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Uninformed Winners Under Adverse Selection

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Abstract:
This paper presents a static model of a market for a quality-differentiated good. In one version quality is observable, in the other it is not. It is shown that some agents who are uninformed when quality is unobservable may have higher utility than they do when it is observable. This is more likely to happen when goods of intermediate quality are scarce.

Keywords: Adverse selection, uninformed agents

Résumé:
Cet article présente un modèle statique où s'échange un bien à qualité différenciable. Dans une version, la qualité est observable ; dans l'autre, elle ne l'est pas. Il est démontré que certains agents qui ne sont pas informés lorsque la qualité est inobservable peuvent atteindre une utilité supérieure à ce qu'ils obtiennent lorsque la qualité est observable. Ceci se produit davantage lorsque les biens de qualité intermédiaire sont rares.

Mots Clés: Sélection adverse, agents non-informés

JEL Classification: D82

1 Introduction

Adverse selection is said to occur "when an informed individual's trading decisions depend on her privately held information in a manner that adversely affects uninformed market participants" [Mas-Colell, Whinston and Green (1995)]. This paper argues that while uninformed agents *as a group* are adversely affected in these circumstances, individual uninformed agents may be positively or negatively affected.

Early models of adverse selection [Akerlof (1970), Wilson (1979, 1980)] did not allude to the possibility of uniformed agents benefitting from quality uncertainty. Since then, leading textbooks have explained adverse selection using models where uninformed agents cannot possibly do better when quality is unobservable [see for example Mas-Colell, Whinston and Green (1995)]. As a result, we may have come to believe that adverse selection is always accompanied by a decrease in welfare for all uninformed agents.

I show here, in a simple supply-demand model, that some uninformed agents may benefit from quality uncertainty if (i) uninformed agents are heterogeneous and (ii) goods of "intermediate" or "average" quality are scarce. It is impossible for *all* uninformed agents (buyers in this case) to do better when quality is unobservable than when it is observable; so it is necessary to have heterogeneous uninformed agents in order that some of them may do better and some of them do worse. Under asymmetric information (unobservable quality), all goods are seen by buyers as having average quality; if this corresponds to an abundance of a certain quality which is rather scarce under symmetric information (observable quality), then some agents may benefit.

The possibility of ignorance being preferable to full information has been raised, but mainly in the context of principal-agent relationships [see Kessler (1998) and references therein].

The rest of the paper is organized as follows. Section 2 presents the bare-bones model to highlight the existence of the phenomenon. Section 3 examines how scarcity of units with average quality causes it. Section 4 concludes.

2 The Model

There are in the economy a measure 1 of sellers and a measure 1 of buyers. Each seller is endowed with one unit of the good, which is indivisible. His utility is

$$u_S = \begin{cases} sx & \text{if he keeps his unit;} \\ p & \text{if he sells it;} \end{cases} \tag{1}$$

where s is the seller's type, x is the unit's quality, and p is the price he can obtain for the unit. Seller types are distributed uniformly over the unit interval. There are two qualities of the good: a measure q of sellers have units of quality x_H, and a measure $1-q$ have units of quality x_L, where $x_H > x_L$. The two distributions (types and qualities) are independent, so that, for example, the measure of sellers of all types $i \leq s$ endowed with units of quality x_H is exactly qs.

Each buyer is endowed with zero units of the good, and can buy at most one unit. His utility is

$$u_B = \begin{cases} bx - p & \text{if he buys a unit;} \\ 0 & \text{if he does not;} \end{cases} \tag{2}$$

where b is his type, x is the quality of the unit he buys, and p is the price he pays. Buyer types are distributed uniformly over the unit interval.

Every agent, whether seller or buyer, prefers quality x_H to quality x_L. His type measures the degree to which quality matters to him. Utility functions of this sort have been used in other adverse selection models [Wilson (1979, 1980), Hendel and Lizzeri (1999), Johnson and Waldman (2003)]. I assume that

$$x_H < 2q(x_H - x_L) \quad . \tag{3}$$

This is done to guarantee an interior solution under asymmetric information, that is to say $S_H \in (0, q)$ and and $S_L \in (0, 1-q)$, where S_H is supply of quality x_H and and S_L is supply of quality x_L.

2.1 Equilibrium with symmetric information

Suppose that buyers can observe quality. Then in equilibrium each quality has its own price. Let us call p_i the price of a unit of quality x_i, with $i = H, L$. Sellers are willing to sell if $sx_i \leq p_i$. Therefore they supply

$$S_H = q \ \min\{p_H/x_H, 1\} \quad ; \tag{4}$$

$$S_L = (1-q) \ \min\{p_L/x_L, 1\} \quad . \tag{5}$$

Buyers have a choice of utilities $bx_H - p_H$, $bx_L - p_L$ or 0. Mapping these utilities (as is done in Figure 1) allows us to see which buyers will do what. Certainly p_H and p_L are endogenous, but it is quite straightforward to determine that, if demand is to equal supply in equilibrium, the schedules must arrange themselves as depicted. That is to say, there must be two numbers b_{HL} and b_0 such that buyers in the interval $[b_{HL}, 1]$ choose to buy quality x_H, those in $[b_0, b_{HL})$ choose quality x_L, while those in $[0, b_0)$ choose to buy nothing. Type b_{HL} is actually indifferent between the two qualities, which fact allows us to calculate

$$b_{HL} = \frac{p_H - p_L}{x_H - x_L} \quad . \tag{6}$$

Similarly b_0 is indifferent between the lower quality and nothing, so

$$b_0 = \frac{p_L}{x_L} \quad . \tag{7}$$

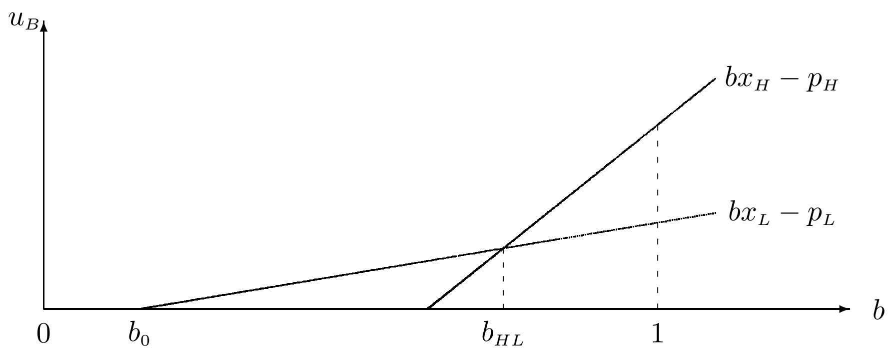

FIGURE 1. Buyers' utilities, symmetric information.

If we denote by D_i the demand for quality x_i, we have

$$D_H = 1 - b_{HL} \quad ; \tag{8}$$
$$D_L = b_{HL} - b_0 \quad . \tag{9}$$

Equilibrium prices are found by setting $S_H = D_H$ and $S_L = D_L$. We suppose for the time being that $p_H/x_H < 1$ and $p_L/x_L < 1$. We calculate

$$p_H = \left[\frac{x_H + (1-q)(x_H - x_L)}{2x_H + q(1-q)(x_H - x_L)}\right] x_H \quad ; \tag{10}$$

$$p_L = \left[\frac{x_H x_L}{2x_H + q(1-q)(x_H - x_L)}\right] \quad . \tag{11}$$

It can be verified that p_H/x_H and p_L/x_L are indeed less than unity.

2.2 Equilibrium with asymmetric information

Suppose now that buyers cannot observe quality. A single price p must clear the market for all units of the good. Buyers base their decisions on this price and the average quality $\bar{x}$ which they suppose will be supplied. I assume that in equilibrium buyers correctly anticipate this average quality.

Much as before, sellers supply

$$S_H = q \ \min\{p/x_H, 1\} \quad ; \tag{12}$$

$$S_L = (1-q) \ \min\{p/x_L, 1\} \quad . \tag{13}$$

If we assume at this point that p/x_L and hence p/x_H will be less than unity in equilibrium, we can compute $\bar{x}$ as

$$\bar{x} = \frac{x_H x_L}{q x_L + (1-q) x_H} \quad ; \tag{14}$$

which is the (weighted) harmonic mean of x_H and x_L. Total supply can be expressed as $S = S_H + S_L = p/\bar{x}$.

Buyers are willing to buy if $b\bar{x} - p \geq 0$. This gives us a demand of

$$D = 1 - p/\bar{x} \quad . \tag{15}$$

Equating demand with total supply yields

$$p = \bar{x}/2 \quad ; \tag{16}$$

where $\bar{x}$ (independent of price in this case) is given by (14). Assumption (3) ensures that p/x_L is indeed less than unity, and hence so is p/x_H.

2.3 Welfare comparison

Figure 2 shows buyers' payoffs under both information structures. Under symmetric information buyers get $bx_H - p_H$, $bx_L - p_L$ or 0; under asymmetric information they get $b\bar{x} - p$ or 0. All prices are endogenous, but again it can be shown that the schedules end up as depicted, i.e. that the $b\bar{x}-p$ schedule passes *over* the intersection of the $bx_H - p_H$ and $bx_L - p_L$ ones. To see this, we compute welfare under both structures for type b_{HL}.

Using the results from Section 2.1, we can pin down the value of b_{HL}:

$$b_{HL} = \frac{(2-q)x_H}{2x_H + q(1-q)(x_H - x_L)} \quad . \tag{17}$$

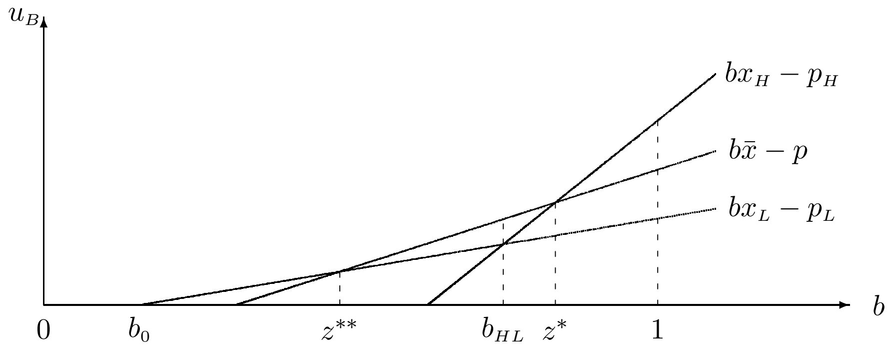

FIGURE 2. Comparison of buyers' utilities.

Let us call $u_B^S(b)$ the equilibrium payoff for a buyer of type b under symmetric information, and $u_B^A(b)$ his equilibrium payoff under asymmetric information. For type b_{HL} we compute

$$u_B^S(b_{HL}) = \frac{(1-q)x_H x_L}{2x_H + q(1-q)(x_H - x_L)} \quad . \tag{18}$$

The same type's equilibrium payoff under asymmetric information is

$$u_B^A(b_{HL}) = (1/2)(1-q)\left[\frac{(2-q)x_H + qx_L}{2x_H + q(1-q)(x_H - x_L)}\right]\bar{x} \tag{19}$$

$$= (1/2)\left[\frac{(2-q)x_H + qx_L}{qx_L + (1-q)x_H}\right]u_B^S(b_{HL}) \tag{20}$$

$$> u_B^S(b_{HL}) \quad . \tag{21}$$

The implication is that some agents, specifically those in the interval (z^{**}, z^*) on the graph, are better off under asymmetric information. Agents in the intervals (b_0, z^{**}) and $(z^*, 1)$ are worse off. The increase in welfare to agents in (z^{**}, z^*) does not offset the loss to the others.

3 Impact of quality distribution

To understand the reasons behind the result just obtained, consider the buyer whose type is b_{HL}. Under symmetric information, he has two qualities from which to choose. On the type spectrum, he is located at the lower edge of the group of buyers who purchase quality x_H and at the upper edge of the group who buy x_L. So broadly speaking, quality x_H is priced to meet the demands of buyers whose types are higher than his; and quality x_L is priced for a lower-type clientele. His own tastes are not being catered to. More precisely, if a third quality, higher than x_L and lower than x_H, were available, he would probably choose that one in equilibrium.

Under symmetric information, only one quality is available, but it fits the description of this hypothetical third quality. Since he is now in the middle of the group of buyers who purchase $\bar{x}$, this quality is, in a sense, aimed more directly at him than either x_H or x_L were. This is reflected in the price p, which to him is relatively low.

It would seem likely, then, that the existence of an intermediate quality x_M would brighten his prospects under symmetric information, while altering his welfare under asymmetric information very little. This section examines in some detail the impact that such an addition to the model would have.

Let us suppose that a measure θ of sellers are endowed with units of quality x_M. In order to keep the total measure of sellers at 1 and leave the relative proportion of sellers with qualities x_H and x_L intact, let us say that a measure $q(1-\theta)$ of sellers are endowed with units of quality x_H, and that a measure $(1-q)(1-\theta)$ are endowed with units of quality x_L. I will assume further that

$$x_M = \frac{x_1 x_2}{q x_2 + (1-q) x_1} \quad ; \tag{22}$$

which is equal to $\bar{x}$ from equation (14), and will in fact be equal to average quality supplied under asymmetric information in the current configuration. This will make a welfare comparison between the two informational structures easy.

Under symmetric information, demand is arranged as one would expect given the previous analysis: there are numbers b_{HM}, b_{ML} and b_0 (shown in Figure 3) such that buyers in the interval $[b_{HM}, 1]$ purchase quality x_H, those in the interval $[b_{ML}, bhm)$ purchase x_M, those in the interval $[b_0, b_{ML})$ purchase x_L, and the rest purchase nothing.

Equilibrium prices are

$$p_H = \left[\frac{Ax_L + \theta AC + [1 + (1-q)(1-\theta)]C}{\theta AB + q(1-\theta)Ax_L + [1+(1-q)(1-\theta)]Bx_H}\right] x_H \quad ; \tag{23}$$

$$p_M = \frac{Ax_H x_L}{\theta AB + q(1-\theta)Ax_L + [1+(1-q)(1-\theta)]Bx_H} \quad ; \tag{24}$$

$$p_L = \frac{x_H x_L [q x_L + (1-q) x_H]}{\theta AB + q(1-\theta)Ax_L + [1+(1-q)(1-\theta)]Bx_H} \quad ; \tag{25}$$

where

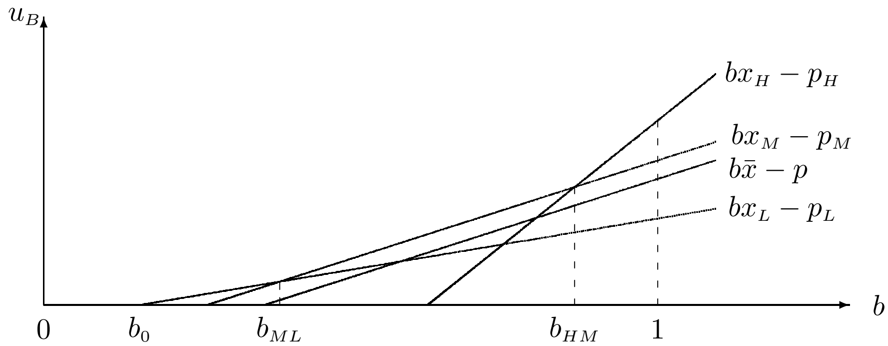

FIGURE 3. Payoff schedules with modified quality distribution.

$$A \equiv x_H + q(1-q)(1-\theta)(x_H - x_L) \quad ; \tag{26}$$

$$B \equiv qx_L + (1-q)x_H + q(1-q)(1-\theta)(x_H - x_L) \quad ; \tag{27}$$

$$C \equiv (1-q)(x_H - x_L) \quad . \tag{28}$$

Under asymmetric information, average quality supplied is $\bar{x} = x_M$ and equilibrium price is $p = x_M/2$. This means that the same quality is available to buyers under both informational structures. Whether buyers are better off under symmetric or asymmetric information now depends on which structure offers them this quality at the lower price. So the question hinges on a comparison of p_M and p.

In Figure 3 the $bx_M - p_M$ and $b\bar{x} - p$ schedules are parallel, reflecting the fact that $\bar{x} = x_M$. Which of the two is positioned higher on the graph depends on which price is lower. The figure shows the case where $p_M < p$. In that case, buying x_M under symmetric information is better for all types than buying $\bar{x}$ under asymmetric information, and therefore no one is better off under asymmetric information. For this to happen, quality x_M must be sufficiently abundant. Figure 4 plots p_M as a function of θ. It can be shown that over $\theta \in (0,1)$ the curve is convex and crosses the horizontal dotted line ($p_M = p$) only once. When quality x_M is scarce, i.e. when θ is less than the threshold value θ^*, p_M is still higher than p. It is only when θ is greater than θ^* that p_M is less than p and thus no one is better off under asymmetric information. Note that at the limit, when $\theta = 1$, all units have quality x_M and there is no difference between the two informational structures.

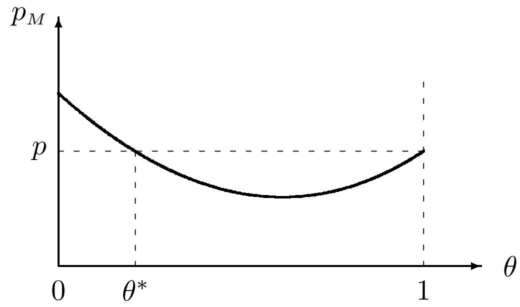

FIGURE 4. p_M as a function of θ.

4 Conclusion

In a very simple static economy where units of a good vary in quality, some buyers can be better off when quality is unobservable (asymmetric information) than when it is observable (symmetric information). This happens when units of "intermediate" quality are scarce relative to those of high and low quality. Under asymmetric information, all units have medium expected quality, and so medium quality is plentiful. It is this disparity between scarcity of medium quality in one information structure and abundance in the other which leads to the result.

The model used here is simple, with a discrete quality space. The result, however, is obtainable with more complex distributions, provided the scarcity just described exists.

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