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## **Labor Supply, Home Production and Welfare Comparisons**

Olivier Donni

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Donni: CIRPÉE & THEMA, Université de Cergy-Pontoise, 33 Boulevard du Port, 95011 Cergy Cedex, France  
[olivier.donni@eco.u-cergy.fr](mailto:olivier.donni@eco.u-cergy.fr)

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**Abstract:** We consider the collective model of labor supply with marketable domestic production (Chiappori, 1997). We first show that, if domestic production is mistakenly ignored by the economist, welfare analyses will be probably distorted. Precisely, the identification of “collective” indirect utilities will be generally biased. The direction and the size of the bias depend on the complementarity/substituability of spouses’ time inputs in the production process. The identification is unbiased if and only if the production function is additive. We then show that, even if domestic labor supplies are not observed by the economist, (i) market labor supplies have to satisfy testable restrictions, (ii) the structure of the model is partially identifiable so that valid welfare comparisons are still possible. Our identification results generalize Chiappori’s (1992) ones.

**Keywords:** Household, Collective Model, Labor Supply, Home Production, Welfare Analysis, Identification

**JEL Classification:** D13, J22

# 1 Introduction

The collective model of labor supply, developed by Chiappori (1988, 1992), is by now a standard tool for analyzing household behavior and its empirical success for the last ten years is considerable. Specifically, the possibility of identifying the intrahousehold sharing of income from the sole observation of labor supplies has turned out to be very attractive. This permits to perform welfare comparisons at the individual level, instead of exclusively concentrating on the distribution of well-being across households as it is generally made with traditional models.

The empirical study presented by Chiappori, Fortin and Lacroix (2001) is very representative of the potentiality of the collective approach. These authors estimate a simple model of household labor supply with a sample of couples extracted from the Panel Study of Income Dynamics. They then observe that a one dollar increase in household non-labor income will increase the wife's share of income by 70 cents and the husband's by 30 cents. They also remark that divorce legislations and the state of the marriage market affect the intrahousehold distribution of income. The relevance of these results for economic policies that aim at targeting a particular person in the household is clear. Lise and Seitz (2004) go even farther and use a collective model of labor supply to investigate recent changes in the distribution of resources in the United-Kingdom. They conclude that the rise in consumption inequality over time at the household level may overstate the degree of consumption inequality at the individual level by 40% between the late 1960s and the present.<sup>1</sup>

One of the most serious criticisms of the collective model of labor supply, however, concerns the treatment of domestic production. In the simplest form of this model, non-market time coincides with pure leisure and homework is ignored. Thus, a low level of market labor supply is automatically interpreted as a large consumption of leisure, whereas it may in fact reflect the specialization of one of the members in home production. Apps and Rees (1997) naturally conclude — even if it is not formally demonstrated in their paper — that the presence of homework may significantly distort welfare analyses that are based on Chiappori's initial identification results. Similar criticisms of the collective model are made by feminist economists;

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<sup>1</sup>Other empirical applications of the collective model of labor supply for various countries include Fortin and Lacroix (1997), Moreau and Donni (2002), Blundell, Chiappori, Magnac and Meghir (2004), Clark, Couprie and Sofer (2004), Vermeulen (2004).

see Grossbard-Schechtman (2000) for instance.

This problem is tackled by Chiappori (1997) who considers a collective model of domestic and market labor supplies. The household consists of a couple without children. Each spouse is characterized by egoistic preferences and, as usual, the decision process results in Pareto-efficient outcomes. There are two (private) consumption goods: a market good, like in the simple model of labor supply, and a domestic good which can be produced within the household from a technology using time input. The latter can be consumed by household members or exchanged on a market at a constant price.<sup>2</sup> Chiappori then shows that, if domestic and market labor supplies are both observed, the spouses' preferences and the outcome of the decision process can be completely identified (except for a constant). In addition, testable restrictions on market and domestic labor supplies can be derived.<sup>3</sup>

In spite of this important theoretical contribution, empirical applications of collective models accounting for home production are surprisingly rare.<sup>4</sup> The most probable reason for that is that these models are very demanding in terms of data and turn out to be difficult to estimate. Time use surveys, even if they are quite broadly available, are generally fragmented and unreliable about wages and incomes. In this case, the main question is to determine what the economist can say about the internal decision process without information on spouses' homework. This is our objective in this paper. To do that, we adopt Chiappori's (1997) framework with marketable domestic production and make the following contributions.

Firstly, we establish the general conditions on preferences and technologies under which household market labor supplies fulfill the restrictions derived from Chiappori's (1988, 1992) original framework. If these conditions are satisfied, the economist is not able to empirically reject the simple model labor supply in favor of a more sophisticated model that incorporates domestic production. That is to say, the 'structure' that rationalizes the 'observ-

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<sup>2</sup>Apps and Rees (1988, 1997) and Chiappori (1997) also consider the case of a non-marketable domestic good. In that case, the price of this good is endogenously determined within the household. Identification raises further difficulties, though.

<sup>3</sup>Other theoretical generalizations of the collective model of labor supply are given by Fong and Zhang (2000), Donni (2003), Chiappori, Blundell and Meghir (2004).

<sup>4</sup>Apps and Rees (1996) and Aronsson, Daunfeldt, Wikstrom (2001) are probably the most notable exceptions. These empirical investigations are directly based upon Apps and Rees's (1997) and Chiappori's (1997) set-up. Couprie (2004) estimates a model of domestic and market labor supplies but she supposes that spouses produce a public good (instead of a private one). The strategy of identification is then completely different.

able' model is not unique. And, if the economist mistakenly ignores domestic production, she may well draw wrong conclusions about the welfare impact of economic policy.

Secondly, we examine this very case, in which there exist two equivalent structures (depending on the omission of domestic production), and show to what extent welfare analyses can be biased. Our result indicates that welfare analyses are unbiased if and only if the profit function is additive. Otherwise, for the non-additive case, the size and the direction of the bias is function of the complementarity/substitutability of spouses' time inputs in the production process. If leisure is a superior good — an uncontroversial assumption — and if spouses' labor are substitutes (resp. complements), the direct-effect of wage on spouses' welfare is underestimated (resp. overestimated) and the cross-effect is overestimated (resp. underestimated). This result is illustrated by an application to the estimations given by Chiappori et alii (2001).

Thirdly, we prove that, even if domestic labor supplies are not observed by the economist, (i) market labor supplies have to satisfy testable restrictions under the form of partial differential equations, and (ii) the most important components of the decision process can be identified so that valid welfare comparisons are still possible. To be precise, we show that the 'collective' indirect utilities, which are generally sufficient to perform welfare analyses, can be recovered (up to composition by an increasing transform).<sup>5</sup> Quite importantly, the system of partial differential equations that defines these utilities in the domestic production context generalizes Chiappori's (1992) system in that the solution of the former reduces to the solution of the latter if the profit function is additive. Since this result does not require more data than what is necessary for estimating a simple collective model of labor supply, we finally advocate a new, more general strategy for performing welfare comparisons with collective models.

The paper is structured as follows. In Section 2, the collective model of labor supply with marketable domestic production is described. In Section 3, the implications of the omission domestic production are discussed. In particular, the form of preferences and technologies that generates non-unique structures is completely characterized. In Section 4, the new identification results, valid in presence of domestic production, are given. In Section 5, a conclusion and a summary are presented.

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<sup>5</sup>Collective indirect utilities measure the level of welfare that individuals *actually* attain in the household when facing a given set of wages, incomes and other variables.

## 2 The model

The model is similar to Chiappori's (1997) and our description will be very brief. We consider a two-person household, consisting of a wife ( $f$ ) and a husband ( $m$ ), which makes decisions about market and domestic labor supplies.<sup>6</sup> Spouse  $i$ 's domestic and market labor supplies are respectively denoted by  $t_i$  and  $h_i$  ( $i = f, m$ ). Each spouse is characterized by specific preferences which can be represented by utilities with usual regularity properties:

$$u_i(T - L_i, C_i, Z_i), \quad (1)$$

where  $L_i = t_i + h_i$ ,  $C_i$  and  $Z_i$  respectively denote spouse  $i$ 's total labor supply, her/his consumption of an aggregate marketable good and her/his consumption of an aggregate domestic good and  $T$  denotes the total endowment in time. The domestic good is produced with the following technology:

$$Z = \phi(t_f, t_m),$$

where  $\phi$  is a strictly concave function. The domestic good can be bought and sold on the market at a constant price.<sup>7</sup> In consequence, we generally have:  $Z \neq Z_f + Z_m$ . We consider the case of cross-sectional data so that the price of both goods can be normalized to one. Spouse  $i$ 's market wage is denoted by  $w_i$  and household income by  $y$ .

Following the basic idea of the collective approach, we assume that the decision process, whatever its true nature, always generates Pareto-efficient outcomes. There is one distribution factor, i.e., one exogenous variable that influences the decision process without affecting preferences or the budget constraint, denoted by  $s$  (the extension to several distribution factors is trivial). From the Theorems of Welfare Economics, the allocation problem can be decentralized. In a first step, spouses determine domestic labor supply in order to maximize profit:

$$\pi(w_f, w_m) = \max_{t_f, t_m} \{ \phi(t_f, t_m) - w_f t_f - w_m t_m \},$$

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<sup>6</sup>The couple is not necessarily married. The terminology is just for convenience.

<sup>7</sup>This assumption is introduced by Gronau (1977) who regards "work at home as a time use that generates services which have a close substitute in the market, while leisure has only poor market substitutes". See Chiappori (1997) for a discussion of its relevance.

where  $\pi(w_f, w_m)$  is a normalized profit function. This function is assumed to be three times continuously differentiable. Then, from the Hotelling Lemma, domestic labor supplies have the following form:

$$t_i = -\frac{\partial \pi(w_f, w_m)}{\partial w_i} = t_i(w_f, w_m),$$

and satisfy a symmetry condition. In a second step, spouses agree on a sharing of total income, defined by

$$\psi = y + \pi(w_f, w_m).$$

Each spouse  $i$  then receives a share  $\rho_i$ , with  $\sum_k \rho_k = \psi$ , and independently maximizes her/his utility subject to a personal budget constraint:

$$\max_{L_i, C_i, Z_i} u_i(T - L_i, C_i, Z_i) \quad (2)$$

subject to

$$C_i + Z_i - w_i L_i = \rho_i(w_f, w_m, y, s).$$

In other words, the function  $\rho_i$  can be seen as the natural generalization of the sharing rule in the case of household production. Total labor supplies that result from this program have the following form:

$$L_i = L_i(w_i, \rho_i),$$

with  $\sum_k \rho_k = \psi$ . Note that  $L_i$  and  $\rho_i$  are assumed to be twice continuously differentiable. Combining these expressions yields market labor supplies:

$$h_i = L_i(w_i, \rho_i) - t_i(w_f, w_m). \quad (3)$$

In what follows, we assume that only the market labor supplies are observed by the economist.

We have to introduce another concept that turns out to be useful in welfare analysis. Since the price of both aggregate goods is normalized to one, the indirect utilities that correspond to (1) can be written as  $v_i(w_i, Tw_i + \rho_i)$ . However, we follow Chiappori (1992) and also define the ‘collective’ indirect utility as:

$$v_i^*(w_f, w_m, y, s) = v_i(w_i, Tw_i + \rho_i(w_f, w_m, y, s)). \quad (4)$$

This expression describes the level of welfare that spouse  $i$  attains in the household when she/he faces a wage-income bundle  $(w_f, w_m, y)$  and a distribution factor  $s$ . The function  $v_i^*$  is generally more convenient than  $v_i$  for making welfare comparisons because it directly gives the actual change in welfare due to a modification in the household environment.

### 3 A first look at the problem

#### 3.1 Definition and characterization

We know that, if there is domestic production, the form of market labor supplies is described by (3). In principle, the economist will not make mistakes in performing welfare analysis since the theoretical restrictions listed by Chiappori (1988, 1992) for the simple model of labor supply have not to be satisfied in a more general context. Hence, if domestic production is mistakenly ignored, the collective approach will be almost always ‘rejected’. This assertion has, however, to be nuanced in practice since data are necessarily imperfect and statistical tests are often misleading. We thus consider a particular class of ‘observable’ market labor supplies that, in spite of domestic production, can be confused with those resulting from Chiappori’s (1988, 1992) original framework and we investigate the nature of the bias that can be made in welfare analysis. This class  $\mathcal{O}$  (say) is defined as follows.

**Definition 1** *Suppose that there is marketable domestic production. A system of observable market labor supplies belongs to class  $\mathcal{O}$  if and only if there exist some twice continuously differentiable functions  $g_i$  and  $\varphi_i$ , with  $\sum_k \varphi_k = y$ , such that market labor supplies can be written as:  $h_i = g_i(w_i, \varphi_i)$ .*

In this definition, the relation  $h_i = g_i(w_i, \varphi_i)$  describes the structure that characterizes market labor supplies in absence of domestic production. Hence, if the system of ‘observed’ market labor supplies belongs to class  $\mathcal{O}$ , the underlying structure is not uniquely defined. To be more explicit, let us consider the following identity:

$$L_i(w_i, \rho_i) - t_i(w_f, w_m) = g_i(w_i, \varphi_i). \quad (5)$$

The ‘true’ structure that accounts for domestic production is in the left-hand-side and the ‘false’ structure that ignores domestic production in the right-hand-side. In this case, the economist is not able to empirically reject the simple model of labor supply in favor of a model with domestic production. And, if the economist mistakenly assumes that there is no domestic production, she may well draw wrong conclusions about the intrahousehold distribution of resources. The consequences of this possible misinterpretation have thus to be carefully examined. The first step is to (almost) completely determine the form of preferences and technologies such that market labor

supplies belong to class  $\mathcal{O}$ , i.e., such that the condition described by (5) is satisfied. This is the role of the following propositions.

**Proposition 2** *A system of observable market labor supplies belongs to class  $\mathcal{O}$  if the profit function is additive, i.e., if there exist some functions  $\pi_i$  such that  $\pi = \pi_f(w_f) + \pi_m(w_m)$ .*

**Proof** If the profit function is additive, then  $L_i(w_i, \rho_i) - t_i(w_i) = g_i(w_i, \varphi_i)$ , with  $\sum_i \rho_i = y + \sum_i \pi_i(w_i)$ . We then define  $\varphi_i = \rho_i - \pi_i(w_i)$  and  $g_i(w_i, \varphi_i) = L_i(w_i, \pi_i(w_i) + \varphi_i) - t_i(w_i)$ .||

In words, this proposition states that, whatever the form of spouses' preferences or the sharing rule, the assumption that spouses' inputs are independent in the production process is sufficient for market labor supplies to satisfy the conditions listed in Chiappori (1988, 1992). In addition, we may remark that the additivity of the profit function is directly implied by the additivity of the production function, i.e.,

$$\phi(t_f, t_m) = \phi_f(t_f) + \phi_m(t_m).$$

Then, a simple consequence is that, if the production function is additive, a test of the collective approach can be implemented without taking account of spouses' homework. We will see below that the additivity assumption has other attractive implications.

The next proposition considers a more general family of production technologies and completes the characterization of class  $\mathcal{O}$ . In this case, Engel curves have to be linear. A regularity condition is necessary.

**Condition R** Preferences and the sharing of income are such that  $\partial L_i / \partial \rho_i \neq 0$  and  $\partial \rho_i / \partial s \neq 0$  for any  $(\rho_i, s)$  in  $\mathbb{R}^2$ .

**Proposition 3** *Suppose that (i) the profit function is non-additive and (ii) R is satisfied. Then, a system of observable market labor supplies belongs to class  $\mathcal{O}$  if and only if there exist some functions  $\alpha_i, \beta_i, \gamma_i$  and  $G$  such that  $L_i(w_i, \rho_i) = \alpha_i(w_i)\rho_i + \beta_i(w_i)$  and*

$$\pi(w_f, w_m) = \gamma_f(w_f) + \gamma_m(w_m) + \frac{G \left( \int \alpha_f(w_f) dw_f - \int \alpha_m(w_m) dw_m \right)}{\exp \left( \int \alpha_f(w_f) dw_f + \int \alpha_m(w_m) dw_m \right)}.$$

The proof of this proposition is postponed until the end of this section since it requires the results that follow. We may note at this stage that the linearity of the Engel curves has well-known implications in terms of preferences. Gorman (1961) shows that, in this case, the indirect utilities have to be of the form:

$$v_i(w_i, Tw_i + \rho_i) = \int \frac{T - \beta_i(w_i)}{\exp(\int \alpha_i(w_i)dw_i)} dw_i + \frac{\rho_i}{\exp(\int \alpha_i(w_i)dw_i)}, \quad (6)$$

where  $\alpha_i$  and  $\beta_i$  are defined as in Proposition 3; see also Pollak and Wales (1992, p. 27). Then, it is easily shown that applying the Roy Identity to (6) gives leisure demands. This assumption may seem a priori restrictive but, in fact, many empirical studies estimate functional forms that are linear in income. Specifically, we will see below that Chiappori et alii's (2001) functional form has this property. The linearity may be regarded as a good approximation in many circumstances.

### 3.2 Identifying the wrong model

In this preliminary step, we suppose that the system of observed market labor supplies belong to class  $\mathcal{O}$  so that a misinterpretation of the reduced-form estimates by the economist cannot be dismissed a priori. We also suppose that condition R is satisfied. The reasoning then follows in three stages.

1. If we differentiate (5) with respect to  $y$  and  $s$ , and solve the resulting system of partial differential equations, we obtain:

$$\frac{\partial g_i}{\partial \varphi_i} = \frac{\partial L_i}{\partial \rho_i}, \quad (7)$$

and

$$\frac{\partial \varphi_i}{\partial y} = \frac{\partial \rho_i}{\partial y}, \quad (8)$$

$$\frac{\partial \varphi_i}{\partial s} = \frac{\partial \rho_i}{\partial s}. \quad (9)$$

2. If we differentiate (5) with respect to  $w_j$  ( $j = f, m$  and  $j \neq i$ ), use (7) and rearrange, we obtain:

$$\frac{\partial \varphi_i}{\partial w_j} = \frac{\partial \rho_i}{\partial w_j} - \nu_i. \quad (10)$$

where

$$\nu_i = \frac{\partial t_i}{\partial w_j} \left( \frac{\partial L_i}{\partial \rho_i} \right)^{-1}$$

represents an error in the estimated derivative of the sharing rule.

3. If we differentiate the adding up restriction  $\sum_k \varphi_k = y$  with respect to  $w_i$  and use (10), we obtain:

$$\frac{\partial \varphi_i}{\partial w_i} = -\frac{\partial \rho_j}{\partial w_i} + \nu_j.$$

Now, we differentiate the adding up restriction  $\sum_k \rho_k = \psi = \pi + y$  with respect to  $w_i$  and use the Hotelling Lemma, we obtain:

$$\frac{\partial \rho_j}{\partial w_i} = -t_i - \frac{\partial \rho_i}{\partial w_i}.$$

All in all, these relations give:

$$\frac{\partial \varphi_i}{\partial w_i} = \frac{\partial \rho_i}{\partial w_i} + (t_i + \nu_j). \quad (11)$$

**Welfare distortions.** The effect of the husband's (say) wage on his own share of income cannot be directly interpreted in terms of welfare variations. To infer the distortions that may result from ignoring homework, we have to use (4). If we differentiate this expression with respect to  $w_j$  ( $j \neq i$ ),  $y$  and  $s$ , we simply obtain:

$$\frac{\partial v_i^*}{\partial y} = \lambda_i \frac{\partial \rho_i}{\partial y}, \quad \frac{\partial v_i^*}{\partial s} = \lambda_i \frac{\partial \rho_i^*}{\partial s}, \quad \frac{\partial v_i^*}{\partial w_j} = \lambda_i \frac{\partial \rho_i}{\partial w_j}, \quad (12)$$

where  $\lambda_i = \partial v_i / \partial \rho_i$  is the marginal utility of money. This indicates that the impact of these variables on spouse  $i$ 's welfare coincides with the derivatives

of the sharing rule up to a multiplicative term. Moreover, if we differentiate (4) with respect to  $w_i$  and use the Roy Identity, we have:

$$\frac{\partial v_i^*}{\partial w_i} = \lambda_i \left( (h_i + t_i) + \frac{\partial \rho_i}{\partial w_i} \right). \quad (13)$$

We can now describe the distortions in welfare analysis that the omission of domestic production implies. To do that, let us define  $\hat{v}_i^*$  as the collective indirect utility that is obtained from the relations:  $h_i = g_i(w_i, \varphi_i)$ . That is, the collective indirect utility that is mistakenly retrieved if the economist ignores domestic production.

**Proposition 4** *Suppose that R is satisfied. If the system of observed market labor supplies belongs to class  $\mathcal{O}$ , then*

$$\begin{aligned} \frac{\partial v_i^*}{\partial y} &= \mu_i \frac{\partial \hat{v}_i^*}{\partial y}, \\ \frac{\partial v_i^*}{\partial s} &= \mu_i \frac{\partial \hat{v}_i^*}{\partial s}, \\ \frac{\partial v_i^*}{\partial w_j} &= \mu_i \frac{\partial \hat{v}_i^*}{\partial w_j} + \eta_i \nu_i, \\ \frac{\partial v_i^*}{\partial w_i} &= \mu_i \frac{\partial \hat{v}_i^*}{\partial w_i} - \eta_i \nu_j \end{aligned}$$

where  $\mu_i$  and  $\eta_i$  are positive functions.

**Proof** Let  $\mu_i = \lambda_i / \hat{\lambda}_i$ , where  $\hat{\lambda}_i$  is the marginal utility of money for the model without domestic production, be the price of utility  $\hat{v}_i^*$  in terms of utility  $v_i^*$  and  $\eta_i = \lambda_i$  be the price of money in terms of utility  $v_i^*$ . Then, the proof straightforwardly results from (8) to (13) and the system of equations which defines collective indirect utilities in Chiappori (1992, p. 451).||

In words, the effect of non-labor income and distribution factors on welfare is correctly estimated by the simple model of labor supply. However, the effect of wages is generally biased and depends on the function  $\nu_i$ . More precisely, the bias resulting from the omission of domestic production is related to the substitutability/complementarity relationship between spouses' time inputs in the production process. If leisure is a superior good — an uncontroversial assumption — and if spouses' labor are substitutes (resp. complements), and consequently  $\nu_i < 0$  (resp.  $\nu_i > 0$ ), the direct-effect of wage on

spouses' welfare is underestimated (resp. overestimated) and the cross-effect is overestimated (resp. underestimated). We also have the very important corollary that follows.

**Corollary 5** *The omission of domestic production does not invalidate welfare comparisons if and only if the profit function is additive.*

This corollary specifies the conditions that the underlying production function has to satisfy to make valid welfare comparisons with Chiappori's (1988, 1992) initial model.<sup>8</sup> The empirical relevance of this condition is examined in the next subsection. Before that, we may consider another interesting application of this corollary. So, if for unspecified reasons the husband's (say) domestic labor supply is perfectly inelastic — this extreme situation certainly represents a convenient approximation for many households in which the husband does not participate to household chores — the welfare comparisons obtained from the simplest form of the collective model are still valid.<sup>9</sup>

### 3.3 A numerical example

The complementarity/substitutability relationship between spouses' time is clearly an empirical issue. What can be learned from data? Consider an activity which is valued primarily for its output rather than its inherent satisfaction — it excludes child caring that, broadly speaking, can be assimilated to leisure. Research then shows that “relative wages of the couple appear to matter some, but that much of the division of labor is independent of wages” (Juster and Stafford, 1991, p. 498). In other words, the number of hours worked at home are not very flexible to variations in wages. Having said that, the hypothesis of substitutability is generally supported by empirical studies in the United-States but conclusive evidence is definitely lacking. In one of these rare investigations, based on the IXth wave of the Panel Study of Income Dynamics, Graham and Green (1984) show that the own-wage elasticity of the wife's domestic labor supply is equal to  $-0.169$  (t-test = 3.593)

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<sup>8</sup>Of course, the direct utilities  $\hat{u}_i$  which are recovered from the relations:  $h_i = g_i(w_i, \varphi_i)$  cannot be interpreted in the usual way, even if the production function is additive. They are a mixture of individual preferences  $u_i$  and individual technologies  $\pi_i$ .

<sup>9</sup>In a certain sense, the implications of the assumptions of the model are overall correct although these assumptions are inaccurate. This is a reminiscence of the instrumentalist view of economics by Friedman (1953).

and the cross-wage elasticity to 0.047 (t-test = 0.888). Hill and Juster (1985) draw a similar conclusion with time use data and a disaggregate framework. See also Gronau (1977) for more results on the United-States.<sup>10</sup>

The previous subsection shows that the distortions in welfare analysis are small if domestic labor supplies are relatively insensitive to partner's wage. On the other hand, empirical evidence suggests that the cross-elasticities of domestic labor supplies are close to zero. Should we necessarily conclude that the simple model of labor supply is sufficient to make precise welfare analysis? Actually, the answer is no. As a matter-of-fact, the size of the bias also depends on the sensitivity of leisure demands. And empirical evidence unambiguously suggests that the impact of income shares on leisure demands is very small. Consequently, the bias in welfare analysis may be, at the end, substantial. The numerical example that follows illustrates this point.

A consequence of what precedes is that any functional form for market labor supplies that satisfies the theoretical restrictions listed in Chiappori (1988, 1992) and implies a linear form such as  $h_i = g_i(w_i, \varphi_i)$  is also compatible with a more sophisticated model accounting for non-additive profit functions. Let us consider for example Chiappori et alii's (2001) functional form for market labor supplies:

$$h_i = a_i + b_i \ln w_f + c_i \ln w_m + d_i \ln w_f \ln w_m + e_i y + d_i f s, \quad (14)$$

where  $a_i, \dots, e_i$  and  $f$  are parameters. This form has linear Engel curves and the slopes  $\alpha_f$  and  $\alpha_m$  are constants, uniquely defined by

$$\alpha_f = e_f - \frac{d_f}{d_m} e_m, \quad \alpha_m = e_m - \frac{d_m}{d_f} e_f.$$

In contrast with Chiappori et alii (2001), however, we suppose that there is domestic production. For example, the underlying profit function may have the following form:

$$\begin{aligned} \pi = & \kappa_0 - \kappa_f w_f - \kappa_m w_m - \frac{\kappa_{ff}}{2} w_f^2 - \frac{\kappa_{mm}}{2} w_m^2 \\ & - \lambda \exp(\alpha_f w_f + \alpha_m w_m)^{-1}, \end{aligned} \quad (15)$$

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<sup>10</sup>However, the most natural application of the model of marketable domestic production concerns agricultural households. It appears that, in this case, the impact of wages on domestic labor supplies is more important. See Singh, Squire and Strauss (1986) and Taylor and Adelman (2004) for surveys.

where  $\kappa_0, \kappa_i, \kappa_{ii}$  and  $\lambda$  are parameters. In this case, the conditions of Proposition 3 are satisfied. From the Hotelling Lemma, the domestic labor supplies that are compatible with (15) are:

$$t_i = \kappa_i + \kappa_{ii}w_i + \alpha_i\lambda \times \exp(\alpha_f w_f + \alpha_m w_m)^{-1}. \quad (16)$$

Then, symmetry is fulfilled and the market labor supplies can be written under the form described by (14).

In what follows, we use the empirical results given by Chiappori et alii (2001) and evaluate the amplitude of the distortions due to the omission of domestic production. The dependent variable is measured by the number of worked hours per year. Firstly, Chiappori et alii (2001, Tables 2 & 4) show that a one-thousand-dollar increase in the yearly wife's (resp. husband's) share of income implies, on average, a decrease in market labor supply by about 10 hours (resp. 20 hours) over the year. That is,  $\alpha_f \simeq 0.01$  and  $\alpha_m \simeq 0.02$ . Secondly, a reasonable and prudent conjecture, based on elasticities that can be found in the literature, is that a one-dollar increase in the wife's (resp. husband's) hourly wage implies an increase in the husband's (resp. wife's) domestic labor supply by about 5 hours.<sup>11</sup> This is, of course, a rough approximation. Hence,  $\nu_f \simeq 500$  and  $\nu_m \simeq 250$ . Let us consider now the wife's welfare. From Chiappori et alii (2001, Table 4), we obtain:<sup>12</sup>

$$\begin{aligned} \partial \hat{v}_f^* / \partial w_f &\simeq 107 \times \hat{\lambda}_f, \\ \partial \hat{v}_f^* / \partial w_m &\simeq 600 \times \hat{\lambda}_f. \end{aligned}$$

The striking point is that the effect of the wife's wage on her welfare is smaller than the effect of the husband's wage. However, using (10) to (13), our correction gives:

$$\begin{aligned} \partial v_f^* / \partial w_f &\simeq (107 + \nu_m) \times \lambda_f = 357 \times \lambda_f, \\ \partial v_f^* / \partial w_m &\simeq (600 - \nu_f) \times \lambda_f = 100 \times \lambda_f. \end{aligned}$$

The wife's welfare is now more sensitive to variations in her own wage than to variations in her partner's wage. This is more in line with intuition. However,

<sup>11</sup>This figure corresponds to an elasticity of 0.05 for a wage equal to \$10 and a number of homeworked hours equal to 1000.

<sup>12</sup>The second line is directly computed from the estimates of the derivatives of the sharing rule at the average point of the sample (600). The first line uses the estimates of the derivatives of the sharing rule (-1634) and the average number of hours worked by women on market (1741). Then,  $107 = 1741 - 1634$ . It is fair to say, however, that the standard deviations for these estimates are quite large.

the conclusion of this example is that, even if domestic labor supplies are quite inelastic, the distortions in welfare comparisons may be important.

### 3.4 Proof of Proposition 3

Let us go back to the proof of Proposition 3. The reader who desires avoiding technicalities may skip this subsection. The proof follows in two steps.

**a) (First Necessary Condition)** The first step is to show that, under condition R, Engel curves have to be linear. To do that, we differentiate (5) with respect to  $w_j$  ( $j \neq i$ ). We obtain:

$$\frac{\partial L_i}{\partial \rho_i} \frac{\partial \rho_i}{\partial w_j} - \frac{\partial t_i}{\partial w_j} = \frac{\partial g_i}{\partial \varphi_i} \frac{\partial \varphi_i}{\partial w_j}.$$

We differentiate this expression again with respect to  $s$  and use (7) to (9). We obtain:

$$\frac{\partial^2 L_i}{\partial \rho_i^2} \frac{\partial \rho_i}{\partial s} \left( \frac{\partial \rho_i}{\partial w_j} - \frac{\partial \varphi_i}{\partial w_j} \right) = 0.$$

The term in parentheses in the right-hand-side is different from zero because of the non-additivity of the profit function (and thus  $\nu_i \neq 0$ ). Then, if  $\partial \rho_i / \partial s \neq 0$ , Engel curves are linear:  $\partial^2 L_i / \partial \rho_i^2 = 0$ . The linearity is thus a first necessary condition in the non-additive case.

**b) (Second Necessary Condition)** Then, we have to prove that the specific form for the profit function given in Proposition 3 is also necessary. If Engel curves are linear, (5) becomes:

$$\alpha_i(w_i) \rho_i + \beta_i(w_i) - t_i(w_f, w_m) = \alpha_i(w_i) \varphi_i + \xi_i(w_i),$$

for some function  $\xi_i(w_i)$ . From this expression, we obtain:

$$\rho_i - \frac{t_i(w_f, w_m)}{\alpha_i(w_i)} + \delta_i(w_i) = \varphi_i, \tag{17}$$

where

$$\delta_i(w_i) = \frac{\beta_i(w_i) - \xi_i(w_i)}{\alpha_i(w_i)},$$

since  $\alpha_i(w_i) \neq 0$  by condition R. We sum up (17) for each spouse and use the Hotelling Lemma. We obtain:

$$\pi(w_f, w_m) + \sum_i \frac{1}{\alpha_i(w_i)} \frac{\partial \pi(w_f, w_m)}{\partial w_i} + \sum_i \delta(w_i) = 0. \quad (18)$$

The fact that the profit function can be seen as a solution to this partial differential equation is a second necessary condition. The explicit solution of this type of equation is given in Lemma 7 in the appendix.

Since this partial differential equation has always a solution, from well-known theorems in partial differential equation theory, the two necessary conditions are sufficient as well. ||

## 4 A new identification result

### 4.1 Result and interpretation

One of the main results presented in the previous section is that a welfare analysis based on a theoretical framework that ignores home production is valid if and only if the profit function is additive. The properties of the profit function is an empirical issue and the economist should leave it to the facts. Still evidence is not clear-cut. Moreover, we show in our numerical illustration that even small deviations from additivity may largely distort welfare conclusions. In this case, the main question is: How can we make welfare analysis without observing domestic labor supplies or making strong assumptions on the profit function?

This important issue is addressed in the present section. Precisely, we show that the most important components of the model can be retrieved from the sole observation of market labor supplies. Hence, making welfare analysis is possible without observing domestic labor supplies. To do that, we introduce the following definitions:

$$\Delta_i = \frac{\partial h_i}{\partial y} - \left( \frac{\partial h_i / \partial s}{\partial h_j / \partial s} \right) \frac{\partial h_j}{\partial y} \quad \text{with } i, j = f, m \text{ and } j \neq i,$$

where denominator is supposed to be different from zero, and the following regularity condition.

**Condition R'** Market labor supplies are such that  $\partial h_i/\partial s \neq 0$ ,  $\Delta_i \neq 0$ ,  $\partial \Delta_i/\partial y \neq 0$  for almost all  $(w_f, w_m, y, s)$  in  $\mathbb{R}_+^2 \times \mathbb{R}^2$ .

The main result is then formally stated in the next proposition.

**Proposition 6** *Suppose collective rationality with marketable domestic production. Market labor supplies are observed. Then, if R' is satisfied,*

1. *Market labor supplies have to satisfy testable constraints under the form of partial differential equations;*
2. *The income share and the domestic labor supply of spouse f (resp. m) can be identified up to an additive function of  $w_f$  (resp.  $w_m$ );*
3. *The indirect collective utility of both spouses can be identified up to composition by an increasing transform.*

Before demonstrating this proposition in the next subsection, we have to make several remarks.

**1.** The second statement can be interpreted as follows. If  $\rho_i^*(w_f, w_m, y, s)$  is a particular solution for the spouse  $i$ 's share that is compatible with both market labor supplies, then the general solution is given by

$$\rho_i(w_f, w_m, y, s) = \rho_i^*(w_f, w_m, y, s) + k^i(w_i),$$

for some unknown function  $k^i(w_i)$ . The interpretation is similar for the third statement (identification of domestic labor supplies). Let us note that, if both domestic labor supplies were observed, the indeterminacy in income shares would consist merely of a constant, instead of a function.

**2.** The identification of income shares is thus incomplete. However, since the functions  $v_i^*(w_f, w_m, y, s)$  — i.e., the most useful concept to make welfare comparisons — are identifiable up to composition by an increasing transform, it is possible to make policy recommendations in almost all circumstances.

**3.** Condition  $R'$  is quite complicated. This condition excludes (i) Marshallian labor supplies that are linear functions of spouses' share and (ii) spouses' shares of income that are linear functions of non-labor income.<sup>13</sup> Thus, the functional form used by Chiappori et alii (2001) does not allow to perform welfare analysis. Our numerical example is thus necessary to evaluate the bias resulting from the omission of domestic production.

**4.** The definitions of the derivatives of the indirect collective utilities with respect to non-labor income or distribution factors are exactly the same as Chiappori's (1992) ones. The definitions of the derivatives of the collective indirect utilities with respect to wages differ but they reduce to Chiappori's definitions if the profit function is additive. Our identification result is thus a generalization of what is perhaps the most famous result on collective models.

**5.** Our recommended strategy is thus the following. To begin with, if she does not have prior information on the underlying domestic technology, the economist should use the general model which allows for domestic production. Then, a statistical test of the validity of the additivity hypothesis can be performed since the cross-derivatives of domestic labor supplies are identifiable. Only if the additivity hypothesis is not rejected by the data, the economist is allowed to use the simplest formulae to retrieve collective indirect utilities when she performs welfare analysis.

## 4.2 Proof of Proposition 6

**Proof of Statements 1 and 2.** If we differentiate (3) with respect to  $y$  and  $s$ , we obtain a system of four partial differential equations of the form:

$$\frac{\partial h_i}{\partial y} = \frac{\partial L_i}{\partial \rho} \frac{\partial \rho_i}{\partial y}, \quad \frac{\partial h_i}{\partial s} = \frac{\partial L_i}{\partial \rho} \frac{\partial \rho_i}{\partial s},$$

with  $\partial \rho_f / \partial y + \partial \rho_m / \partial y = 1$  and  $\partial \rho_f / \partial s + \partial \rho_m / \partial s = 0$ . Chiappori et alii (2001) shows that, if  $R$  is satisfied, this system can be solved with respect

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<sup>13</sup>Unsurprisingly, condition  $R'$  is related to the conditions on preferences and technologies listed in Propositions 2 and 3: if the underlying structure is not uniquely defined, it is obvious that some identification problems —harmless or serious — appear. Condition  $R'$  is also related to Condition  $R$ .

to the derivatives of the sharing rule and total labor supplies. Using the notation defined above, the solutions are:

$$\frac{\partial L_i}{\partial \rho} = \Delta_i, \quad (19)$$

and

$$\frac{\partial \rho_i}{\partial y} = \frac{\partial h_i / \partial y}{\Delta_i}, \quad \frac{\partial \rho_i}{\partial s} = \frac{\partial h_i / \partial s}{\Delta_i}. \quad (20)$$

The cross-derivative restrictions applied to (20) imply that

$$\mathbf{C1:} \quad \frac{\partial h_i}{\partial y} \frac{\partial \Delta_i}{\partial s} = \frac{\partial h_i}{\partial s} \frac{\partial \Delta_i}{\partial y} \text{ for } i = f, m.$$

Consider now the derivatives of (3) with respect to  $w_j$  ( $j = f, m$  and  $j \neq i$ ). We obtain:

$$\frac{\partial h_i}{\partial w_j} = \frac{\partial L_i}{\partial \rho} \frac{\partial \rho_i}{\partial w_j} - \frac{\partial t_i}{\partial w_j}. \quad (21)$$

If we differentiate this expression with respect to  $y$ , and use (19), we obtain:

$$\frac{\partial \rho_i}{\partial w_j} = \frac{\partial \Delta_i / \partial w_j}{\partial \Delta_i / \partial y} \frac{\partial h_i / \partial y}{\Delta_i}. \quad (22)$$

The cross-derivative restrictions applied to (20) and (22) imply that

$$\mathbf{C2:} \quad \frac{\partial^2 \Delta_i}{\partial y \partial w_j} \frac{\partial \Delta_i}{\partial y} \frac{\partial h_i}{\partial y} + \frac{\partial \Delta_i}{\partial w_j} \frac{\partial \Delta_i}{\partial y} \frac{\partial^2 h_i}{\partial y^2} - \frac{\partial \Delta_i}{\partial w_j} \frac{\partial^2 \Delta_i}{\partial y^2} \frac{\partial h_i}{\partial y} - \left( \frac{\partial \Delta_i}{\partial y} \right)^2 \frac{\partial^2 h_i}{\partial y \partial w_j} = 0$$

for  $i, j = f, m$  and  $j \neq i$ .

$$\mathbf{C3:} \quad \frac{\partial^2 \Delta_i}{\partial s \partial w_j} \frac{\partial \Delta_i}{\partial s} \frac{\partial h_i}{\partial s} + \frac{\partial \Delta_i}{\partial w_j} \frac{\partial \Delta_i}{\partial s} \frac{\partial^2 h_i}{\partial s^2} - \frac{\partial \Delta_i}{\partial w_j} \frac{\partial^2 \Delta_i}{\partial s^2} \frac{\partial h_i}{\partial s} - \left( \frac{\partial \Delta_i}{\partial s} \right)^2 \frac{\partial^2 h_i}{\partial s \partial w_j} = 0$$

for  $i, j = f, m$  and  $j \neq i$ .

Introducing (22) in (21) yields the cross-derivatives of the domestic labor supplies:

$$\frac{\partial t_i}{\partial w_j} = \frac{\partial \Delta_i / \partial w_j}{\partial \Delta_i / \partial y} \frac{\partial h_i}{\partial y} - \frac{\partial h_i}{\partial w_j}.$$

The symmetry implies that

$$\mathbf{C4:} \quad \frac{\partial h_f}{\partial w_m} - \frac{\partial h_f}{\partial y} \frac{\partial \Delta_f / \partial w_m}{\partial \Delta_f / \partial y} = \frac{\partial h_m}{\partial w_f} - \frac{\partial h_m}{\partial y} \frac{\partial \Delta_m / \partial w_f}{\partial \Delta_m / \partial y}.$$

**Proof of Statement 3.** If we differentiate (4) with respect to  $y$  and  $s$ , we obtain:

$$\frac{\partial v_i^*}{\partial y} = \lambda_i \frac{\partial h_i / \partial y}{\Delta_i}, \quad \frac{\partial v_i^*}{\partial s} = \lambda_i \frac{\partial h_i / \partial s}{\Delta_i}.$$

Similarly, if we differentiate (4) with respect to  $w_j$  ( $j = f, m$  and  $j \neq i$ ), we obtain:

$$\frac{\partial v_i^*}{\partial w_j} = \lambda_i \frac{\partial \Delta_i / \partial w_j}{\partial \Delta_i / \partial y} \frac{\partial h_i / \partial y}{\Delta_i}.$$

From the constraint  $\sum_k \rho_k = \psi$ , and the Hotelling Lemma, we have:

$$\frac{\partial \rho_i}{\partial w_i} = - \left( t_i + \frac{\partial \Delta_j / \partial w_i}{\partial \Delta_j / \partial y} \frac{\partial h_j / \partial y}{\Delta_j} \right). \quad (23)$$

If we differentiate (4) with respect to  $w_i$ , and use (23) and the Roy Identity, we obtain:

$$\frac{\partial v_i^*}{\partial w_i} = \lambda_i \left( h_i - \frac{\partial \Delta_j / \partial w_i}{\partial \Delta_j / \partial y} \frac{\partial h_j / \partial y}{\Delta_j} \right).$$

Since the partial derivatives are defined up to a multiplicative function  $\lambda_i$ , the collective indirect utilities are defined up to composition by an increasing transform. ||

## 5 Summary and conclusion

The present paper can be regarded as a toolbox for applied economists who are interested in performing welfare comparisons at the individual level. A synthesis of our main conclusions may be useful at this stage. Let us consider a sample of couples and assume there is domestic production. Then,

1. A simple model of market labor supplies, which does not allow for domestic production, may conveniently fit the data if and only if (i) the profit function is additive or (ii) Engel curves are linear and the profit function has a particular, not necessarily additive form.
2. If (i) or (ii) is satisfied, then the economist is not able to empirically reject the simple model of labor supply. Suppose the economist arbitrarily decides to ignore domestic production.

3. If (i) is satisfied, welfare analyses the economist makes are valid. If (ii) is satisfied, welfare analyses are biased: if spouses' labor are substitutes (resp. complements), the direct-effect of wage on spouses' welfare is underestimated (resp. overestimated) and the cross-effect is overestimated (resp. underestimated).
4. There exists a simple method for retrieving collective indirect utilities that is robust to household production. This method necessitates the observation of the sole market labor supplies, allows to make correct welfare analysis and, if the profit function is additive, reduces to the traditional method.

To sum up, the main — and probably the most unexpected — result in this paper is that the economist can get out of observing homework when performing welfare comparisons at the individual level. This opens up new horizons for empirical investigations. Quite importantly, our results crucially depend on the assumption that domestic goods are marketable. Admittedly, the goods trade on outside market are likely imperfect substitutes for goods produced within the household — except in agricultural households, for which domestic goods have a simple, natural interpretation. Our argument is that the assumption of marketability is certainly less restrictive than the straight exclusion of domestic production as in the simple collective model of labor supply. Future researches should, however, investigate the case of non-marketable domestic goods.

## A Appendix — A Useful Lemma

**Lemma 7** *Consider the following partial differential equation in  $f$  :*

$$f(x, y) + \frac{1}{a(x)} \frac{\partial f(x, y)}{\partial x} + \frac{1}{b(y)} \frac{\partial f(x, y)}{\partial y} + c(x) + d(y) = 0, \quad (24)$$

*where functions  $a(x)$ ,  $b(y)$  are continuously differentiable in  $\mathbb{R}$  and do not vanish simultaneously at any point of  $\mathbb{R}$ , and functions  $c(x)$ ,  $d(y)$  are continuous in  $\mathbb{R}$ . Then, the general solution of (24) on  $\mathbb{R}^2$  is*

$$f(x, y) = \frac{G(A(x) - B(y))}{\exp(A(x) + B(y))} + \frac{C(x)}{\exp A(x)} + \frac{D(y)}{\exp B(y)}$$

for some function  $G$ , where

$$\begin{aligned} A(x) &= \int a(x) \cdot dx, \\ B(y) &= \int b(y) \cdot dy, \\ C(x) &= \int a(x)c(x) \exp A(x) \cdot dx, \\ D(y) &= \int b(y)d(y) \exp B(y) \cdot dy. \end{aligned}$$

**Proof.** The idea of the proof is well-known in the theory of partial differential equations (see Zachmanoglou and Thoe, 1976, for instance). It follows in stages. We consider the solution of the homogenous partial differential equation that corresponds to (24), i.e.,

$$f(x, y) + \frac{1}{a(x)} \frac{\partial f(x, y)}{\partial x} + \frac{1}{b(y)} \frac{\partial f(x, y)}{\partial y} = 0,$$

and use a particular solution of (24) to solve the non-homogenous case.

**Homogenous case.** We have to introduce new coordinates,  $r$  and  $t$ , in terms of which (24) takes the form of an ordinary differential equation that can be easily solved. Let the new coordinates be related to the old ones by the equations:  $r = r(x, y)$  and  $t = t(x, y)$ . We require that the functions  $r(x, y)$  and  $t(x, y)$  are continuously differentiable and their Jacobian is different from zero, i.e.,

$$J_{\text{pde}} \equiv \frac{\partial r}{\partial x} \frac{\partial t}{\partial y} - \frac{\partial r}{\partial y} \frac{\partial t}{\partial x} \neq 0.$$

If this condition is satisfied at the point  $(x_0, y_0)$ , we also have in the neighborhood of  $(x_0, y_0)$  the inverse relations:  $x = x(r, t)$  and  $y = y(r, t)$ . Substituting these expressions into (24) and using the Chain Rule, we obtain the following equation:

$$\zeta(r, t) + \alpha(r, t) \frac{\partial \zeta(r, t)}{\partial r} + \beta(r, t) \frac{\partial \zeta(r, t)}{\partial t} = 0,$$

where  $\zeta(r, t) = f(x(r, t), y(r, t))$  and

$$\alpha = \frac{1}{a} \frac{\partial r}{\partial x} + \frac{1}{b} \frac{\partial r}{\partial y}, \quad \beta = \frac{1}{a} \frac{\partial t}{\partial x} + \frac{1}{b} \frac{\partial t}{\partial y}.$$

We see that  $\alpha = 0$  if  $r$  is a solution of the following partial differential equation:

$$\frac{1}{a} \frac{\partial r}{\partial x} + \frac{1}{b} \frac{\partial r}{\partial y} = 0. \quad (25)$$

This is the characteristic equation of the partial differential equation. There are infinitely many solutions. Supposing for example that  $a(x) \neq 0$ , the characteristic direction is given by:

$$\frac{dx}{dy} = \frac{b(y)}{a(x)}.$$

Then, for any change of variable such that this condition is satisfied, we have:  $\alpha = 0$ . The characteristic curves, which satisfy by definition this condition, can be obtained by solving the differential equation:

$$b(y) \cdot dy = a(x) \cdot dx.$$

That is:

$$\int b(y) \cdot dy - \int a(x) \cdot dx = k,$$

where  $k$  is a constant of integration. We choose thus the following change of variable:

$$r = \int b(y) \cdot dy - \int a(x) \cdot dx.$$

It is easy to show that (25) is satisfied for this change of variable. The second change of variable can be arbitrarily assigned such that  $J_{\text{pde}} \neq 0$ . We choose:

$$t = y.$$

Using this change of variable gives a simple ordinary differential equation in  $\zeta$ . That is:

$$\zeta(r, t) + \frac{1}{b(t)} \frac{\partial \zeta(r, t)}{\partial t} = 0.$$

The general solution to this ordinary differential equation is:

$$\zeta(r, t) = g(r) \times \exp\left(-\int b(t) \cdot dt\right),$$

where  $g$  is any function. The general solution of the original (homogeneous) equation is:

$$f(x, y) = g \left( \int b(y) \cdot dy - \int a(x) \cdot dx \right) \times \exp \left( - \int b(y) \cdot dy \right)$$

or

$$f(x, y) = \frac{G(A(x) - B(y))}{\exp(A(x) + B(y))}$$

for some function  $G$ .

**Non-homogenous case.** The second step is obtain a solution in the non-homogeneous case. If we add a particular solution of the non-homogenous partial differential equation to the general solution which is derived above, we obtain the general solution of the non-homogeneous partial differential equation. In particular, a solution is:

$$\phi(x, y) = \frac{C(x)}{\exp(A(x))} + \frac{D(y)}{\exp(B(y))}.$$

This concludes the proof of the Lemma.||

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