \( x \). Thus, \( \hat{x} \) is biased towards the prior mean. Perfect accuracy in adjudication only obtains when the arbiter does not care about submission costs, i.e., \( \lim_{\theta \to 0} \hat{x} = x \). Conversely, if he cares a lot about submission costs, \( \hat{x} \) approaches the prior mean, i.e., \( \lim_{\theta \to \infty} \hat{x} = \mu \). The extent of lying is maximal when perfect accuracy obtains and minimal when the mean is adjudicated, i.e., \( \lim_{\theta \to 0} |x_i - \hat{x}| = 1/(\gamma_A + \gamma_B) \) and \( \lim_{\theta \to \infty} |x_i - \hat{x}| = 0, \ i = A, B \).6

Expected error costs and total submission costs are
\[ \bar{l}_J = \frac{\theta^2}{2(\gamma_A + \gamma_B + \theta)^2} \text{ and } \]
\[ C_J = 2\gamma + \frac{\gamma_A + \gamma_B}{2(\gamma_A + \gamma_B + \theta)^2}. \]

Society’s total loss is
\[ \bar{L}_J = \bar{l}_J + \theta C_J = \frac{\theta}{2(\gamma_A + \gamma_B + \theta)} + 2\theta \gamma. \] (5)

Only one party submits

Under procedure \( d = S \) only one party is heard. Assume \( B \) is more credible, i.e., \( \gamma_A < \gamma_B \), and he is therefore the party required to submit. The adjudication function is now \( \hat{x} = bx_B + c \). Minimizing \( \bar{L}_S \) with respect to \( b \) and \( c \) subject to the constraint that \( x_B \) maximizes \( B \)’s payoff yields
\[ b = \frac{\gamma_B}{\gamma_B + \theta}, \quad c = \frac{b^2}{\gamma_B} - \mu(b - 1), \]

6In Parisi (2002) the judge decides what weight he will give to his own appointed experts (similar to our priors) and what weight he gives to the parties’ testimonies. The greater the latter weight, the greater the incentives for the parties to engage in rent-seeking. The weight Parisi gives to the parties’ testimony plays a similar role as does the weight \( \theta \) we give to submission costs.
and the adjudicated value is

$$\hat{x} = bx_B + c = \frac{\gamma_B}{\gamma_B + \theta} x + \frac{\theta}{\gamma_B + \theta} \mu. \quad (6)$$

Expected error costs and submission costs are

$$\bar{l}_S = \frac{\theta^2}{2(\gamma_B + \theta)^2} \quad \text{and} \quad C_S = \gamma + \frac{\gamma_B}{2(\gamma_B + \theta)^2}.$$ 

The total expected loss is

$$\bar{L}_S = \bar{l}_S + \theta C_S = \frac{\theta}{2(\gamma_B + \theta)} + \theta \gamma. \quad (7)$$

The extent of lying by $B$ is now $|x_B - x| = 1/(\gamma_B + \theta)$. This is greater than the amount of lying by $B$ when both parties are heard. The reason is that greater weight is now given to the party’s submission, thereby inducing him to falsify more. Thus, confronting the parties in adversarial hearings induces either of them to distort the evidence less than when only one testimony is heard. Yet when both parties are heard, both are involved in boosting the evidence.

No party submits

Under procedure $d = N$, no party testifies and submission costs are therefore zero. The arbiter then minimizes expected error costs solely on the basis of the priors $f(\cdot)$, implying $\hat{x} = \mu$. The expected total loss is $\bar{L}_N = 1/2$, i.e., half the variance of $x$.

4 Optimal Procedure

Let us now determine the optimal number of parties to submit evidence. The arbiter chooses whether no party $N$, only party $B$ under procedure $S$, or both
parties $J$ are required to submit evidence so as to minimize the expected loss. We assume $\gamma_A < \gamma_B$ throughout.

**Proposition 1:** i) If $\gamma \leq \gamma_A/(2(\gamma_B + \theta)(\gamma_A + \gamma_B + \theta)) := \bar{\gamma}$, the judge requires joint submission $J$ and adjudicates according to (4);

ii) If $\bar{\gamma} < \gamma \leq \gamma_B/(2(\gamma_B + \theta)) := \hat{\gamma}$, single submission $S$ by party $B$ is optimal and the judge adjudicates according to (6);

iii) If $\gamma > \hat{\gamma}$, no submission $N$ minimizes the expected social loss and $\hat{x} = \mu$.

The result follows because $\bar{L}_J \leq \bar{L}_S$ for $\gamma \leq \bar{\gamma}$ and $\bar{L}_S \leq \bar{L}_N$ for $\gamma \leq \hat{\gamma}$. Moreover, since only the more credible party $B$ is heard under procedure $S$, $\bar{\gamma} < \hat{\gamma}$ so that there is never a direct switch from $J$ to $N$.

Figure 1 shows in the $(\gamma, \theta)$ plane the regions where the arbiter requires both, only one, or no party to submit evidence. If, for example, $\gamma = 0$ so that non-falsified submissions generate no costs, it is optimal to have both parties submit, irrespective of their credibility and irrespective of $\theta$. The intuition is straightforward. Comparing (4) and (6), it is readily seen that the arbiter puts less weight on his priors under joint submissions than when only a single, albeit the most credible party, submits. The expected error cost is accordingly smaller under joint submissions. Furthermore, when $\gamma = 0$, total submission costs are also smaller under joint submissions since each party’s testimony is accorded an appropriately small weight. Put differently, competition between advocates reduces falsification because each advocate has less influence on the outcome.

When $\gamma > 0$, duplication imposes a dead weight loss. However, when $\gamma$ is not too large, it remains optimal to hear both parties. Although we may now have $C_J > C_S$, the fact that $\bar{I}_J < \bar{I}_S$ may more than compensate. Finally, requiring parties to submit evidence is never optimal if $\gamma$ is sufficiently large.
and \( \theta > 0 \). The adjudicated value then rests only on priors, as the submission costs of either \( J \) or \( S \) exceed the error cost of \( N \).

Note that the threshold \( \bar{\gamma} \) is increasing in \( \gamma_A \), party \( A \)'s credibility. The more the parties are alike in terms of credibility (recall that \( \gamma_A < \gamma_B \)), the larger the range over which \( J \) is better than \( S \). A larger \( \gamma_A \) lowers the error costs and thus the social loss from joint submission, making \( J \) more attractive relative to \( S \), the value of which is independent of \( \gamma_A \).

## 5 No Commitment

So far we have assumed that once a procedure has been chosen the arbiter can commit to the announced adjudication rule. However, it is clear that the true \( x \) can be inferred from the evidence submitted under either procedure \( J \) or \( S \). An arbiter seeking to minimize error and submission costs would then be tempted to renege on the announced adjudication rule. Ex post, submission costs are sunk and the sequentially optimal action, from the arbiter’s point of view, is to adjudicate \( \hat{x} = x \).

The inability to commit restricts the set of feasible adjudication rules. A rule must now be part of an equilibrium where, at the last stage of the game the arbiter adjudicates sequentially rationally after up-dating his beliefs. Under procedure \( J \), the adjudication function must satisfy

\[
\hat{x}(x_A, x_B) = \arg \min_{\hat{x}} E \left[ \frac{1}{2} (\hat{x} - x)^2 \mid x_A, x_B \right] = E \left[ x \mid x_A, x_B \right].
\]

If reports are monotone in the true \( x \), the latter can be inferred from any party’s action, i.e., along the equilibrium path the posterior distribution is degenerate and we have

\[
\hat{x} \left[ x_A(x), x_B(x) \right] \equiv x \quad (8)
\]

Since error costs will be zero, the best adjudication rule under procedure \( J \) is the one which minimizes total submission costs subject to (8). We now
have

\[ \hat{x}(x_A, x_B) = ax_A + bx_B + c \]
\[ = a \left( x + \frac{a}{\gamma_A} \right) + b \left( x - \frac{b}{\gamma_B} \right) + c \equiv x, \]

where we substituted for \( x_A \) and \( x_B \) from the equilibrium conditions (1) and imposed constraint (8). This implies

\[ a + b = 1 \quad \text{and} \quad c = \frac{b}{\gamma_B} - \frac{a}{\gamma_A}. \]

These constraints reflect the fact that the adjudication rule (through the parameters \( a, b, \) and \( c \)) can be picked only to the extent that the arbiter’s strategy, at the last stage of the game, is part of a sequential equilibrium. Note that \( a \) and \( b \) now sum to unity, by contrast with the commitment case.\(^7\)

Choosing \( a \) and \( b \) to minimize total submission costs, subject to the foregoing constraints, now yields

\[ a = \frac{\gamma_A}{\gamma_A + \gamma_B} \quad \text{and} \quad b = \frac{\gamma_B}{\gamma_A + \gamma_B}. \]

Compared to the specification (2) and (3) for the commitment case, it is readily seen that the arbiter gives more weight to each party’s submission,

\(^7\)These parameters reflect how the arbiter interprets out-of-equilibrium moves. Substituting for \( c \) and \( b \), the adjudicated value is

\[ \hat{x} = a \left[ x_A - \frac{a}{\gamma_A} \right] + (1 - a) \left[ x_B + \frac{1 - a}{\gamma_B} \right] \]

Along the equilibrium path, the expressions in both square brackets equal \( x \). If one party deviates from his equilibrium strategy, the expressions will differ and one will differ from the true \( x \). Observing the discrepancy between the two expressions, the arbiter will not know which party deviated. Parameter \( a \) is the probability he ascribes to the possibility that \( A \) did not deviate (i.e., to the deviation arising from \( B \)). Hence, \( a \) is the weight given to party \( A \)’s submission, duly corrected for the party’s overstatement.
thereby inducing more falsification. A similar argument for procedure $S$, under which only party $B$ is heard, yields

$$b = 1 \quad \text{and} \quad c = \frac{b}{\gamma_B}.$$  

Under procedure $J$ or $S$ the total loss is now (using the superscript $n$ to denote non commitment):

$$\bar{L}_n^J = \theta C_n^J = \theta \left( 2\gamma + \frac{1}{2(\gamma_A + \gamma_B)} \right) \quad \text{and} \quad (11)$$

$$\bar{L}_n^S = \theta C_n^S = \theta \left( \gamma + \frac{1}{2\gamma_B} \right). \quad (12)$$

Under the procedure $N$ the loss is of course the same as before.

The inability to commit on the part of the arbiter influences the choice of procedure. Whenever he receives evidence, the arbiter is now unable to put any weight on his priors and the adjudication rule has too much “power”. The consequence, as shown in the following proposition, is that the arbiter will now be more inclined to choose procedure $N$ rather than $J$ or $S$ and procedure $J$ rather than $S$.

**Proposition 2:** Under no commitment $\hat{x} = x$ if evidence is submitted; otherwise, $\hat{x} = \mu$. Let $\hat{\theta} = \gamma_B(\gamma_A + \gamma_B)/(2\gamma_A + \gamma_B)$.

(i) For $\theta \leq \hat{\theta}$, the optimal procedure is $J$ if $\gamma \leq \hat{\gamma}_n$, $S$ if $\hat{\gamma}_n < \gamma \leq \hat{\gamma}_n$, and $N$ otherwise, where

$$\hat{\gamma}_n = \frac{\gamma_A}{2\gamma_B(\gamma_A + \gamma_B)} \quad \text{and} \quad \frac{\gamma_B - \theta}{2\theta\gamma_B} := \hat{\gamma}_n.$$  

(ii) For $\theta > \hat{\theta}$, the optimal procedure is $J$ if $\gamma \leq \tilde{\gamma}_n$ and $N$ otherwise, where

$$\tilde{\gamma}_n = \frac{\gamma_A + \gamma_B - \theta}{4\theta(\gamma_A + \gamma_B)} < \hat{\gamma}_n.$$
This result follows because $\bar{L}_J^n < \bar{L}_S^n$ for $\gamma \leq \bar{\gamma}^n$. For $\theta \leq \hat{\theta}$, $\bar{L}_S^n \leq 1/2$ for $\gamma \leq \hat{\gamma}^n$. For $\theta > \hat{\theta}$, $1/2 < \bar{L}_J^n$ for $\gamma > \hat{\gamma}^n$.

With commitment hearing both parties is optimal when $\gamma$ is sufficiently small, independently of $\theta$. Without commitment, this is no longer true. See Figure 2. It is now better not to receive any submission at all if $\theta > \gamma_A + \gamma_B$. Thus, no commitment implies a larger parameter region for which the parties should not be heard. In the commitment case a large $\theta$ means that the arbiter gives little weight to the agents’ testimony. Without commitment the arbiter cannot commit to give little weight to the reports. He can only give full weight to the reports or no weight at all by choosing $N$ in the first place.

Consider first the case where $\theta \leq \hat{\theta}$. The critical values of $\gamma$ at which the procedure switches from $J$ to $S$ and from $S$ to $N$ differ from the commitment case. Specifically, for $\theta > 0$,

$$\bar{\gamma} < \tilde{\gamma}^n < \hat{\gamma}^n < \hat{\gamma}.$$

Accordingly, $\bar{\gamma} < \tilde{\gamma}^n$ implies procedure $J$ is now preferred to $S$ over a larger set of values. Without commitment perfect accuracy is attained under $J$ and $S$. Under both procedures the amount of lying is independent of $\theta$. Hence, $\tilde{\gamma}^n$ is independent of $\theta$. With commitment perfect accuracy also obtains for $\theta$ sufficiently small. Therefore, $\lim_{\theta \to 0} \hat{\gamma} = \pi^n$. Yet, $\tilde{\gamma}$ decreases with $\theta$. The higher $\theta$, the more $\hat{x}$ under commitment is pushed towards the mean—less weight is given to the parties’ submissions. There is no such effect under no commitment because the true $x$ is always adjudicated.

Moreover, the region over which procedure $S$ rather than $N$ is used is smaller. Since under no commitment the amount of lying by $B$ doesn’t decrease with $\theta$, the arbiter switches to $N$ for lower values of $\theta$. To put it differently, when $\theta$ is not too large, under no commitment it is less likely that only a single party will be heard; it is more likely that both parties will be heard or that no submissions will be required.
For $\theta > \hat{\theta}$, hearing only a single party is never desirable. The arbiter switches directly from $J$ to $N$. $S$ is dominated by either $J$ or $N$. $\tilde{\gamma}^n$ is decreasing because the higher $\theta$ the more the arbiter weighs the submission costs in choosing the procedure.

To summarize: When the adjudicator cannot commit, he knows ex ante he will be giving too much weight to the parties’ submissions, i.e., allow them to influence the adjudication too much. Thus, he knows they will invest “a lot” in falsifying. The only way out of this excessive falsifying may be to refuse to hear the parties at all. At other times, it is better to hear both parties even though only one should be heard under commitment. Under commitment, when the fixed submission cost becomes sufficiently large, it becomes more attractive to hear one rather than both parties. However, the more the decision is then pushed towards the mean, thereby reducing incentives to falsify. Without commitment perfect accuracy always obtains so that switching to a single party is less attractive.

6 Concluding Remarks

In this paper we analyze a stylized model of the trade-off between accuracy in adjudication and misrepresentation costs. We show that the cost of misrepresentation (net of fixed submission costs) is lower when both parties are heard than when only one party submits evidence: hearing both parties duplicates fixed submission costs but lowers misrepresentation costs. Accordingly, it is preferable to hear both parties when fixed costs are low. We therefore qualify Tullock’s (1975) statement that adversarial systems are inferior to inquisitorial systems due to the duplication of misrepresentation costs.

We also point out the crucial role of commitment. When the judge cannot commit not to infer (and adjudicate) the truth from the parties’ statements, it is more likely that he hears both or no party at all than if he can commit. The inability to commit, which is presumably more likely than commitment,
makes it more attractive to hear two rather than one party.

A few qualifications and remarks are in order. Under commitment, the linear adjudication scheme allowed us to readily obtain our results. Nevertheless, relaxing this restriction will not change the qualitative nature of our results. The truly optimal adjudication rule is not linear and it will depend on the prior \( f(x) \). Yet, the same qualitative results hold: if they are called to submit evidence, the parties always falsify; the procedure should switch from joint to single and to no submissions as the fixed cost of submitting gets larger.\(^8\)

Under no commitment, once the procedure has been chosen, the arbiter becomes a player in the game. His adjudication strategy must then be part of a sequential (or perfect Bayesian) equilibrium. There are many such equilibria. Choosing the best adjudication “rule” amounts to picking the best equilibrium in terms of minimizing submission costs. It can be shown that the linear strategy described in the text does indeed characterize the best sequential equilibrium. Hence, in the no commitment case the linearity assumption plays no role.

Similarly, the quadratic cost functions allowed us to obtain closed form solutions. Our conjecture is that most of our results also hold under more general falsification cost functions.

Our results are driven by the fact that the arbiter can only adjudicate one value that one party loses and the other party gains. If we relax this adding-up constraint, the arbiter could obviously do better. The judge could use, for example, the following mechanism: if both parties make the same report, he adjudicates this value. If the parties report different values, the judge punishes both of them heavily for perjury.\(^9\) In reality, however, perjury cases

---

\(^8\)Using the revelation principle, the optimal scheme can be obtained by considering a direct truthful mechanism \( \tilde{F}(t_A, t_B), x_A(t_A), x_B(t_B) \) where \( t_A \) and \( t_B \) are the messages sent by the parties (messages differ from the costly submissions themselves).

\(^9\)See Demski and Sappington (1984) for an analysis of information extraction in a multiagent context.
are very rare and there is plenty of evidence indicating that slanted testimony is endemic in courts.\textsuperscript{10} Since perjury law seems to be ineffective, we didn’t include this possibility in the adjudication function. Moreover, non-judicial proceedings—e.g., regulatory hearings—usually have no such provisions.

References


\textsuperscript{10}For example, in a continuing scandal in New York City, police engaged in a pattern of perjury so common that they called it “testiliing;” in impeachment proceedings former President Clinton admitted making misleading statements about his sexual conduct while steadfastly denying that he committed perjury. For more evidence on slanted testimony see Cooter and Emons (2003).


Tullock, G. On the Efficient Organization of Trials, Kyklos 28 (1975), 745-762.

Figure 1: Optimal procedures in the commitment case
Figure 2: Optimal procedures in the no commitment case