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Vehicle and Fleet Random Effects in a Model of Insurance Rating for Fleets of Vehicles

Jean-François Angers
Denise Desjardins
Georges Dionne
François Guertin

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Angers, Desjardins, Guertin: CRT, University of Montreal
Dionne: HEC Montréal, CRT, CIRPÉE, and THEMA (France)

Abstract:
We are proposing a parametric model to rate insurance for vehicles belonging to a fleet. The tables of premiums presented take into account past vehicle accidents, observable characteristics of the vehicles and fleets, and violations of the road-safety code committed by drivers and carriers. The premiums are also adjusted according to accidents accumulated by the fleets over time. The model proposed accounts directly for explicit changes in the various components of the probability of accidents. It represents an extension of bonus-malus-type automobile insurance models for individual premiums (Lemaire, 1985 ; Dionne and Vannase, 1989 and 1992 ; Pinquet, 1997 and 1998 ; Frangos and Vrontos, 2001 ; Purcaru and Denuit, 2003). The extension adds a fleet effect to the vehicle effect so as to account for the impact that the unobservable characteristics or actions of carriers can have on truck accident rates. This form of rating makes it possible to visualize what impact the behaviors of owners and drivers can have on the predicted rate of accidents and, consequently, on premiums.

Keywords: Fleet of vehicles, random effects, vehicle effect, fleet effect, insurance pricing, behaviors of owners and drivers, Poisson, gamma, Dirichlet

Résumé:
Nous proposons un modèle paramétrique de tarification de l'assurance de véhicules routiers appartenant à une flotte. Les tables de primes qui y sont présentées tiennent compte des accidents passés des véhicules, des caractéristiques observables des véhicules et des flottes et des infractions au code de la sécurité routière des conducteurs et des transporteurs. De plus, les primes sont ajustées en fonction des accidents accumulés par les flottes dans le temps. Il s'agit d'un modèle qui prend en compte directement des changements explicites des différentes composantes des probabilités d'accidents. Il représente une extension aux modèles d'assurance automobile de type bonus-malus pour les primes individuelles (Lemaire, 1985; Dionne et Vannase, 1989 et 1992; Pinquet, 1997 et 1998; Frangos et Vrontos, 2001; Purcaru et Denuit, 2003). L'extension ajoute un effet flotte à l'effet véhicule pour tenir compte des caractéristiques ou des actions non observables des transporteurs sur les taux d'accidents des camions. Cette forme de tarification comporte plusieurs avantages. Elle permet de visualiser l'impact des comportements des propriétaires des flottes et des conducteurs des véhicules sur les taux d'accidents prédits et, par conséquent, sur les primes. Elle mesure l'influence des infractions et des accidents accumulés sur les primes d'assurance mais d'une façon différente. En effet, les effets des infractions sont obtenus via la composante de régression, alors que les effets des accidents proviennent des résidus non expliqués de la régression sur les accidents des camions via un modèle bayésien de tarification.

Mots Clés: Tarification de l'assurance, flotte de véhicules, modèle bayésien, sécurité routière, bonus-malus

JEL Classification: D81, G22

1 INTRODUCTION

Very few studies have analyzed systematically the risks of accidents for vehicle fleets. Marie-Jeanne (1994) developed a rating model based on the size of the fleet and Teugels and Sundt (1991) proposed rating based on the aggregated loss of the fleet. Other researchers have confined themselves to studying the drivers of vehicles to obtain a portrait of the risks posed by a carrier (Dionne et al., 2001). This amounts to forgetting that firms' owners or management can also affect the accident rates of their vehicles. Decisions regarding driving hours, spending on vehicle maintenance, and guidelines for loading or securing cargo in vehicles can have repercussions on road safety. Dionne, Desjardins, and Pinquet (1999 and 2001) developed bonus-malus models that use a semi-parametric approach to take into account the behaviors of both the drivers and owners of vehicles. In this article, we propose a parametric model.

Measuring the risks associated with fleets of vehicles is difficult for a number of reasons. For one, the units composing the fleets must be defined. Should we do this by observing drivers or vehicles? We answered that question by opting for vehicles: For, with information readily available from insurers, the link between vehicles and carriers can be made continuously. Linking information on drivers to carriers is, in contrast, very costly, since the movements of drivers from one fleet to another are not systematically recorded, whereas licensing and insurance contracting keep track of vehicles as they move among fleets. The vehicles are taken to represent different individual risks. These risks are influenced by the observable and unobservable characteristics of the vehicles, the drivers using them, and the carriers who own or lease them. It is thus essential to use care in modeling these different sources of information.

Another difficulty is weighting the information obtained on individuals and fleets for rating purposes. An adequate model for rating the risks of fleets must integrate the behaviors of drivers with those of owners so as to introduce incentives for safety tailored to the various levels of decisions to be made when facing hierarchical moral hazard (Moses and Savage, 1994, 1996; Fluet, 1999; Winter, 2000).

We are proposing a new rating model for vehicles belonging to a fleet. The model is a parametric one which can account directly for both observable and unobservable characteristics of the vehicles, drivers, and owners associated with a particular vehicle fleet. The model proposed is a direct extension of bonus-malus-type automobile insurance models (Lemaire 1985, 1995; Dionne and Vanasse 1989 and 1992; Pinquet 1997, 1998; Frangos and Vrontos 2001; Purcaru and Denuit 2003) to individual premiums (see Pinquet, 2000, for a review of the literature). The extension adds a random fleet effect to the vehicle effect in the model, in order to take into account the unobservable effects of carriers, vehicles, and their drivers on truck accident rates in the Bayesian or *a posteriori* calculation of premiums. Observable variables characterizing vehicles, fleets, and the road-safety behavior of both drivers and carriers are used in evaluating the *a priori* risks of different vehicles.

In the following section, we present statistical models for estimating accident probabilities for vehicles belonging to fleets of various sizes. Section 3 develops the optimal bonus-malus system integrating both fleet and vehicle effects. Section 4 proposes different premium tables, while section 5 discusses possible extensions of the model.

2 STATISTICAL MODELS

Our methodology is divided into two steps. In the first step, we use an econometric model to evaluate the accident probabilities for the vehicles of carriers. As *a priori* information, we shall use estimated parameters to calculate insurance premiums. These parameters take into account the information available on the observable characteristics of vehicles and fleets as well as on traffic violations by drivers and carriers. In order to take unobservable characteristics and actions into account for purposes of rating, we shall use the residuals of the econometric estimations. One of the article's contributions consists in proposing a new model for estimating accident probabilities, a model capable of explicitly isolating the fleet effect from the vehicle effect. In a second step, we propose a bonus-malus system which can use both the *a priori* information obtained from the estimated parameters and the *a posteriori* information obtained from residuals of the estimations of vehicle accident distributions. In order to show what contribution the different effects make to insurance premiums, we shall distinguish between one-vehicle and two-vehicle carriers and then generalize the model to carriers with more than two vehicles.

2.1 ECONOMETRIC MODEL FOR ESTIMATING DISTRIBUTIONS OF VEHICLE ACCIDENTS

Most econometric models applied to discrete (or countable) variables are based on the Poisson distribution, where probability P that a vehicle i belonging to fleet f will be involved in accidents at period j can be represented by the following expression:

$$P\left(y_{fi}^{j} \mid \lambda_{fi}^{j}\right)=\frac{e^{-\lambda_{fi}^{j}}\left(\lambda_{fi}^{j}\right)^{y_{fi}^{j}}}{y_{fi}^{j}!}.$$

With the Poisson law, we obtain that the mathematical expectation of the number of accidents (E) is equal to the variance (Var), $E\left(Y_{fi}^{j}\right)=Var\left(Y_{fi}^{j}\right)=\lambda_{fi}^{j}$ where Y_{fi}^{j} is the number of accidents for truck *i* belonging to fleet *f* at period *j* and λ_{fi}^{j} is the parameter of the Poisson distribution. This modeling implicitly supposes that the distribution of accidents can be entirely explained by observable heterogeneity, which cancels any need for a bonus-malus system.

Let us now suppose that an unobservable heterogeneity exists owing to certain characteristics or actions non observable by the insurer. Suppose that $\lambda_{fi}^{j}=\gamma_{fi}^{j}\alpha_{f}\theta_{fi}$ with $\gamma_{fi}^{j}=d_{fi}^{j}e^{X_{fi}^{j}\beta}$ where d_{fi}^{j} measures the number of days that vehicle *i* of fleet *f* is authorized to circulate during period *j*, divided by the number of total days in period *j*. This measures the exposure to the risk of accident in period *j*. Using the exponential to define γ_{fi}^{j} allows us to ensure the non-negativity of λ_{fi}^{j}. The vector $X_{fi}^{j}=\left(x_{fi1}^{j},\cdots,x_{fip}^{j}\right)$ contains the *p* characteristics of truck *i* in fleet *f* observed at period *j*; this vector contains specific information on the vehicle and other characteristics on the fleet. β is a vector of parameters to be estimated. Parameter α_{f} is the random effect associated with fleet *f*, that is, the unobservable risk attributable to the fleet; whereas parameter θ_{fi} is the random effect of truck *i* in fleet *f*. We suppose that $\sum_{i=1}^{I_f}\theta_{fi}=1$ where I_f is the total number of vehicles in fleet *f*. In other terms, θ_{fi} is the proportion of the risk for fleet *f* which can be attributed to vehicle *i*; the total unobservable risk for vehicle *i* of fleet *f* is defined by $\alpha_{f}\theta_{fi}$. It should be noted that when fleet *f* has only one vehicle such that $I_f = 1$, $\theta_{f1} = 1$ by definition. This means that the risk attributable to vehicle corresponds to that of the fleet, from which it follows that $\lambda_{f1}^j = \gamma_{f1}^j \alpha_f$.

We make the hypothesis that θ_{fi} will follow a Dirichlet parametric distribution with parameters $(\nu_1, \nu_2, \cdots, \nu_{I_f})$ and that α_f will follow a gamma distribution with parameters $\left(I_f \kappa_f^{-1}, \kappa_f^{-1}\right)$. This parametization permits to obtain a mean fleet effect that increases with the number of vehicles in the fleet.

2.1.1 Size-1 carrier

For period *j*, the distribution of the number of accidents for a fleet with one vehicle is given by:

$$P\left(y_{f1}^j\right) = \int_0^\infty P\left(y_{f1}^j \mid \alpha_f\right) f\left(\alpha_f\right) d\alpha_f,$$

which, assuming that α_f follows a gamma distribution $\left(\kappa_f^{-1}, \kappa_f^{-1}\right)$, can be rewritten as follows:

$$P\left(y_{f1}^j\right) = \frac{\Gamma\left(y_{f1}^j + \kappa_f^{-1}\right)}{\Gamma\left(y_{f1}^j + 1\right)\Gamma\left(\kappa_f^{-1}\right)} \left(\frac{\kappa_f^{-1}}{\kappa_f^{-1} + \gamma_{f1}^j}\right)^{\kappa^{-1}} \left(\frac{\gamma_{f1}^j}{\kappa_f^{-1} + \gamma_{f1}^j}\right)^{y_{f1}^j}. \qquad (1)$$

This distribution has been used fairly often in the literature (Lemaire, 1985; Dionne and Vanasse, 1989; Hausman et al., 1984; Gouriéroux, 1999). It is capable of modeling unobservable heterogeneity and of introducing a bonus-malus system for individual observations. On the other hand, it is not directly applicable when estimating the probability of accidents for vehicles belonging to a fleet, as it cannot isolate the fleet effect from the vehicle effect. We now present our generalization of this basic model, starting with the simple case of a fleet composed of two vehicles.

2.1.2 Carrier with 2 vehicles

The joint probability of the number of accidents at period *j* for the two vehicles in fleet *f* is given by:

$$P\left(y_{f1}^{j}, y_{f2}^{j}\right) = \int_{0}^{1} P\left(y_{f1}^{j}, y_{f2}^{j} \mid \theta_f\right) f\left(\theta_f\right) d\theta_f,$$

where

$$\theta_f = \theta_{f1} \text{ and } 1-\theta_f = \theta_{f2}.$$

Conditionally on θ_f, the joint probability of accident is equal to:

$$P\left(y_{f1}^{j}, y_{f2}^{j} \mid \theta_f\right) = \left[\frac{\left(\gamma_{f1}^{j}\right)^{y_{f1}^{j}} \left(\gamma_{f2}^{j}\right)^{y_{f2}^{j}} \left(\theta_f\right)^{y_{f1}^{j}} \left(1-\theta_f\right)^{y_{f2}^{j}} \kappa_f^{-2\kappa_f^{-1}}}{\Gamma\left(y_{f1}^{j}+1\right)\Gamma\left(y_{f2}^{j}+1\right)\Gamma\left(2\kappa_f^{-1}\right)}\right] \int_{0}^{\infty} \left[\left(\alpha_f\right)^{\sum_{i=1}^{2} y_{fi}^{j}+2\kappa_f^{-1}-1}\right]\left[e^{-\alpha_f\left(\theta_f\gamma_{f1}^{j}+\left(1-\theta_f\right)\gamma_{f2}^{j}+\kappa_f^{-1}\right)}\right] d\alpha_f. \quad (2)$$

By integrating (2) and substituting the value of $P\left(y_{f1}^{j}, y_{f2}^{j} \mid \theta_f\right)$ in $P\left(y_{f1}^{j}, y_{f2}^{j}\right)$, we obtain:

$$P\left(y_{f1}^{j}, y_{f2}^{j}\right) = \int_{0}^{1} \left[\frac{\left(\gamma_{f1}^{j}\right)^{y_{f1}^{j}} \left(\gamma_{f2}^{j}\right)^{y_{f2}^{j}} \left(\theta_f\right)^{y_{f1}^{j}} \left(1-\theta_f\right)^{y_{f2}^{j}}}{\Gamma\left(y_{f1}^{j}+1\right)\Gamma\left(y_{f2}^{j}+1\right)}\right]\left[\frac{\kappa_f^{-2\kappa_f^{-1}}}{\Gamma\left(2\kappa_f^{-1}\right)}\right] \frac{\Gamma\left(2\kappa_f^{-1}+\sum_{i=1}^{2} y_{fi}^{j}\right)}{\left(\kappa_f^{-1}+\theta_f\gamma_{f1}^{j}+\left(1-\theta_f\right)\gamma_{f2}^{j}\right)^{2\kappa_f^{-1}+\sum_{i=1}^{2} y_{fi}^{j}}} f\left(\theta_f\right) d\theta_f. \quad (3)$$

In order to estimate the probabilities of accident with a parametric approach, we must now make the distribution of θ_f more explicit. As indicated above, we suppose that the vehicle effect will follow a Dirichlet distribution. By replacing the density function $f(\theta_f)$ in equation (3) with the density of a parametric Dirichlet (ν_1, ν_2),

$$f(\theta_f) = \frac{\Gamma\left(\sum_{i=1}^{2} \nu_i\right)}{\prod_{i=1}^{2} \Gamma(\nu_i)} (\theta_f)^{\nu_1 - 1} (1-\theta_f)^{\nu_2 - 1},$$

we obtain:

$$P\left(y_{f1}^j, y_{f2}^j\right) = \frac{\left(\gamma_{f1}^j\right)^{y_{f1}^j} \left(\gamma_{f2}^j\right)^{y_{f2}^j}}{\Gamma\left(y_{f1}^j+1\right)\Gamma\left(y_{f2}^j+1\right)} \left[\frac{\Gamma\left(2\kappa_f^{-1} + \sum_{i=1}^{2} y_{fi}^j\right)\kappa_f^{-2\kappa_f^{-1}}}{\Gamma\left(2\kappa_f^{-1}\right)}\right] \left(\frac{\Gamma(\nu_1+\nu_2)}{\Gamma(\nu_1)\Gamma(\nu_2)}\right) \int_0^1 \left[\left(\frac{(\theta_f)^{y_{f1}^j+\nu_1-1}(1-\theta_f)^{y_{f2}^j+\nu_2-1}}{\left(\kappa_f^{-1} + \theta_f\gamma_{f1}^j + (1-\theta_f)\gamma_{f2}^j\right)^{2\kappa_f^{-1}+\sum_{i=1}^{2} y_{fi}^j}}\right)\right] d\theta_f. \quad (4)$$

To obtain a value of the joint probability in (4), we must compute the integral:

$$\int_0^1 \frac{(\theta_f)^{y_{f1}^j+\nu_1-1}(1-\theta_f)^{y_{f2}^j+\nu_2-1}}{\left(\kappa_f^{-1} + \theta_f\gamma_{f1}^j + (1-\theta_f)\gamma_{f2}^j\right)^{2\kappa_f^{-1}+\sum_{i=1}^{2} y_{fi}^j}} \, d\theta_f .$$

To do so, let's write the expression $\kappa_f^{-1} + \theta_f\gamma_{f1}^j + (1-\theta_f)\gamma_{f2}^j$ of the denominator as follows:

$$\left(\kappa_f^{-1} + \gamma_{f2}^j\right)\left[1 - \left(\frac{\gamma_{f2}^j - \gamma_{f1}^j}{\kappa_f^{-1} + \gamma_{f2}^j}\right)\theta_f\right],$$

which permits us to rewrite the integral in (4):

$$\int_0^1 \frac{(\theta_f)^{y_{f1}^j+\nu_1-1}(1-\theta_f)^{y_{f2}^j+\nu_2-1}}{\left(\left(\kappa_f^{-1}+\gamma_{f2}^j\right)\left[1-\left(\frac{\gamma_{f2}^j-\gamma_{f1}^j}{\kappa_f^{-1}+\gamma_{f2}^j}\right)\theta_f\right]\right)^{2\kappa_f^{-1}+\sum_{i=1}^{2} y_{fi}^j}} d\theta_f = \frac{1}{\left(\kappa_f^{-1}+\gamma_{f2}^j\right)^{2\kappa_f^{-1}+\sum_{i=1}^{2} y_{fi}^j}} \left[\frac{\Gamma\left(y_{f1}^j+\nu_1\right)\Gamma\left(y_{f2}^j+\nu_2\right)}{\Gamma\left(y_{f1}^j+y_{f2}^j+\nu_1+\nu_2\right)}\right]$$

$$\times {}_2F_1\left(y_{f1}^j+\nu_1; 2\kappa_f^{-1}+y_{f1}^j+y_{f2}^j; \nu_1+\nu_2+y_{f1}^j+y_{f2}^j; \left(\frac{\gamma_{f2}^j-\gamma_{f1}^j}{\kappa_f^{-1}+\gamma_{f2}^j}\right)\right).$$

$_2F_1$ is a hypergeometric function whose value is equal to:

$$1+\sum_{\ell=1}^{\infty}\left[\frac{\left(y_{f1}^{j}+\nu_1\right)^{[\ell]}\left(2\kappa_f^{-2}+\sum_{i=1}^{2}y_{fi}^{j}\right)^{[\ell]}}{\left(\sum_{i=1}^{2}\left(y_{fi}^{j}+\nu_i\right)\right)^{[\ell]}}\times\frac{\left(\frac{\gamma_{f2}^{j}-\gamma_{f1}^{j}}{\kappa_f^{-1}+\gamma_{f2}^{j}}\right)^{\ell}}{\ell!}\right],$$

with $h^{[\ell]}=h(h-1)\cdots(h-\ell+1)$, a decreasing factorial function.

The distribution of the number of accidents observed at period *j* for the two vehicles in fleet *f* is now given by:

$$P\left(y_{f1}^{j},y_{f2}^{j}\right)=\frac{\left(\gamma_{f1}^{j}\right)^{y_{f1}^{j}}\left(\gamma_{f2}^{j}\right)^{y_{f2}^{j}}}{\Gamma\left(y_{f1}^{j}+1\right)\Gamma\left(y_{f2}^{j}+1\right)}\left[\frac{\Gamma\left(2\kappa_f^{-1}+\sum_{i=1}^{2}y_{fi}^{j}\right)\kappa_f^{-2\kappa_f^{-1}}}{\Gamma\left(2\kappa_f^{-1}\right)}\right]\left(\frac{\Gamma\left(\nu_1+\nu_2\right)}{\Gamma\left(\nu_1\right)\Gamma\left(\nu_2\right)}\right)$$
$$\times\frac{1}{\left(\kappa_f^{-1}+\gamma_{f2}^{j}\right)^{2\kappa_f^{-1}+\sum_{i=1}^{2}y_{fi}^{j}}}\left[\frac{\Gamma\left(y_{f1}^{j}+\nu_1\right)\Gamma\left(y_{f2}^{j}+\nu_2\right)}{\Gamma\left(y_{f1}^{j}+y_{f2}^{j}+\nu_1+\nu_2\right)}\right]{}_2F_1\left(y_{f1}^{j}+\nu_1;2\kappa_f^{-1}+y_{f1}^{j}+y_{f2}^{j};\nu_1+\nu_2+y_{f1}^{j}+y_{f2}^{j};\left(\frac{\gamma_{f2}^{j}-\gamma_{f1}^{j}}{\kappa_f^{-1}+\gamma_{f2}^{j}}\right)\right).$$

We now generalizes the model to a fleet of I_f vehicles.

2.1.3 Carrier with more than 2 vehicles

The joint distribution of the number of accidents at period *j* for the I_f vehicles in fleet *f* is given by:

$$P\left(y_{f1}^{j},\cdots,y_{fI_f}^{j}\right)=\int_0^1\cdots\int_0^1 P\left(y_{f1}^{j},\cdots,y_{fI_f}^{j}\mid\theta_{f1},\cdots,\theta_{fI_f-1}\right)f\left(\theta_{f1}\cdots\theta_{fI_f}\right)d\theta_{f1}\cdots d\theta_{fI_f-1}\qquad(5)$$

where

$$\theta_{fI_f} = 1 - \sum_{i=1}^{I_f - 1} \theta_{fi} \, .$$

We can rewrite the conditional probability in (5) as:

$$P\left(y_{f1}^j, \cdots, y_{fI_f}^j \mid \theta_{f1}, \cdots, \theta_{fI_f - 1}\right) = \int_0^\infty P\left(Y_{f1}^j, \cdots, Y_{fI_f}^j \mid \alpha_f, \theta_{f1}, \cdots, \theta_{fI_f - 1}\right) f\left(\alpha_f\right) d\alpha_f$$

and by integrating with respect to α_f, we obtain a negative binomial distribution whose joint conditional probability of accidents is equal to:

$$P\left(y_{f1}^j, \cdots, y_{fI_f}^j \mid \theta_{f1}, \cdots, \theta_{fI_f - 1}\right) = \left[\prod_{i=1}^{I_f} \left(\frac{\left(\gamma_{f1}^j\right)^{y_{fi}^j} \left(\theta_{fi}\right)^{y_{fi}^j}}{\Gamma\left(y_{fi}^j + 1\right)} \right)\right] \left[\frac{\kappa_f^{-I_f \kappa_f^{-1}}}{\Gamma\left(I_f \kappa_f^{-1}\right)} \right] \frac{\Gamma\left(I_f \kappa_f^{-1} + \sum_{i=1}^{I_f} y_{fi}^j\right)}{\left(\kappa_f^{-1} + \sum_{i=1}^{I_f} \theta_{fi} \gamma_{fi}^j\right)^{I_f \kappa_f^{-1} + \sum_{i=1}^{I_f} y_{fi}^j}} \, . \quad (6)$$

Thus, by replacing $P\left(y_{f1}^j, \cdots, y_{fI_f}^j \mid \theta_{f1}, \cdots, \theta_{fI_f - 1}\right)$ in equation (5) by its value given in (6) and by replacing the density function $f\left(\theta_{f1}, \cdots, \theta_{fI_f}\right)$ by the density of a parametric Dirichlet $\left(\nu_1, \nu_2, \cdots, \nu_{I_f}\right)$, we obtain the following expression:

$$P\left(y_{f1}^j, \cdots, y_{fI_f}^j\right) = \left[\prod_{i=1}^{I_f} \left(\frac{\left(\gamma_{fi}^j\right)^{y_{fi}^j}}{\Gamma\left(y_{fi}^j + 1\right)} \right)\right] \frac{\kappa_f^{-I_f \kappa_f^{-1}} \Gamma\left(I_f \kappa_f^{-1} + \sum_{i=1}^{I_f} y_{fi}^j\right) \Gamma\left(\sum_{i=1}^{I_f} \nu_i\right)}{\Gamma\left(I_f \kappa_f^{-1}\right) \prod_{i=1}^{I_f} \Gamma\left(\nu_i\right)} \int \underset{\sum_{i=1}^{I_f} \theta_{fi} = 1}{\cdots} \int \frac{\prod_{i=1}^{I_f} \left(\theta_{fi}\right)^{y_{fi}^j + \nu_i - 1}}{\left(\kappa_f^{-1} + \sum_{i=1}^{I_f} \theta_{fi} \gamma_{fi}^j\right)^{I_f \kappa_f^{-1} + \sum_{i=1}^{I_f} y_{fi}^j}} d\theta_{f1} \cdots d\theta_{fI_f - 1} \, . \quad (7)$$

Once again, we must estimate the multidimensional integral:

$$\int_{\sum_{i=1}^{I_f}\theta_{fi}=1}\cdots\int \frac{\prod_{i=1}^{I_f}(\theta_{fi})^{y_{fi}^j+v_i-1}}{\left(\kappa_f^{-1}+\sum_{i=1}^{I_f}\theta_{fi}\gamma_{fi}^j\right)^{I_f\kappa_f^{-1}+\sum_{i=1}^{I_f}y_{fi}^j}} d\theta_{f1}\cdots d\theta_{fI_f-1} \tag{8}$$

of equation (8) in order to estimate the model's parameters. Three possibilities are now open. They are discussed in detail in Angers et al. (2004). Here we summarize the main results.

1. The first possibility, which greatly simplifies the calculations, is to suppose that all the γ_{fi}^j of the I_f vehicles are identical.

This first scenario supposes implicitly that all the vehicles in the fleet represent identical *a priori* risks, which is probably a very strong hypothesis since, as we shall see, several variables distinguishing the vehicles and the behaviors of drivers are significant in estimating the probabilities of accidents. Another possibility is to divide the vehicles into different risk groups, as is done by insurers when classifying risks.

2. Under the second possibility, we can separate the vehicles into two groups, for example, and define $G_1 = 1,\cdots,g$ as all the vehicles in the first group with $\gamma_{fg1}^j = \frac{\sum_{i=1}^{g}\gamma_{fi}^j}{g}$ and $G_2 = g+1,\cdots,I_f$ as all the vehicles in the second group with $\gamma_{fg2}^j = \frac{\sum_{i=g+1}^{I_f}\gamma_{fi}^j}{I_f - g}$. The integral of equation (8) can thus be approximated by:

$$\int_0^1 \cdots \int_0^1 \frac{\left[\prod_{i=1}^{g} (\theta_{fi})^{c_i-1} \prod_{i=g+1}^{I_f} (\theta_{fi})^{c_i-1}\right]}{\left(\kappa_f^{-1} + \gamma_{fg1}^{j} \sum_{i=1}^{g} \theta_{fi} + \gamma_{fg2}^{j} \sum_{i=g+1}^{I_f} \theta_{fi}\right)^{d}} d\theta_{f1} \cdots d\theta_{fI_f-1} \tag{9}$$

with

$$c_i = y_{fi}^j + \nu_i \quad \text{and} \quad d = I_f \kappa_f^{-1} + \sum_{i=1}^{I_f} y_{fi}^j .$$

Taking that

$$u_i = \frac{\theta_{fi}}{\sum_{i=1}^{g} \theta_{fi}} \quad i = 1, \cdots, g-1 \,;\; v = \sum_{i=1}^{g} \theta_{fi} \text{ and } w_i = \frac{\theta_{fi}}{1 - \sum_{i=1}^{g} \theta_{fi}} \quad i = g+1, \cdots, I_f ,$$

we can rewrite (9) and substitute the new expression in equation (7) to obtain an approximation for the distribution of the number of accidents at period *j* of the vehicles in fleet *f*:

$$P\left(y_{f1}^j, \cdots, y_{fI_f}^j\right) \approx$$

$$\frac{\Gamma\left(\sum_{i=1}^{I_f} y_{fi}^j + I_f \kappa_f^{-1}\right)}{\Gamma\left(I_f \kappa_f^{-1}\right)} \left(\kappa_f^{-1}\right)^{I_f \kappa_f^{-1}} \left(\frac{1}{\kappa_f^{-1} + \gamma_{fg2}^j}\right)^{I_f \kappa_f^{-1} + \sum_{i=g+1}^{I_f} y_{fi}^j} \prod_{i=1}^{I_f} \left(\frac{\Gamma\left(y_{fi}^j + \nu_i\right)\left(\gamma_{fi}^j\right)^{y_{fi}^j}}{\Gamma\left(y_{fi}^j + 1\right)\Gamma\left(\nu_i\right)}\right) \times \frac{\Gamma\left(\sum_{i=1}^{I_f} \nu_i\right)}{\Gamma\left(\sum_{i=1}^{I_f} \left(y_{fi}^j + \nu_i\right)\right)} \tag{10}$$

$$\times {}_2F_1\left(\sum_{i=1}^{g} \left(y_{fi}^j + \nu_i\right), \left(I_f \kappa_f^{-1} + \sum_{i=1}^{I_f} y_{fi}^j\right), \sum_{i=1}^{I_f} \left(y_{fi}^j + \nu_i\right), \left(\frac{\gamma_{fg2}^j - \gamma_{fg1}^j}{\kappa_f^{-1} + \gamma_{fg2}^j}\right)\right),$$

where ${}_2F_1$ is a hypergeometric function as defined in section 2.1.2. This procedure in estimating the integral can be generalized to several homogeneous groups, but it is not obvious that the precision gained would be greater than that corresponding to a Monte Carlo approximation of the multivariate integral of equation (8).

3. We can estimate the integral in (8) by the Monte Carlo method (see Angers et al., 2004, for details). This estimation could also be used to verify the precision of the hypergeometric approximation.

2.2 ECONOMETRIC ESTIMATIONS

2.2.1 Descriptive statistics

The data come from the files of the *Société d'assurance automobiles du Québec* (henceforth referred to as the SAAQ), dating from 1997 to 1998 (for a detailed description of the data base see Dionne, Desjardins, and Pinquet, 1999, 2001). We had access to data on the two years from 43,679 carriers of merchandise by truck. More than two thirds of the carriers have only one vehicle. At 31 December 1997 and 31 December 1998, these small carriers owned about 30% of the 103,848 heavy trucks with authorization to circulate at least one day, so the econometric estimation was made with 73,328 trucks from 13,159 carriers. We use the 1998 data for information on accidents and characteristics of vehicles and fleets and the 1997 data for traffic violations, so as to respect the SAAQ's rating policy. Moreover, this approach reduces the problem of simultaneity between the "violations" and "accidents" variables.

It should be mentioned that a vehicle is not necessarily authorized to circulate 365 days in 1998. On average, a vehicle is authorized to circulate 88.5% of 1998. Depending on the size of the fleet, this percentage will vary between 86.7 and 93.9%. To obtain an annual statistic, we calculated the number of trucks in trucks-year, by adding the number of days each truck was authorized to circulate and then dividing by 365 days. The average frequency of total accidents per truck-year is

0.1592. This average increases as the size of the fleet increases, but decreases when the fleet contains more than 150 trucks.

2.2.2. Estimation of parameters

We used the maximum likelihood method to estimate the unknown parameters, $\kappa_f^{-1}, (\nu_1, \nu_2, \cdots, \nu_{I_f}), \beta = (\beta_1 \ , \cdots, \ \beta_p)$. We used SAS/IML software to apply the optimization algorithm. The results for all size of fleets with 2 vehicles and more are presented in Table 1 in the Appendix. For this estimation, the vehicles of fleets with more than two trucks were divided into two risk groups, according to the average number of accidents per truck predicted by the negative binomial distribution model. For fleets with two vehicles, we used the exact model of Section 2.1.2. The variance-covariance matrix was estimated based on the SAS/NLFPDD subroutine. We used the 10% threshold (*p* lower than or equal to 0.10) to consider a statistical coefficient different from zero.

We note in Table 1 that the vehicles with more experience (number of years as carrier) have fewer accidents. The results also indicate that the factors explaining accidents include: the carrier's size and sector of activity; the type of use to which the vehicle was put; the type of fuel; the number of cylinders; and the number of axles. Vehicles with fleet violations (violations of trucking standards) in 1997 are more at risk for accidents in 1998 than those without these types of offenses. Moreover, vehicles whose drivers have accumulated demerit points for violations in 1997 represent higher risks for accidents in 1998 than those without such points.

Table 1 also reports the results on the parameters for random effects distributions. Regression indicates that the κ_f^{-1} parameters of the negative binomial are significant, which means that we can reject the Poisson distribution and apply a bonus-malus insurance rating model to these fleets. It is important to mention that we estimated seven κ parameters because these parameters are affected by fleet size. The ν parameter is also significant at the 99% threshold. It is not affected by the fleet size. These results signal that both the vehicle and fleet effects can be used in calculating premiums.

In conclusion, the β coefficients will be very useful in estimating *a priori* risks when calculating insurance premiums, whereas coefficients κ_f^{-1} and ν will be useful in adjusting premiums to fit the past accident records of vehicles and fleets in the bonus-malus model.

3. BONUS-MALUS

3.1 OPTIMAL BONUS-MALUS SYSTEM

To construct an optimal bonus-malus system (Lemaire, 1985; Dionne and Vanasse, 1989, 1992) based on the number of past accidents recorded for a truck as well as those observed for its fleet, we must calculate the premium for a truck of a given fleet at period *t+1* using the following mathematical expectation relation:

$$\gamma_{fi}^{t+1}\left(\frac{E\left(\theta_{fi}\alpha_f \mid y_f, X_f\right)}{E\left(\theta_{fi}\alpha_f\right)}\right).$$

The term γ_{fi}^{t+1} corresponds to the part of the mathematical expectation obtained from the econometric regressions. It is equal to $d_{fi}^{t+1}e^{X_{fi}^{t+1}\beta}$ where d_{fi}^{t+1} is the number of days that vehicle *i* of fleet *f* is authorized to circulate in period *t+1* divided by the total number of days in period *t+1*. As already indicated, this variable measures exposure to risk. The regression component corresponds to $X_{fi}^{t+1}\beta$ where the vector of coefficients (β) is estimated by means of econometric models and $X_{fi}^{t+1} = \left(x_{fi1}^{t+1},\cdots,x_{fip}^{t+1}\right)$ represents the observable *p* characteristics of truck *i* in fleet *f* at the beginning of period *t*+1. $X_f = \left(X_{f1}^1,\cdots,X_{fI_f}^1,\cdots,X_{f1}^{t+1},\cdots,X_{fI_f}^{t+1}\right)$ gives the *p* characteristics of all the trucks in fleet *f* up to the *t+1* period. The vector $y_f = \left(y_{f1}^1,\cdots,y_{fI_f}^1,\cdots,y_{f1}^t,\cdots,y_{fI_f}^t\right)$ represents the accidents of vehicles in fleet *f* up to period *t* and $E\left(\theta_{fi}\alpha_f \mid y_f, X_f\right)$ designates the mathematical expectation of the fleet and vehicle effects attributable to vehicle *i*, based on past experience as measured by accidents accumulated over the preceding *t* periods. As we shall see, the modeling proposed will take into account both the accidents of vehicle *i* and those of its fleet *f*. These effects account for the unobservable factors which can affect the accidents of trucks and fleets: α_f is the effect associated with fleet *f* and θ_{fi} is the weight truck *i* in fleet *f* actually exerts on this fleet effect. Finally, $E\left(\theta_{fi}\alpha_f\right)$ gives the mathematical expectation of the two effects attributable to truck *i* not conditional on accidents. The last term is used to normalize the bonus-malus factor at 1 when the insurer has no experience with a particular vehicle.

The preceding equation comes from a Bayesian analysis of the evolution of accidents over time. We are now going to show its explicit form under the hypotheses of statistical distribution for the two random effects. We know that the true mathematical expectation of the number of accidents for truck *i* of fleet *f* at period *t+1* is equal to $\lambda_{fi}^{t+1}\left(X_f,\alpha_f,\theta_{fi}\right)$. It is a function of the vector for the observable characteristics of the vehicle up to period *j* and of the random factors for fleet α_f and vehicle θ_{fi}, which are supposed to be independent of time.

Given the observations obtained up to period *t*+1, the optimal estimator of this mathematical expectation at period *t*+1, $\hat{\lambda}_{fi}^{t+1}\left(y_f,X_f\right)$ can be calculated as follows:

$$\hat{\lambda}_{fi}^{t+1}\left(y_f,X_f\right)=\gamma_{fi}^{t+1}\left(\frac{E\left(\alpha_f\theta_{fi}\mid y_f,X_f\right)}{E\left(\alpha_f\right)E\left(\theta_{fi}\right)}\right)=\gamma_{fi}^{t+1}\left(\frac{E\left(\theta_{fi}E\left(\alpha_f\mid\theta_{f1},\cdots,\theta_{fI_f},y_f,X_f\right)\mid y_f,X_f\right)}{E\left(\alpha_f\right)E\left(\theta_{fi}\right)}\right).$$

We know that:

$$E\left(\theta_{fi}E\left(\alpha_f\mid\theta_{f1},\cdots,\theta_{fI_f},y_f,X_f\right)\mid y_f,X_f\right)=\int\cdots\int_{\sum_{i=1}^{I_f}\theta_{fi}=1}\theta_{fi}E\left(\alpha_f\mid\theta_{f1},\cdots,\theta_{fI_f},y_f,X_f\right)f\left(\theta_{f1},\cdots,\theta_{fI_f}\mid y_f,X_f\right)d\theta_{f1}\cdots d\theta_{fI_f}$$

with:

$$f\left(\theta_{f1},\cdots,\theta_{fI_f}\mid y_f,X_f\right)=\frac{P\left(y_f\mid\theta_{f1},\cdots,\theta_{fI_f},X_f\right)f\left(\theta_{f1}\cdots\theta_{fI_f}\right)}{\int\cdots\int_{\sum_{i=1}^{I_f}\theta_{fi}=1}P\left(y_f\mid\theta_{f1},\cdots,\theta_{fI_f},X_f\right)f\left(\theta_{f1}\cdots\theta_{fI_f}\right)d\theta_{f1}\cdots d\theta_{fI_f}}.$$

Similarly, we can calculate:

$$E\left(\alpha_f\mid\theta_{f1},\cdots,\theta_{fI_f},y_f,X_f\right)=\int_0^{\infty}\alpha_f f\left(\alpha_f\mid y_f,X_f,\theta_{f1},\cdots,\theta_{fI_f}\right)d\alpha_f$$

with:

$$f\left(\alpha_f\mid\theta_{f1},\cdots,\theta_{fI_f},y_f,X_f\right)=\frac{P\left(y_f\mid\alpha_f,\theta_{f1},\cdots,\theta_{fI_f},X_f\right)f\left(\alpha_f\right)}{\int_0^{\infty}P\left(y_f\mid\alpha_f,\theta_{f1},\cdots,\theta_{fI_f},X_f\right)f\left(\alpha_f\right)d\alpha_f}.$$

Now let’s see how we can apply this Bayesian rating formula to carriers of different sizes.

3.1.1 Size-1 carrier

In this situation, the conditional accident probability for the fleet is given by:

$$P\left(y_{f1}^{1},\cdots,y_{f1}^{t} \mid \alpha_f, X_f\right)=\prod_{j=1}^{t}\left[\frac{\left(\gamma_{f1}^{j}\alpha_f\right)^{y_{f1}^{j}} e^{-\gamma_{f1}^{j}\alpha_f}}{y_{f1}^{j}\,!}\right]=\left[\prod_{j=1}^{j}\frac{\left(\gamma_{f1}^{j}\right)^{y_{f1}^{j}}}{y_{f1}^{j}\,!}\right]\left[\left(\alpha_f\right)^{\sum_{j=1}^{t} y_{f1}^{j}}\right]\left[e^{-\left(\alpha_f \sum_{j=1}^{t}\gamma_{f1}^{j}\right)}\right]. \qquad (11)$$

Given past accidents observed up to period *t*, the mathematical expectation estimator of the number of accidents for the truck in fleet *f* at period *t+1* is equal to:

$$\gamma_{f1}^{t+1}\left(\frac{E\left(\alpha_f \mid y_{f1}^{1},\cdots,y_{f1}^{t},\cdots,X_{f1}^{1},\cdots,X_{f1}^{t+1}\right)}{E\left(\alpha_f\right)}\right)=\gamma_{f1}^{t+1}\left[\frac{\kappa_f^{-1}+\sum_{j=1}^{t} y_{f1}^{j}}{\kappa_f^{-1}+\sum_{j=1}^{t}\gamma_{f1}^{j}}\right]. \qquad (12)$$

Equation (12) is the formula used in the literature (Lemaire, 1985; Dionne and Vanasse, 1989, 1992) for individual vehicles and does not have to account for the fleet effect since the fleet *is* the vehicle.

3.1.2. Carrier with 2 vehicles

In this situation, the conditional accident probability for the fleet is given by:

$$P\left(y_f \mid \theta_f, \alpha_f, X_f\right) = \left[\prod_{j=1}^{t}\prod_{i=1}^{I_f}\frac{\left(\gamma_{fi}^j\right)^{y_{fi}^j}}{\Gamma\left(y_{fi}^j\right)}\right]\left[\left(\theta_f\right)^{\sum_{j=1}^{t} y_{f1}^j}\left(1-\theta_f\right)^{\sum_{j=1}^{t} y_{f2}^j}\right]\left[\left(\alpha_f\right)^{\sum_{j=1}^{t}\sum_{i=1}^{I_f} y_{fi}^j}\right]\left[e^{-\left(\alpha_f\left(\theta_f\sum_{j=1}^{t}\gamma_{f1}^j+\left(1-\theta_f\right)\sum_{j=1}^{t}\gamma_{f2}^j\right)\right)}\right]. \tag{13}$$

We know that, given the past accidents observed up to period *t* and due to the values assigned to the random effects of the 2 trucks in fleet *f*, the *a posteriori* density function for α_f corresponds to a gamma density with parameters:

$$\left(\sum_{j=1}^{t}\sum_{i=1}^{2} y_{fi}^j + 2\kappa_f^{-1},\ \kappa_f^{-1} + \theta_f\sum_{j=1}^{t}\gamma_{f1}^j + \left(1-\theta_f\right)\sum_{j=1}^{t}\gamma_{f2}^j\right).$$

So:

$$f\left(\alpha_f \mid y_f, X_f, \theta_f\right) = \frac{\left(\kappa^{-1} + \theta_f\sum_{j=1}^{t}\gamma_{f1}^j + \left(1-\theta_f\right)\sum_{j=1}^{t}\gamma_{f2}^j\right)^{\sum_{j=1}^{t}\sum_{i=1}^{2} y_{fi}^j + 2\kappa^{-1}}}{\Gamma\left(2\kappa^{-1} + \sum_{j=1}^{t}\sum_{i=1}^{2} y_{fi}^j\right)}\left(\alpha_f\right)^{\sum_{j=1}^{t}\sum_{i=1}^{2} y_{fi}^j + 2\kappa^{-1} - 1}\left[e^{-\alpha_f\left(\kappa^{-1}+\theta_f\sum_{j=1}^{t}\gamma_{f1}^j+\left(1-\theta_f\right)\sum_{j=1}^{t}\gamma_{f2}^j\right)}\right].$$

Given the past accidents observed up to period *t* and due to the values assigned to the random effects of the 2 trucks in fleet *f*, the mathematical expectation of α_f is equal to:

$$E\left(\alpha_f \mid y_f, X_f, \theta_f\right) = \frac{2\kappa_f^{-1} + \sum_{j=1}^{t}\sum_{i=1}^{2} y_{fi}^j}{\kappa_f^{-1} + \theta_f\sum_{j=1}^{t}\gamma_{f1}^j + \left(1-\theta_f\right)\sum_{j=1}^{t}\gamma_{f2}^j}. \tag{14}$$

Given the past accidents observed up to period *t* for the two trucks of fleet *f*, the density function of θ_f is equal to:

$$f\left(\theta_f \mid y_f, X_f\right) = D \times \frac{\left[\left(\theta_f\right)^{\nu_1 - 1 + \sum_{j=1}^{t} y_{f1}^j} \left(1-\theta_f\right)^{\nu_2 - 1 + \sum_{j=1}^{t} y_{f2}^j}\right]}{\left(\kappa_f^{-1} + \theta_f \sum_{j=1}^{t} \gamma_{f1}^j + \left(1-\theta_f\right) \sum_{j=1}^{t} \gamma_{f2}^j\right)^{2\kappa_f^{-1} + \sum_{j=1}^{t}\sum_{i=1}^{2} y_{fi}^j}} \tag{15}$$

where:

$$D^{-1} = \frac{\prod_{i=1}^{2} \Gamma\left(\nu_i + \sum_{j=1}^{t} y_{fi}^j\right)}{\Gamma\left(\sum_{i=1}^{2}\left(\nu_i + \sum_{j=1}^{t} y_{fi}^j\right)\right)\left(\kappa_f^{-1} + \sum_{j=1}^{t} \gamma_{f2}^j\right)^{2\kappa_f^{-1} + \sum_{j=1}^{t}\sum_{i=1}^{2} y_{fi}^j}} \times {}_2F_1\left(\nu_1 + \sum_{j=1}^{t} y_{f1}^j; 2\kappa_f^{-1} + \sum_{j=1}^{t}\sum_{i=1}^{2} y_{fi}^j; \sum_{i=1}^{2}\left(\nu_i + \sum_{j=1}^{t} y_{fi}^j\right); \frac{\sum_{j=1}^{t}\gamma_{f2}^j - \sum_{j=1}^{t}\gamma_{f1}^j}{\kappa_f^{-1} + \sum_{j=1}^{t}\gamma_{f2}^j}\right).$$

Given the past accidents observed up to period *t* for the two trucks of fleet *f*, the mathematical expectation estimator of the number of accidents for truck *i* in fleet *f* at period *t+1* is thus equal to $\theta_{fi} = \theta_f$ if $i = 1$ and $1-\theta_f$ if $i = 2$:

$$\gamma_{fi}^{t+1} E\left(\alpha_f \theta_{fi} \mid y_f, X_f\right) = \gamma_{fi}^{t+1}\left(2\kappa_f^{-1} + \sum_{j=1}^{t}\sum_{i=1}^{2} y_{fi}^j\right) E\left(\frac{\theta_{fi}}{\kappa_f^{-1} + \theta_f \sum_{j=1}^{t}\gamma_{f1}^j + \left(1-\theta_f\right)\sum_{j=1}^{t}\gamma_{f2}^j} \mid y_f, X_f\right).$$

It remains to calculate the expression:

$$E\left(\frac{\theta_{fi}}{\kappa_f^{-1} + \theta_f \sum_{j=1}^{t}\gamma_{f1}^j + \left(1-\theta_f\right)\sum_{j=1}^{t}\gamma_{f2}^j} \mid y_f, X_f\right).$$

By definition,

$$E\left(\frac{\theta_{fi}}{\kappa_f^{-1}+\theta_f\sum_{j=1}^{t}\gamma_{f1}^{j}+\left(1-\theta_f\right)\sum_{j=1}^{t}\gamma_{f2}^{j}}\mid y_f,X_f\right)=\int\frac{\theta_{fi}}{\kappa_f^{-1}+\theta_f\sum_{j=1}^{t}\gamma_{f1}^{j}+\left(1-\theta_f\right)\sum_{j=1}^{t}\gamma_{f2}^{j}}f\left(\theta_f\mid y_f,X_f\right)d\theta_f\cdot$$

By replacing $f\left(\theta_f \mid y_f, X_f\right)$ by its value given in (15), we obtain that:

$$E\left(\frac{\theta_{fi}}{\kappa_f^{-1}+\theta_f\sum_{j=1}^{t}\gamma_{f1}^{j}+\left(1-\theta_f\right)\sum_{j=1}^{t}\gamma_{f2}^{j}}\mid y_f,X_f\right)=D\int\frac{\theta_{fi}\left[\left(\theta_f\right)^{\nu_1-1+\sum_{j=1}^{t}y_{f1}^{j}}\left(1-\theta_f\right)^{\nu_2-1+\sum_{j=1}^{t}y_{f2}^{j}}\right]}{\left(\kappa_f^{-1}+\theta_f\sum_{j=1}^{t}\gamma_{f1}^{j}+\left(1-\theta_f\right)\sum_{j=1}^{t}\gamma_{f2}^{j}\right)^{2\kappa_f^{-1}+\sum_{j=1}^{t}\sum_{i=1}^{2}y_{fi}^{j}+1}}d\theta_f\cdot$$

Calculating the integral, we obtain:

$$E\left(\frac{\theta_{fi}}{\kappa_f^{-1}+\theta_f\sum_{j=1}^{t}\gamma_{f1}^{j}+\left(1-\theta_f\right)\sum_{j=1}^{t}\gamma_{f2}^{j}}\mid y_f,X_f\right)$$

$$=\left[\frac{\left(\nu_i+\sum_{j=1}^{t}y_{fi}^{j}\right)}{\kappa_f^{-1}+\sum_{j=1}^{t}\gamma_{f2}^{j}}\right]\left[\frac{{}_2F_1\left(I+\nu_1+\sum_{j=1}^{t}y_{f1}^{j};1+2\kappa_f^{-1}+\sum_{j=1}^{t}\sum_{i=1}^{2}y_{fi}^{j};1+\sum_{i=1}^{2}\left(\nu_i+\sum_{j=1}^{t}y_{fi}^{j}\right);\frac{\sum_{j=1}^{t}\gamma_{f2}^{j}-\sum_{j=1}^{t}\gamma_{f1}^{j}}{\kappa_f^{-1}+\sum_{j=1}^{t}\gamma_{f2}^{j}}\right)}{\sum_{i=1}^{2}\left(\nu_i+\sum_{j=1}^{t}y_{fi}^{j}\right)\times{}_2F_1\left(\nu_1+\sum_{j=1}^{t}y_{f1}^{j};2\kappa_f^{-1}+\sum_{j=1}^{t}\sum_{i=1}^{2}y_{fi}^{j};\sum_{i=1}^{2}\left(\nu_i+\sum_{j=1}^{t}y_{fi}^{j}\right);\frac{\sum_{j=1}^{t}\gamma_{f2}^{j}-\sum_{j=1}^{t}\gamma_{f1}^{j}}{\kappa_f^{-1}+\sum_{j=1}^{t}\gamma_{f2}^{j}}\right)}\right] \quad (16)$$

with $\theta_{fi}=\theta_f$ if $i=1$ and $1-\theta_f$ if $i=2$ and the indicative function $I = 1$ if $i = 1$ and $I = 0$ if $i = 2$.

Thus, the optimal estimator of vehicle *i* is equal to:

$$\gamma_{fi}^{t+1}\left(\frac{E\left(\theta_{fi}\alpha_f \mid y_f, X_f\right)}{E\left(\alpha_f\right)E\left(\theta_{fi}\right)}\right)=\gamma_{fi}^{t+1}\left(\sum_{j=1}^{t}\sum_{i=1}^{2}y_{fi}^{j}+2\kappa_f^{-1}\right)\frac{1}{2}\times\frac{\nu_1+\nu_2}{\nu_i}E\left(\frac{\theta_{fi}}{\kappa_f^{-1}+\theta_f\sum_{j=1}^{t}\gamma_{fi}^{j}+\left(1-\theta_f\right)\sum_{j=1}^{t}\gamma_{f2}^{j}}\mid y_f, X_f\right)$$

$$=\gamma_{fi}^{t+1}\times\frac{1}{2}\times\frac{\nu_1+\nu_2}{\nu_i}\left[\frac{\left(\sum_{j=1}^{t}y_{fi}^{j}+\nu_i\right)}{\sum_{j=1}^{t}\gamma_{f2}^{j}+\kappa_f^{-1}}\right]\left[\frac{\sum_{j=1}^{t}\sum_{i=1}^{2}y_{fi}^{j}+2\kappa_f^{-1}}{\sum_{i=1}^{2}\left(\sum_{j=1}^{t}y_{fi}^{j}+\nu_i\right)}\right]\left[\frac{{}_2F_1\left(\sum_{j=1}^{t}y_{f1}^{j}+\nu_1+1;\sum_{j=1}^{t}\sum_{i=1}^{2}y_{fi}^{j}+2\kappa_f^{-1}+1;\sum_{i=1}^{2}\left(\nu_i+\sum_{j=1}^{t}y_{fi}^{j}\right)+1;\frac{\sum_{j=1}^{t}\gamma_{f2}^{j}-\sum_{j=1}^{t}\gamma_{f1}^{j}}{\kappa_f^{-1}+\sum_{j=1}^{t}\gamma_{f2}^{j}}\right)}{{}_2F_1\left(\sum_{j=1}^{t}y_{f1}^{j}+\nu_1;\sum_{j=1}^{t}\sum_{i=1}^{2}y_{fi}^{j}+2\kappa_f^{-1};\sum_{i=1}^{2}\left(\nu_i+\sum_{j=1}^{t}y_{fi}^{j}\right);\frac{\sum_{j=1}^{t}\gamma_{f2}^{j}-\sum_{j=1}^{t}\gamma_{f1}^{j}}{\kappa_f^{-1}+\sum_{j=1}^{t}\gamma_{f2}^{j}}\right)}\right].$$

We note that for each vehicle *i*, the optimal estimator for accidents at period *t+1* is a function of the following factors: the parameters observable when the insurance policy is being renewed at period *t+1*; the accidents accumulated by vehicle *i* over the preceding *t* periods; the total accidents of the fleet over the same periods; the observable characteristics of the two vehicles over the preceding *t* periods; and the gamma and Dirichlet parameters. We shall apply this formula to our data in section 4. But let's now see how it is possible to generalize this insurance rating formula to a fleet of I_f vehicles.

3.1.3. Carrier with more than 2 vehicles

This section is divided into three subsections corresponding to the three approximation hypotheses for the multiple integral discussed in section 2.1.3.

- All the γ_{fi}^{j} for the I_f vehicles are identical

In this situation, the conditional accident probability for the fleet is given by:

$$P\left(y_f \mid \theta_{f1,}\cdots,\theta_{fI_f-1},\alpha_f,X_f\right)=\prod_{j=1}^{t}\prod_{i=1}^{I_f}\left[\frac{\left(\gamma_f^j\theta_{fi}\alpha_f\right)^{y_{fi}^j}e^{-\gamma_f^j\theta_{fi}\alpha_f}}{y_{fi}^j!}\right]$$
$$=\left[\prod_{j=1}^{t}\prod_{i=1}^{I_f}\frac{\left(\gamma_f^j\right)^{y_{fi}^j}}{y_{fi}^j!}\right]\left[\prod_{i=1}^{I_f}\left(\theta_{fi}\right)^{\sum_{j=1}^{t}y_{fi}^j}\right]\left[\left(\alpha_f\right)^{\sum_{j=1}^{t}\sum_{i=1}^{I_f}y_{fi}^j}\right]\left[e^{-\left(\alpha_f\sum_{j=1}^{t}\gamma_f^j\right)}\right]. \tag{17}$$

The optimal estimator $\hat{\lambda}_{fi}^{t+1}$ is thus equal to:

$$\frac{\gamma_{fi}^{t+1}E\left(\theta_{fi}\alpha_f \mid y_f,X_f\right)}{E\left(\theta_{fi}\alpha_f\right)}=\gamma_{fi}^{t+1}\left(\frac{\sum_{j=1}^{t}y_{fi}^j+\nu_i}{\sum_{j=1}^{t}\gamma_{f1}^j+\kappa_f^{-1}}\right)\left(\frac{\sum_{j=1}^{t}\sum_{i=1}^{I_f}y_{fi}^j+I_f\kappa_f^{-1}}{\sum_{j=1}^{t}\sum_{i=1}^{I_f}y_{fi}^j+\sum_{i=1}^{I_f}\nu_i}\right)\frac{\sum_{i=1}^{I_f}\nu_i}{I_f\nu_i}. \tag{18}$$

This formula compares rather well with the one presented in equation (12) for a carrier with a single vehicle. Here, as all the vehicles are identical in terms of the observable variables, differentiation of the two formulas will be principally the work of the experience variables. On the one hand, all the accidents of the fleet come into play and, on the other hand, the weight of past accidents takes into account the parameters of the Dirichlet distribution, on an individual basis ν_i for each vehicle and on an aggregated basis $\sum_{i=1}^{I_f}\nu_i$ for all the vehicles.

- Divide the vehicles into 2 groups

If we now have different vehicles, we can form groups with homogeneous characteristics or risks to obtain an explicit formula. In fact, insurers form more or less homogeneous risk classes by using different classification variables such as the type of car, the territory… Past experience serves to pinpoint the differences which are not observable *a priori*. If we limit ourselves to two groups, the conditional accident probability for the fleet is given by:

$$P\left(y_f \mid \theta_{f1},\cdots,\theta_{fI_f-1},\alpha_f,X_f\right)=\prod_{j=1}^{t}\prod_{i=1}^{I_f}\left[\frac{\left(\gamma_{fi}^{j}\theta_{fi}\alpha_f\right)^{y_{fi}^{j}}e^{-\gamma_{fi}^{j}\theta_{fi}\alpha_f}}{y_{fi}^{j}!}\right]$$

$$=\prod_{j}\left(\prod_{i}\frac{\gamma_{fg}^{j}}{y_{fi}^{j}!}\right)\left[\prod_{i=1}^{I_f}\left(\theta_{fi}\right)^{\sum_{j=1}^{t}y_{fi}^{j}}\right]\left[\left(\alpha_f\right)^{\sum_{j=1}^{t}\sum_{i=1}^{I_f}y_{fi}^{j}}\right]\left[e^{-\left(\alpha_f\left(\sum_j\gamma_{fg1}^{j}\sum_{i=1}^{g}\theta_{fi}+\sum_j\gamma_{fg2}^{j}\sum_{i=g+1}^{I_f}\theta_{fi}\right)\right)}\right]$$

with

$$\gamma_{fg1}^{j}=\left(\frac{\sum_{i=1}^{g}\gamma_{fi}^{j}}{g}\right)\quad \text{et}\quad \gamma_{fg2}^{j}=\left(\frac{\sum_{i=g+1}^{I_f}\gamma_{fi}^{j}}{I_f-g}\right)$$

for the two groups respectively.

The optimal estimator $\hat{\lambda}_{fi}^{t+1}$ is thus equal to:

$$\gamma_{fi}^{t+1}\left(\frac{E\left(\theta_{fi}\alpha_f \mid y_f, X_f\right)}{E\left(\alpha_f\right)E\left(\theta_{fi}\right)}\right)=\gamma_{fi}^{t+1}\left(\sum_{i=1}^{I_f}\sum_{j=1}^{t} y_{fi}^{j}+I_f\kappa_f^{-1}\right)\frac{1}{I_f}\times\frac{\sum_{i=-1}^{I_f}\nu_i}{\nu_i}E\left(\frac{\theta_{fi}}{\kappa_f^{-1}+\gamma_{fg1}^{j}\sum_{m=1}^{g}\theta_{fm}+\gamma_{fg2}^{j}\sum_{m=g+1}^{I_f}\theta_{fm}}\mid y_f, X_f\right)$$

$$=\gamma_{fi}^{t+1}\times\frac{1}{I_f}\times\frac{\sum_{i=-1}^{I_f}\nu_i}{\nu_i}\left[\frac{\left(\sum_{j=1}^{t}y_{fi}^{j}+\nu_i\right)}{\sum_{j=1}^{t}\gamma_{fg2}^{j}+\kappa_f^{-1}}\right]\left[\frac{\sum_{i=1}^{I_f}\sum_{j=1}^{t}y_{fi}^{j}+I_f\kappa_f^{-1}}{\sum_{i=1}^{I_f}\left(\sum_{j=1}^{t}y_{fi}^{j}+\nu_i\right)}\right]\left[\frac{{}_2F_1\left(\sum_{i=1}^{g}\left(\sum_{j=1}^{t}y_{fi}^{j}+\nu_i\right)+1;\sum_{i=1}^{I_f}\sum_{j=1}^{t}y_{fi}^{j}+I_f\kappa_f^{-1}+1;\sum_{i=1}^{I_f}\left(\sum_{j=1}^{t}y_{fi}^{j}+\nu_i\right)+1;\frac{\sum_{j=1}^{t}\gamma_{fg2}^{j}-\sum_{j=1}^{t}\gamma_{fg1}^{j}}{\kappa_f^{-1}+\sum_{j=1}^{t}\gamma_{fg2}^{j}}\right)}{{}_2F_1\left(\sum_{i=1}^{g}\left(\sum_{j=1}^{t}y_{fi}^{j}+\nu_i\right);\sum_{i=1}^{I_f}\sum_{j=1}^{t}y_{fi}^{j}+I_f\kappa_f^{-1};\sum_{i=1}^{I_f}\left(\sum_{j=1}^{t}y_{fi}^{j}+\nu_i\right);\frac{\sum_{j=1}^{t}\gamma_{fg2}^{j}-\sum_{j=1}^{t}\gamma_{fg1}^{j}}{\kappa_f^{-1}+\sum_{j=1}^{t}\gamma_{fg2}^{j}}\right)}\right]$$

(19)

where the indicative function:

$$I = \begin{cases} 1 & \text{if the truck belongs to group 1} \\ 0 & \text{if the truck belongs to group 2.} \end{cases}$$

This formula is very difficult to generalize to more than two groups. If the fleet has several more or less homogeneous groups of vehicles, it may be more advantageous to rely on a Monte Carlo simulation approach.

- Monte Carlo simulation approach

In the general case, the conditional accident probability for the fleet is given by:

$$P\left(y_f, \mid \theta_{f1}, \cdots, \theta_{fI_f-1}, \alpha_f, X_f\right)=\left[\prod_{j=1}^{t}\prod_{i=1}^{I_f}\left(\frac{\left(\gamma_{fi}^{j}\right)^{y_{fi}^{j}}}{\Gamma\left(y_{fi}^{j}+1\right)}\right)\right]\left[\prod_{i=1}^{I_f}\left(\theta_{fi}\right)^{\sum_{j=1}^{t}y_{fi}^{j}}\right]\left[\left(\alpha_f\right)^{\sum_{j=1}^{t}\sum_{i=1}^{I_f}y_{fi}^{j}}\right]\left[e^{-\left(\alpha_f\sum_{i=1}^{I_f}\theta_{fi}\sum_{j=1}^{t}\gamma_{fi}^{j}\right)}\right]. \quad (20)$$

We also obtain that:

$$E\left(\alpha_f \mid y_f, X_f, \theta_{f1}, \cdots, \theta_{fI_f-1}\right) = \frac{I_f \kappa_f^{-1} + \sum_{i=1}^{I_f} \sum_{j=1}^{t} y_{fi}^j}{\kappa_f^{-1} + \sum_{i=1}^{I_f} \theta_{fi} \sum_{j=1}^{t} \gamma_{fi}^j} \tag{21}$$

and

$$f\left(\theta_{f1}, \cdots, \theta_{fI_f-1} \mid y_f, X_f\right) = \frac{\left[\dfrac{\prod_{i=1}^{I_f}\left(\left(\kappa_f^{-1} + \sum_{j=1}^{t} \gamma_{fi}^j\right)\theta_{fi}\right)^{\sum_{j=1}^{t} y_{fi}^j + \nu_i - 1}}{\left(\sum_{i=1}^{I_f}\left(\kappa_f^{-1} + \sum_{j=1}^{t} \gamma_{fi}^j\right)\theta_{fi}\right)^{I_f \kappa_f^{-1} + \sum_{i=1}^{I_f}\sum_{j=1}^{t} y_{fi}^j}}\right]}{\int \underset{\sum_{i=1}^{I_f} \theta_{fi}=1}{\cdots} \int \left[\dfrac{\prod_{i=1}^{I_f}\left(\left(\kappa_f^{-1} + \sum_{j=1}^{t} \gamma_{fi}^j\right)\theta_{fi}\right)^{\sum_{j=1}^{t} y_{fi}^j + \nu_i - 1}}{\left(\sum_{i=1}^{I_f}\left(\kappa_f^{-1} + \sum_{j=1}^{t} \gamma_{fi}^j\right)\theta_{fi}\right)^{I_f \kappa_f^{-1} + \sum_{i=1}^{I_f}\sum_{j=1}^{t} y_{fi}^j}}\right] d\theta_{f1} \cdots d\theta_{fI_f-1}} \tag{22}$$

We can estimate the multiple integral:

$$\int_0^1 \cdots \int_0^1 \frac{\left[\prod_{i=1}^{I_f} \left(\theta_{fi}\right)^{\sum_{j=1}^{t} y_{fi}^j + \nu_i - 1}\right]}{\left(\sum_{i=1}^{I_f}\left(\kappa_f^{-1} + \sum_{j=1}^{t} \gamma_{fi}^j\right)\theta_{fi}\right)^{I_f \kappa_f^{-1} + \sum_{i=1}^{I_f}\sum_{j=1}^{t} y_{fi}^j}} d\theta_{f1} \cdots d\theta_{fI_f-1}$$

of equation (22) with the Monte Carlo method by using the importance function (weighting) (Lange, 1999) $h\left(\underset{\sim}{\theta}\right)$ where $\underset{\sim}{\theta} = \theta_{f1}, \cdots, \theta_{fI_f}$ such that:

$$\int \underset{\sum_{i=1}^{I_f} \theta_{fi}=1}{\cdots} \int g\left(\underset{\sim}{\theta}\right) d\underset{\sim}{\theta} = \int \underset{\sum_{i=1}^{I_f} \theta_{fi}=1}{\cdots} \int \frac{g\left(\underset{\sim}{\theta}\right)}{h\left(\underset{\sim}{\theta}\right)} h\left(\underset{\sim}{\theta}\right) d\underset{\sim}{\theta} = \int \underset{\sum_{i=1}^{I_f} \theta_{fi}=1}{\cdots} \int w\left(\underset{\sim}{\theta}\right) h\left(\underset{\sim}{\theta}\right) d\underset{\sim}{\theta} \approx \frac{1}{N} \sum_{\ell=1}^{N} w\left(\underset{\sim}{\theta}_\ell\right)$$

with

$$w\left(\underset{\sim}{\theta}\right) = \frac{g\left(\underset{\sim}{\theta}\right)}{h\left(\underset{\sim}{\theta}\right)}.$$

By taking:

$$h(\underset{\sim}{\theta}) = \frac{\Gamma\left(\sum_{i=1}^{I_f}\left(\sum_{j=1}^{t} y_{fi}^{j} + \nu_i\right)\right)}{\prod_{i=1}^{I_f}\Gamma\left(\sum_{j=1}^{t} y_{fi}^{j} + \nu_i\right)} \prod_{i=1}^{I_f}\left(\theta_{fi}\right)^{\sum_{j=1}^{t} y_{fi}^{j} + \nu_i - 1},$$

the optimal estimator $\hat{\lambda}_{fi}^{t+1}$ is approximately equal to:

$$\gamma_{fi}^{t+1}\left(\frac{E\left(\theta_{fi}\alpha_f \mid y_f, X_f\right)}{E\left(\theta_{fi}\alpha_f\right)}\right) \approx \gamma_{fi}^{t+1}\left(\frac{\left(I_f\kappa_f^{-1} + \sum_{i=1}^{I_f}\sum_{j=1}^{t} y_{fi}^{j}\right)}{I_f}\right)\left(\frac{\sum_{i=1}^{I_f}\nu_i}{\nu_i}\right)\left[\frac{\sum_{\ell=1}^{N}\left(\frac{\theta_{\ell fi}}{\left(\sum_{m=1}^{I_f}\left(\kappa_f^{-1} + \sum_{j=1}^{t}\gamma_{fm}^{j}\right)\theta_{\ell fm}\right)^{I_f\kappa_f^{-1} + \sum_{i=1}^{I_f}\sum_{j=1}^{t} y_{fi}^{j} + 1}}\right)}{\sum_{\ell=1}^{N}\left(\frac{1}{\left(\sum_{m=1}^{I_f}\left(\kappa_f^{-1} + \sum_{j=1}^{t}\gamma_{fm}^{j}\right)\theta_{\ell fm}\right)^{I_f\kappa_f^{-1} + \sum_{i=1}^{I_f}\sum_{j=1}^{t} y_{fi}^{j}}}\right)}\right] \quad (23)$$

with

$$\theta_{\ell fi} = \frac{a_{\ell fi}}{\sum_{i=1}^{I_f} a_{\ell fi}}$$

where the $a_{\ell fi}$ are values of a Gamma:

$$G\left(\sum_{j=1}^{t} y_{fi}^{j} + \nu_i, 1\right) \text{ for } i = 1, \ldots, I_f \text{ and } \ell = 1, \ldots, N.$$

4. APPLICATION OF THE BONUS-MALUS SYSTEM

In this section, we propose premium tables over several years, representing extensions of those proposed in the literature on automobile insurance for individual vehicles. Given that we did not model the conditional distribution for the cost of claims, we suppose that the average cost of claims is $10,000, seemingly a reasonable value for accidents involving trucks in North America (Dionne, Laberge-Nadeau et al., 1999).

4.1 FLEET OF 2 TRUCKS

Table 2 presents an example of premiums calculated for a truck belonging to a fleet of two trucks. The first line of the table (Fleet accidents) gives the sum of the accidents for the fleet over three years. The maximum indicated is 2 accidents but it could be higher. The second line (Truck accidents) gives the sum of accidents for the truck in question. For example, in the third column where the fleet accumulates two accidents, the truck concerned may have had 0, 1 or 2 accidents. Thus each corresponding scenario of premiums depends on the truck's and the fleet's own experience. If we use the result of Table 1 showing that $\nu_1 = \nu_2 = \nu$, a truck has a bonus-malus factor (BMF) equal to:

$$BMF=\left[\frac{\left(\hat{\nu}+\sum_{j=1}^{t} y_{fi}^{j}\right)}{\hat{\kappa}_f^{-1}+\sum_{j=1}^{t}\hat{\gamma}_{f2}^{j}}\right]\left[\frac{2\hat{\kappa}_f^{-1}+\sum_{i=1}^{2}\sum_{j=1}^{t} y_{fi}^{j}}{2\hat{\nu}+\sum_{i=1}^{2}\sum_{j=1}^{t} y_{fi}^{j}}\right]\left[\frac{{}_2F_1\left(I+\hat{\nu}+\sum_{j=1}^{t} y_{f1}^{j};1+2\hat{\kappa}_f^{-1}+\sum_{i=1}^{2}\sum_{j=1}^{t} y_{fi}^{j};1+2\hat{\nu}+\sum_{i=1}^{2}\sum_{j=1}^{t} y_{fi}^{j};\frac{\sum_{j=1}^{t}\hat{\gamma}_{f2}^{j}-\sum_{j=1}^{t}\hat{\gamma}_{f1}^{j}}{\hat{\kappa}_f^{-1}+\sum_{j=1}^{t}\hat{\gamma}_{f2}^{j}}\right)}{{}_2F_1\left(\hat{\nu}+\sum_{j=1}^{t} y_{f1}^{j};2\hat{\kappa}_f^{-1}+\sum_{i=1}^{2}\sum_{j=1}^{t} y_{fi}^{j};2\hat{\nu}+\sum_{i=1}^{2}\sum_{j=1}^{t} y_{fi}^{j};\frac{\sum_{j=1}^{t}\hat{\gamma}_{f2}^{j}-\sum_{j=1}^{t}\hat{\gamma}_{f1}^{j}}{\hat{\kappa}_f^{-1}+\sum_{j=1}^{t}\hat{\gamma}_{f2}^{j}}\right)}\right],$$

where the indicative function I is defined as before.

The estimated values of the parameters are equal to $\hat{\kappa}^{-1}=0.6404$ and $\hat{\nu}=2.2056$ (Table 1). Let's take the column "No accident" for the fleet and the truck. We note that the premium for the truck decreases over time. The following column gives the variations in the premiums if the fleet does have an accident and depending on whether or not the truck has an accident. We note that the premium for the truck increases in comparison to the first column even if the truck did not have an accident, for it is penalized by the fleet effect. But the increase is less than the one corresponding to the case where it did incur an accident.

Table 2: Table of insurance premium for vehicles belonging to a size-2 fleet

Fleet accidents		$\sum_{j=1}^{t}\sum_{i=1}^{2} y_{fi}^{j} = 0$		$\sum_{j=1}^{t}\sum_{i=1}^{2} y_{fi}^{j} = 1$				$\sum_{j=1}^{t}\sum_{i=1}^{2} y_{fi}^{j} = 2$					
Truck accidents		$\sum_{j=1}^{t} y_{fi}^{j} = 0$		$\sum_{j=1}^{t} y_{fi}^{j} = 0$		$\sum_{j=1}^{t} y_{fi}^{j} = 1$		$\sum_{j=1}^{t} y_{fi}^{j} = 0$		$\sum_{j=1}^{t} y_{fi}^{j} = 1$		$\sum_{j=1}^{t} y_{fi}^{j} = 2$	
t	γ_{fi}^{t+1}	BMF	γ_{fi}^{t+1} BMF × \$10,000	BMF	γ_{fi}^{t+1} BMF × \$10,000	BMF	γ_{fi}^{t+1} BMF × \$10,000	BMF	γ_{fi}^{t+1} BMF × \$10,000	BMF	γ_{fi}^{t+1} BMF × \$10,000	BMF	γ_{fi}^{t+1} BMF × \$10,000
0	0.114	1.000	\$1,143										
1	0.114	0.849	\$970	1.232	\$1,408	1.790	\$2,046	1.496	\$1,709	2.174	\$2,484	2.852	\$3,259
2	0.114	0.737	\$842	1.070	\$1,223	1.555	\$1,777	1.299	\$1,485	1.888	\$2,158	2.477	\$2,831
3	0.114	0.651	\$744	0.945	\$1,081	1.374	\$1,571	1.148	\$1,312	1.668	\$1,907	2.189	\$2,502
4	0.114	0.583	\$667	0.847	\$968	1.231	\$1,407	1.028	\$1,175	1.495	\$1,708	1.961	\$2,241
5	0.114	0.528	\$604	0.767	\$877	1.115	\$1,274	0.931	\$1,065	1.354	\$1,547	1.776	\$2,030
6	0.114	0.483	\$552	0.701	\$801	1.019	\$1,164	0.851	\$973	1.237	\$1,414	1.623	\$1,855
7	0.114	0.445	\$508	0.645	\$738	0.938	\$1,072	0.784	\$896	1.139	\$1,302	1.494	\$1,708
8	0.114	0.412	\$471	0.598	\$683	0.869	\$993	0.726	\$830	1.055	\$1,206	1.384	\$1,582
9	0.114	0.384	\$439	0.557	\$637	0.810	\$925	0.676	\$773	0.983	\$1,123	1.289	\$1,474

4.2 FLEET OF SEVERAL TRUCKS

4.2.1. All vehicles in fleet have the same risk characteristics

In this situation, the insurance premium estimated for truck *i* belonging to carrier *f* is given by:

$$\hat{\gamma}_{fi}^{t+1}\left[\frac{I_f\hat{\kappa}_f+\sum_{j=1}^{t}\sum_{i=1}^{I_f}y_{fi}^{j}}{I_f\hat{\nu}+\sum_{j=1}^{t}\sum_{i=1}^{I_f}y_{fi}^{j}}\right]\left[\frac{\hat{\nu}+\sum_{j=1}^{t}y_{fi}^{j}}{\hat{\kappa}_f^{-1}+\sum_{j=1}^{t}\hat{\gamma}_{fi}^{j}}\right]=\hat{\gamma}_{fi}^{t+1}\text{BMF}$$

which is (18) when $\sum_{i=1}^{I_f}\nu_i = I_f\hat{\nu}$.

Table 3 presents this example for a fleet of 10 identical trucks with $\hat{\nu}$ = 2.2056 and $\hat{\kappa}_f$ = 6.4867 (see Table 1). Suppose that the carrier accumulates 2 accidents over the next period, with 6 trucks incurring no accident nor speeding violation; 2 trucks incurring no accident but one speeding violation; 1 truck incurring an accident but no speeding violation; and 1 truck incurring an accident as well as a speeding violation. Still supposing that the average cost of claims is \$10,000, the *a priori* insurance premium for a vehicle when no account is taken of past experience is established at \$1,850 (0. 185 × 1 × \$10,000). Since all the vehicles of the fleet are identical in terms of observable risk, they all have the same $\gamma_{fi}^{t} = 0.185$ and a BMF equal to 1 at the start of the insurance contract. The total premium for the fleet is established at \$18,500 (10×\$1,850). In the following period (t+1), the insurance premiums for each of the records of the vehicles in the fleet are given in Table 3.

Table 3: Table of insurance premiums for vehicles belonging to a size-10 fleet when the fleet accumulates 2 accidents a year

γ_{fi}^{t}	*Accumulation of accidents*	*Speeding violation*	γ_{fi}^{t+1}	BMF	γ_{fi}^{t+1}BMF × \$10,000	*Number of trucks*	
0.185	1	1	0.324	1.391	\$4,507	1	\$4,507
0.185	1	0	0.185	1.391	\$2,573	1	\$2,573
0.185	0	1	0.324	0.957	\$3,101	2	\$6,202
0.185	0	0	0.185	0.957	\$1,770	6	\$10,620
Total	**2**	**2**				**10**	**\$23,902**

We note that accidents affect the bonus-malus factor (BMF) of all the vehicles (fleet effect), whereas speeding violations affect the *a priori* risk via the regression component for the vehicles which accumulate them. The detailed calculation of the BMF for the accident accumulation of a truck involved in the accident corresponds to:

$$\text{BMF} = \left[\frac{10\times 0.1542+2}{10\times 2.2056+2}\right]\left[\frac{2.2056+1}{0.1542+0.185}\right] = 1.391.$$

We note that the BMF is higher for vehicles having had an accident than for those which did not. We also notice that the *a priori* risk measurement γ_{fi}^{t+1} increases significantly for vehicles which have accumulated a speeding violation. If none of the 10 vehicles in the fleet had been involved in an accident nor had been charged with speeding, the total premium would have decreased from \$18,500 to \$8,440 (10×\$844), for the BMF would be equal to 0.456 and the individual truck premium to \$8,440 (0.185 × 0.456 × \$10,000 = \$844). However, in our example, the total premium goes from \$18,500 to \$23,902 based on the accumulated experience of the 10 vehicles.

Now, if the carrier has accumulated 3 past accidents and has 9 trucks with no accident and no speeding violation and 1 truck with 3 accidents but no speeding violation, the total premium is \$24,776. The insurance premiums of the fleet for each of the experiences are given in Table 4.

Table 4: Table of insurance premiums for vehicles belonging to a size-10 fleet when the fleet has accumulated 3 accidents

γ_{fi}^{t}	*Accumulation of accidents*	γ_{fi}^{t+1}	BMF	γ_{fi}^{t+1}BMF × \$10,000	*Number of trucks*	
0.185	0	0.185	1.179	\$2,181	9	\$19,629
0.185	3	0.185	2.782	\$5,147	1	\$5,147
Total	**3**				**10**	**\$24,776**

The detailed calculation of the BMF for the accumulated 3 accidents is equal to:

$$\text{BMF} = \left[\frac{10\times 0.1542+3}{10\times 2.2056+3}\right]\left[\frac{2.2056+3}{0.1542+0.185}\right] = 2.782\,.$$

We note that the premium for a vehicle with no accident nor speeding violation is \$2,181 when it belongs to a fleet having accumulated 3 accidents and drops to \$1,770 if it belongs to a fleet having accumulated 2 accidents, while retaining the same characteristics (Table 3). This result is explained by the fact that the BMFs of all the vehicles are affected by the fleet's accumulation of accidents. We also note that, when the vehicle comes from a fleet having accumulated 2 accidents, accumulating 3 accidents increases the insurance premium more ($5,147) than accumulating one accident and one speeding violation. ($4,507).

4.2.3. Dividing the vehicles into 2 groups

In this situation, the estimated insurance premium of a truck *i* belonging to a carrier *f* is given by (19) with $\sum_{i=1}^{I_f} \nu_i = I_f \hat{\nu}$.

Suppose that the accidents accumulated by the carrier over the next period is 0, with 4 trucks belonging to group 1 (a priori expected number of accidents below or equal to 0.14345) and 6 trucks belonging to group 2 (a priori expected number of accidents above 0.14345). By supposing that the average cost of claims is $10,000, the insurance premiums for the history of each of the fleet's vehicles in the following period are given in Table 5.

Table 5: Table of insurance premiums for vehicles belonging to a 10-truck fleet when the fleet has not accumulated a single accident

Group	$\hat{\gamma}^{t}_{fgi}$	*Accumulation of accidents*	γ^{t+1}_{fi}	BMF	γ^{t+1}_{fi}BMF × $10,000	*Number of trucks*	
1	0.1305	0	0.1305	0.455	$594	4	$2,376
2	0.2331	0	0.2331	0.440	$1,026	6	$6,156
Total		0				**10**	**$8,532**

The detailed calculation of the BMF for a truck belonging to group 1 corresponds to:

$$\text{BMF} = \left[\frac{0+2.2056}{0.2331+0.1542}\right]\left[\frac{10\times 0.1542+0}{10\times 2.2056+0}\right]\left[1.1434\right] = 0.455$$

and that for a truck belonging to group 2 is given by

$$\text{BMF} = \left[\frac{0+2.2056}{0.2331+0.1542}\right]\left[\frac{10\times 0.1542+0}{10\times 2.2056+0}\right]\left[1.1063\right] = 0.440\,.$$

Now, if the fleet has accumulated 1 accident and if the vehicle involved in the accident belongs to group 2, the insurance premiums for the fleet's vehicles are given in Table 6.

Table 6: Table of insurance premiums for vehicles belonging to a 10-truck fleet when the fleet has accumulated 1 accident (in group 2)

Group	$\hat{\gamma}^{t}_{fgi}$	*Accumulation of accidents*	γ^{t+1}_{fi}	BMF	γ^{t+1}_{fi}BMF × \$10,000	*Number of trucks*	
1	0.1305	0	0.1305	0.720	\$940	4	\$3,760
2	0.2331	0	0.2331	0.689	\$1,606	5	\$8,030
2	0.2331	1	0.2331	1.001	\$2,333	1	\$2,333
Total		1				**10**	**\$14,123**

The detailed calculation of the BMF for a truck belonging to group 1 corresponds to:

$$\text{BMF} = \left[\frac{0+2.2056}{0.2331+0.1542}\right]\left[\frac{10\times 0.1542+1}{10\times 2.2056+1}\right]\left[1.1466\right] = 0.720\,.$$

That of a truck belonging to group 2 and not having had any accident is given by:

$$\text{BMF} = \left[\frac{0+2.2056}{0.2331+0.1542}\right]\left[\frac{10\times 0.1542+1}{10\times 2.2056+1}\right]\left[1.0974\right] = 0.689\,,$$

whereas that of a truck in group 2 having had 1 accident is equal to:

$$\text{BMF} = \left[\frac{1+2.2056}{0.2331+0.1542}\right]\left[\frac{10\times 0.1542+1}{10\times 2.2056+1}\right]\left[1.0974\right] = 1.001\,.$$

In contrast, if the vehicle involved in the accident belongs to group 1, we obtain the values shown in Table 7.

Table 7: Table of insurance premiums for vehicles belonging to a 10-truck fleet when the fleet has accumulated 1 accident (in group 1)

Group	$\hat{\gamma}^{t}_{fgi}$	*Accumulation of accidents*	γ^{t+1}_{fi}	BMF	γ^{t+1}_{fi}BMF × \$10,000	*Number of trucks*	
1	0.1305	0	0.1305	0.728	\$950	3	\$2,850
1	0.1305	1	0.1305	1.058	\$1,381	1	\$1,381
2	0.2331	0	0.2331	0.697	\$1,625	6	\$9,750
Total		1				**10**	**\$13, 981**

The detailed calculation of the BMF for a truck belonging to group 1 corresponds to:

$$\text{BMF} = \left[\frac{0+2.2056}{0.2331+0.1542}\right]\left[\frac{10\times 0.1542+1}{10\times 2.2056+1}\right]\left[1.1598\right] = 0.728$$

and that of a truck in group 1 having had 1 accident is equal to:

$$\text{BMF} = \left[\frac{1+2.2056}{0.2331+0.1542}\right]\left[\frac{10\times 0.1542+1}{10\times 2.2056+1}\right]\left[1.1598\right] = 1.058\,.$$

Finally, the BMF for a truck belonging to group 2 is given by:

$$\text{BMF} = \left[\frac{0+2.2056}{0.2331+0.1542}\right]\left[\frac{10\times 0.1542+1}{10\times 2.2056+1}\right]\left[1.1095\right] = 0.697\,.$$

Table 8 sums up all the cases (numbers not in parentheses).

Table 8: Table of insurance premiums for vehicles belonging to a 10-truck fleet separated into two risk groups

Fleet accidents	$\sum_{j=1}^{t}\sum_{i=1}^{10} y_{fi}^{j}=0$		$\sum_{j=1}^{t}\sum_{i=1}^{10} y_{fi}^{j}=1$				$\sum_{j=1}^{t}\sum_{i=1}^{10} y_{fi}^{j}=2$					
Group 1 accidents	$\sum_{j=1}^{t}\sum_{i=1}^{4} y_{fi}^{j}=0$		$\sum_{j=1}^{t}\sum_{i=1}^{4} y_{fi}^{j}=0$		$\sum_{j=1}^{t}\sum_{i=1}^{4} y_{fi}^{j}=1$		$\sum_{j=1}^{t}\sum_{i=1}^{4} y_{fi}^{j}=0$		$\sum_{j=1}^{t}\sum_{i=1}^{4} y_{fi}^{j}=1$		$\sum_{j=1}^{t}\sum_{i=1}^{4} y_{fi}^{j}=2$	
γ_{fi}^{t+1}	BMF	γ_{fi}^{t+1} BMF × $10,000	BMF	γ_{fi}^{t+1} BMF × $10,000	BMF	γ_{fi}^{t+1} BMF × $10,000	BMF	γ_{fi}^{t+1} BMF × $10,000	BMF	γ_{fi}^{t+1} BMF × $10,000	BMF	γ_{fi}^{t+1} BMF × $10,000
Group 1 0.1305												
$\sum_{j=1}^{t} y_{fi}^{j}=0$	0.455 (0.456)	$594 ($594)	0.720 (0.720)	$940 ($940)	0.728 (0.728)	$950 ($950)	0.964	$1,258	0.974	$1,271	0.985	$1,285
$\sum_{j=1}^{t} y_{fi}^{j}=1$					1.058 (1.059)	$1,381 ($1,382)			1.416	$1,848	1.431	$1,867
$\sum_{j=1}^{t} y_{fi}^{j}=2$											1.878	$2,451
Groupe 2 0,2331												
$\sum_{j=1}^{t} y_{fi}^{j}=0$	0.440 (0.441)	$1,026 ($1,026)	0.689 (0.689)	$1,606 ($1,606)	0.697 (0.697)	$1,625 ($1,625)	0.914	$2,131	0.923	$2,152	0.932	$2,172
$\sum_{j=1}^{t} y_{fi}^{j}=1$			1.001 (1.000)	$2,333 ($2,331)			1.328	$3,096	1.341	$3,126		
$\sum_{j=1}^{t} y_{fi}^{j}=2$							1.742	$4,061				

It should be denoted that the Monte Carlo computations of premiums are identical to those with the hypergeometric approximation when we assume that all trucks are identical inside the two groups. They correspond to the numbers in parenthesis in Table 8. One advantage of Monte Carlo simulations is that we can consider all trucks as different in a given fleet. We now present results for ten different trucks using Monte Carlo simulations to make a posteriori computations. We still use the econometric results of Table 1 for a priori evaluations. The simulations are repeated 500,000 times; this takes about 10 minutes for a scenario like the ones presented in Table 9 in the Appendix, whereas the hypergeometric approximations are instantaneous.

Table 9 presents the premium evolution over five years for three scenarios. The a priori expected number of accidents for these three scenarios is 0.192. Scenario 1 is for a fleet that accumulates many accidents over time. In the first column, we observe the ten different a priori values. In the third column, we have the corresponding starting premiums for the three scenarios which amount to a total premium of $19,206 for the fleet. Accumulating eighteen accidents over five years yields a total premium of $33,190 for the next period. In scenario 2, the fleet accumulates only five accidents over the five years and the total premium drops to $11,196. Finally, in scenario 3, the fleet has two accidents each year (its average), resulting in an almost constant premium over time.

In Figure 1, we graphically represent the three scenarios with solid lines. The dotted lines correspond to the case where the fleet effect is not computed in both the regression and the premium computations (see the numbers in Table 10 in the Appendix). The differences are significant. Introducing the fleet effect increases the fluctuations in the premiums and should introduce more incentives for road safety.

(Figure 1 here)

5. CONCLUSION

In this article, we have developed a parametric model for rating insurance premiums for fleets of vehicles. We have shown how taking into account both fleet and vehicle effects can affect the Bayesian calculation of insurance premiums over time. The model proposed was estimated using data over a single period. An important extension would be to model a panel effect which would take into account the repetitions of information on fleets and vehicles over time (see Abowd et al., 1999, for a first analysis of this type of model).

The rating formula developed presupposes a decentralized management of road safety as regards carriers. In effect, charging different premiums for each of the vehicles in a fleet based on the experience of both the fleet and its trucks will prompt road-safety managers themselves to keep a close eye on road-safety policy and to set up institutional incentives motivating drivers and carriers to adopt prudent behaviors. Indeed, knowing which drivers and carriers are risky, these managers can then assign sliding premiums according to the risk levels of the different drivers and trucks.

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Corresponding addresses:

Georges Dionne, Canada Research Chair in Risk Management, HEC Montréal, 3000, Chemin de la Côte-Sainte-Catherine, Montreal (Qc) Canada, H3T 2A7. Telephone: (514)340-6596. Fax: (514)340-5019. georges.dionne@hec.ca.

Jean-François Angers, Denise Desjardins, and François Guertin, Center for Research on Transportation, University of Montreal, C.P. 6128, Succ. Centre-Ville, Montreal, Canada, H3C 3J7.

APPENDIX

Table 1: Estimation of parameters to predict the number of accidents for trucks in all the fleets by dividing the trucks into two groups for fleets with more than 2 trucks

Explanatory variables	Coefficient	Statistic *t*	P
Constant	-3.4049	-34.8132	< 0.0001
Number of years as carrier as of 31 December 1998	-0.0596	-10.5814	< 0.0001
Sector of activity in 1998			
Other sectors	-0.2021	-0.5936	0.5528
General public trucking	0.1675	2.2648	0.0235
Bulk public trucking		Reference group	
Private trucking	0.0810	1.3897	0.1646
Short-term rental firm	0.6476	4.4103	< 0.0001
Size of fleet			
2		Reference group	
3	0.1805	3.6976	0.0002
4 to 5	0.2076	4.3908	< 0.0001
6 to 9	0.3117	6.5705	< 0.0001
10 to 20	0.4111	8.3910	< 0.0001
21 to 50	0.4358	6.9676	< 0.0001
More than 50	0.5907	6.0927	< 0.0001
Number of days authorized to circulate in 1997	1.8679	25.5989	< 0.0001
Number of violations of trucking standards in 1997			
For overload	0.1803	4.6352	< 0.0001
For excessive size	0.6118	2.1543	0.0312
For poorly secured cargo	0.4611	4.0156	0.0001
For failure to respect service hours	0.2761	2.0195	0.0434
For failure to pass mechanical inspection	0.3630	3.8630	0.0001
For other reasons	0.3980	2.2359	0.0254
Type of vehicle use			
Commercial use including transport of goods without C.T.Q. permit	-0.1163	-2.1748	0.0296
Transport of other than "bulk" goods	-0.1376	-2.1048	0.0353
Transport of "bulk" materials		Reference group	
Type of fuel			
Diesel		Reference group	
Gas	-0.4402	-10.2683	< 0.0001
Others	-0.1813	-1.0208	0.3074
Number of cylindres			
1 to 5 cylindres	0.1814	1.7689	0.0769
6 to 7 cylindres	0.3548	10.0550	< 0.0001
8 or more than 10 cylindres		Reference group	
Number of axles			
2 axles (3,000 to 4,000 kg)	-0.3480	-6.5823	< 0.0001
2 axles (more than 4,000 kg)	-0.3143	-8.4974	< 0.0001
3 axles	-0.1677	-4.6494	< 0.0001
4 axles	-0.1442	-2.9576	0.0031
5 axles	-0.1913	-4.5111	< 0.0001
6 axles or more		Reference group	
Number of violations with demerit points in 1997			
For speeding	0.2648	11.0069	< 0.0001
For driving under suspension	0.4725	3.4847	0.0005
For running a red light	0.4031	6.1732	< 0.0001
For ignoring stop sign or traffic agent	0.5134	7.5045	< 0.0001
For not wearing a seat belt	0.1741	1.6445	0.1001
For other offenses	1.1218	14.5857	< 0.0001
ν	2.2056	10.5147	< 0.0001
κ (fleets of 2 trucks)	1.5615	5.8126	< 0.0001
κ (fleets of 3 trucks)	2.1061	6.0286	< 0.0001
κ (fleets of 4 to 5 trucks)	3.0853	7.6211	< 0.0001
κ (fleets of 6 to 9 trucks)	3.5167	7.5446	< 0.0001
κ (fleets of 10 to 20 trucks)	6.4867	9.2521	< 0.0001
κ (fleets of 21 to 50 trucks)	15.9511	8.1146	< 0.0001
κ (fleets of more than 50 trucks)	118.4366	7.5069	< 0.0001
Log-likelihood		-30,494	
Number of carriers		13,159	
Number of vehicles		73,328	

Scenario 1 (($\nu = 2.2056; \;\; \kappa^{-1} = 0.1542$)

Table 9: Monte Carlo Simulations[1]

t	0		1		2		3		4		5	
γ_{fi}	BMF	Premium $	BMF	Premium $	BMF	Premium $	BMF	Premium $	BMF	Premium $	BMF	Premium $
0.1190	1.000	1,190	1.213	1,444	1.349	1,605	1.839*	2,188	2.388*	2,842	2.860*	3,404
0.1207	1.000	1,207	1.762*	2,127	2.566*	3,098	2.407	2,905	2.945*	3,554	2.853	3,444
0.1408	1.000	1,408	1.199	1,688	2.502**	3,523	2.320	3,267	2.273	3,200	2.181	3,070
0.1415	1.000	1,415	1.197	1,693	1.312	1,857	1.216	1,720	1.730*	2,448	2.696**	3,814
0.1633	1.000	1,633	1.179	1,925	1.274	2,081	1.701*	2,777	1.649	2,693	1.567	2,559
0.2281	1.000	2,281	1.652*	3,767	2.249*	5,130	2.014	4,594	1.900	4,333	1.773	4,045
0.2301	1.000	2,301	1.646*	3,788	1.711	3,937	1.531	3,523	1.445	3,326	1.766*	4,064
0.2421	1.000	2,421	1.126	2,725	1.161	2,812	1.030	2,494	1.415*	3,426	1.310	3,172
0.2633	1.000	2,633	1.111	2,924	1.134	2,986	1.456*	3,833	1.359	3,577	1.257	3,309
0.2717	1.000	2,717	1.105	3,002	1.123	3,052	0.990	2,691	0.922	2,506	0.850	2,309
	Total	19,206	Total	25,083	Total	30,079	Total	29,991	Total	31,906	Total	33,190

Scenario 2

t	0		1		2		3		4		5	
γ_{fi}	BMF	Premium $	BMF	Premium $	BMF	Premium $	BMF	Premium $	BMF	Premium $	BMF	Premium $
0.1190	1.000	1,190	0.724	862	0.934*	1,111	0.866	1,030	0.690	821	0.789	939
0.1207	1.000	1,207	0.724	874	0.641	774	0.595	718	0.474	572	0.542	655
0.1408	1.000	1,408	0.719	1,013	0.632	890	0.584	823	0.463	653	0.529	744
0.1415	1.000	1,415	0.718	1,015	0.631	894	0.584	827	0.465	657	0.766*	1,083
0.1633	1.000	1,633	0.710	1,159	0.622	1,015	0.831*	1,357	0.660	1,078	0.975*	1,593
0.2281	1.000	2,281	1.004*	2,289	0.867	1,977	0.785	1,789	0.622	1,419	0.686	1,565
0.2301	1.000	2,301	0.691	1,591	0.594	1,368	0.539	1,240	0.426	981	0.470	1,082
0.2421	1.000	2,421	0.687	1,662	0.591	1,430	0.534	1,292	0.422	1,021	0.463	1,120
0.2633	1.000	2,633	0.681	1,794	0.582	1,533	0.524	1,380	0.414	1,091	0.454	1,194
0.2717	1.000	2,717	0.680	1,846	0.579	1,574	0.521	1,414	0.411	1,117	0.449	1,220
	Total	19,206	Total	14,106	Total	12,565	Total	11,870	Total	9,409	Total	11,196

Scenario 3

t	0		1		2		3		4		5	
γ_{fi}	BMF	Premium $	BMF	Premium $	BMF	Premium $	BMF	Premium $	BMF	Premium $	BMF	Premium $
0.1190	1.000	1,190	0.970	1,154	0.928	1,104	0.897	1,068	0.861	1,024	1.200*	1,428
0.1207	1.000	1,207	0.969	1,170	0.927	1,119	0.896	1,082	0.855	1,032	0.824	994
0.1408	1.000	1,408	0.959	1,350	0.911	1,282	0.873	1,229	0.828	1,166	0.792	1,116
0.1415	1.000	1,415	0.961	1,360	1.323*	1,872	1.662*	2,352	1.956*	2,768	1.868	2,643
0.1633	1.000	1,633	0.946	1,545	0.891	1,454	1.233*	2,014	1.162	1,898	1.104	1,802
0.2281	1.000	2,281	1.330*	3,033	1.224	2,792	1.141	2,602	1.057	2,412	0.987	2,252
0.2301	1.000	2,301	0.915	2,107	0.840	1,932	0.782	1,799	0.724	1,666	0.677	1,557
0.2421	1.000	2,421	1.325*	3,207	1.208	2,925	1.122	2,718	1.036	2,509	1.270*	3,075
0.2633	1.000	2,633	0.901	2,373	1.187*	3,126	1.094	2,879	1.320*	3,476	1.225	3,227
0.2717	1.000	2,717	0.896	2,435	0.810	2,202	0.746	2,028	0.685	1,860	0.635	1,725
	Total	19,206	Total	19,734	Total	19,808	Total	19,770	Total	19,813	Total	19,820

[1] One * indicates that the truck had one accident during the previous period while two * is for two accidents during the previous period.

Scenario 1($\kappa^{-1} = 0.9289$) **Table 10: Monte Carlo Simulations Without the Fleet Effect in both the Premium Computation and Parameter Estimation**

t	0		1		2		3		4		5	
γ_{fi}	BMF	Premium $	BMF	Premium $	BMF	Premium $	BMF	Premium $	BMF	Premium $	BMF	Premium $
0.1190	1.000	1,190	0.886	1,055	0.796	947	1.500*	1,785	2.085*	2,481	2.578*	3,068
0.1207	1.000	1,207	1.838*	2,218	2.503*	3,021	2.269	2,738	2.783*	3,359	2.564	3,095
0.1408	1.000	1,408	0.868	1,223	2.420*	3,407	2.167	3,052	1.963	2,764	1.794	2,526
0.1415	1.000	1,415	0.852	1,206	0.743	1,051	0.658	931	1.227*	1,736	2.672*	3,781
0.1633	1.000	1,633	0.850	1,389	0.740	1,208	1.360*	2,220	1.219	1,991	1.105	1,805
0.2281	1.000	2,281	1.667*	3,803	2.115*	4,823	1.816	4,141	1.591	3,628	1.415	3,228
0.2301	1.000	2,301	1.664*	3,830	1.389	3,195	1.191	2,741	1.043	2,400	1.409*	3,241
0.2421	1.000	2,421	0.793	1,920	0.657	1,591	0.561	1,359	1.017*	2,461	0.902	2,183
0.2633	1.000	2,633	0.779	2,051	0.638	1,680	1.122*	2,955	0.973	2,562	0.859	2,262
0.2717	1.000	2,717	0.774	2,102	0.631	1,714	0.533	1,447	0.461	1,252	0.406	1,103
	Total	19,206	Total	20,797	Total	22,639	Total	23,370	Total	24,635	Total	26,292

Scenario 2

t	0		1		2		3		4		5	
γ_{fi}	BMF	Premium $	BMF	Premium $	BMF	Premium $	BMF	Premium $	BMF	Premium $	BMF	Premium $
0.1190	1.000	1,190	0.886	1,055	1.653*	1,967	1.500	1,785	1.373	1,634	1.266	1,506
0.1207	1.000	1,207	0.885	1,068	0.794	958	0.720	868	0.658	794	0.606	732
0.1408	1.000	1,408	0.868	1,223	0.767	1,080	0.687	968	0.623	877	0.569	801
0.1415	1.000	1,415	0.852	1,206	0.743	1,051	0.658	931	0.591	836	1.113*	1,575
0.1633	1.000	1,633	0.850	1,389	0.740	1,208	1.360*	2,220	1.219	1,991	1.678*	2,740
0.2281	1.000	2,281	1.667*	3,803	1.393	3,177	1.196	2,727	1.048	2,390	0.932	2,126
0.2301	1.000	2,301	0.801	1,844	0.669	1,539	0.574	1,320	0.502	1,156	0.447	1,028
0.2421	1.000	2,421	0.793	1,920	0.657	1,591	0.561	1,359	0.490	1,185	0.434	1,051
0.2633	1.000	2,633	0.779	2,051	0.638	1,680	0.540	1,423	0.469	1,234	0.414	1,089
0.2717	1.000	2,717	0.774	2,102	0.631	1,714	0.533	1,447	0.461	1,252	0.406	1,103
	Total	19,206	Total	17,662	Total	15,966	Total	15,049	Total	13,348	Total	13,752

Scenario 3

t	0		1		2		3		4		5	
γ_{fi}	BMF	Premium $	BMF	Premium $	BMF	Premium $	BMF	Premium $	BMF	Premium $	BMF	Premium $
0.1190	1.000	1,190	0.886	1,055	0.796	947	0.722	860	0.661	787	1.266*	1,506
0.1207	1.000	1,207	0.885	1,068	0.794	958	0.720	868	0.658	794	0.606	732
0.1408	1.000	1,408	0.868	1,223	0.767	1,080	0.687	968	0.623	877	0.569	801
0.1415	1.000	1,415	0.852	1,206	1.543*	2,183	2.075*	2,937	2.499*	3,536	2.267	3,208
0.1633	1.000	1,633	0.850	1,389	0.740	1,208	1.360*	2,220	1.219	1,991	1.105	1,805
0.2281	1.000	2,281	1.667*	3,803	1.393	3,177	1.196	2,727	1.048	2,390	0.932	2,126
0.2301	1.000	2,301	0.801	1,844	0.669	1,539	0.574	1,320	0.502	1,156	0.447	1,028
0.2421	1.000	2,421	1.647*	3,988	1.365	3,305	1.165	2,821	1.017	2,461	1.369*	3,314
0.2633	1.000	2,633	0.779	2,051	1.325*	3,489	1.122	2,955	1.478*	3,891	1.304	3,434
0.2717	1.000	2,717	0.774	2,102	0.631	1,714	0.533	1,447	0.461	1,252	0.406	1,103
	Total	19,206	Total	19,729	Total	19,600	Total	19,123	Total	19,134	Total	19,058

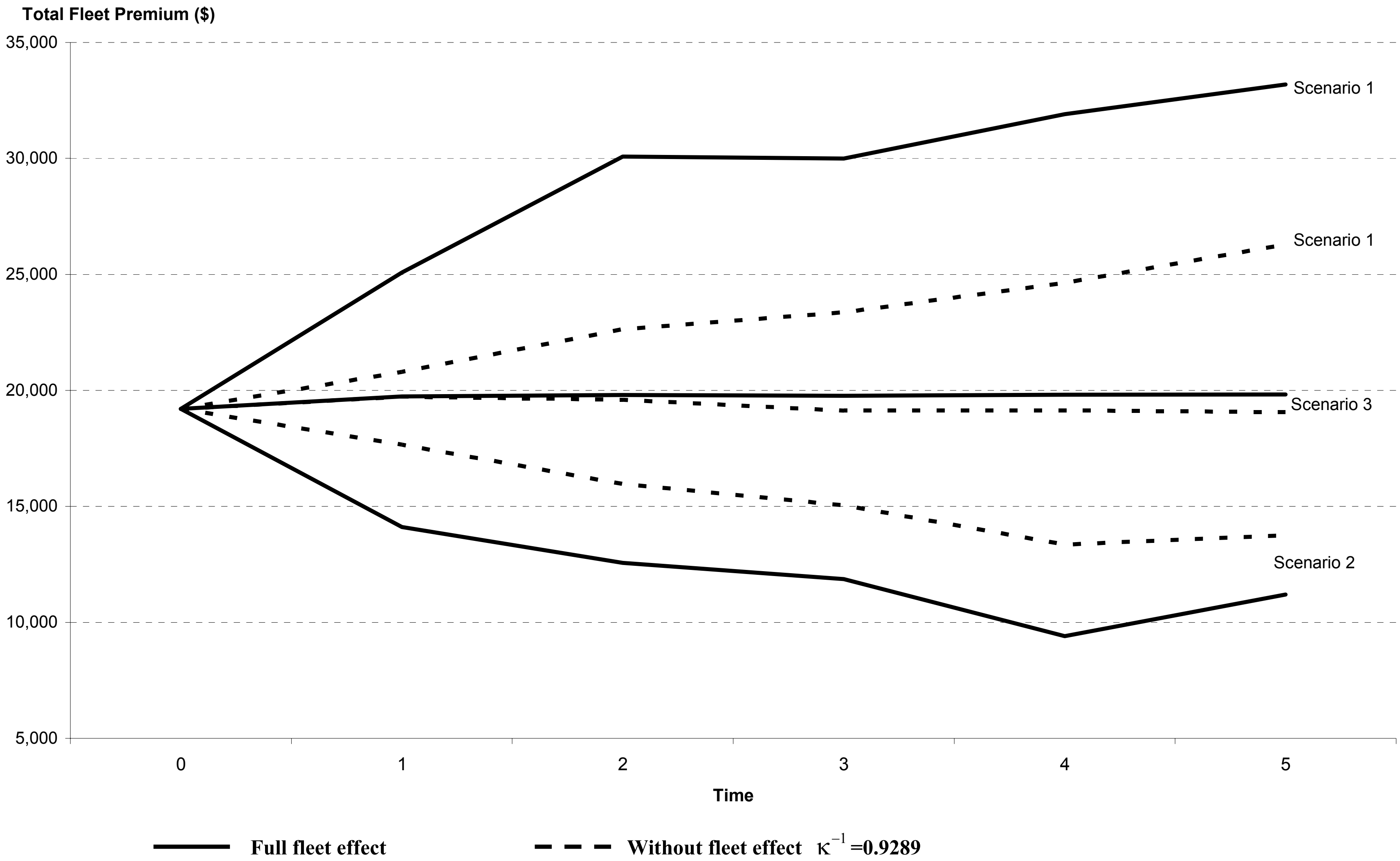

Figure 1: Simulation with and without the fleet effect