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Inequity Aversion in Tournaments

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Abstract: We consider the cost of providing incentives through tournaments when workers are inequity averse and performance evaluation is costly. The principal never benefits from empathy between the workers, but he may benefit from their propensity for envy depending on the costs of assessing performance. More envious employees are preferred when these costs are high, less envious ones when they are low.

1. Introduction

Reward schemes based on relative performance are widely used and often constitute the main motivating device, as with employees in large organizations competing for bonuses or promotions. Following Lazear and Rosen (1981), the properties and raison d'être of tournaments have been extensively discussed.[1] The present paper analyzes their profitability, from the principals' point of view, when agents have a distaste for unequal payoffs relative to their reference group.

A standard justification for tournaments is that relative comparisons are often easier to make than absolute judgments. Tournaments also commit an organization to a fixed prize structure. When performance evaluation is subjective, this is useful in eliminating incentives to underreport performance so as to avoid paying bonuses; conversely, it counteracts the leniency bias of evaluators reluctant to distinguish between good and bad performance. Tournaments motivate agents by creating situations where ranking cannot be avoided. This generates strong incentives, but it also confronts contestants with the certainty of unequal payoffs between peers.

In the usual formulation, agents care about ranking only to the extent that it affects their own absolute payoff. There is no room for rivalry per se, for instance the satisfaction from outperforming rivals. Neither is there room for the possibility that individuals resent earning less than their peers or conversely that they feel uneasy when earning more through mere luck. A large empirical literature – not to mention a long tradition in social psychology – suggests that such concerns matter for the individuals' well being.[2] They may also affect behavior in important ways. As shown by Fehr and Schmidt (1999) and Bolton and Ockenfels (2000) who draw on findings from the experimental literature, a distaste for inequality in payoff distributions has much explanatory power.

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[1] See Nalebuff and Stiglitz (1983), Malcomson (1984) and additional references in Gibbons and Waldman (1999) and Prendergast (1999).

[2] See Loewenstein, Thomson and Bazerman (1989), Clark and Oswald (1996) and the numerous references therein, as well as those in Akerlof and Yellen (1990) for instance.

We consider the implications on the cost of using tournaments as incentive schemes when agents are concerned with relative payoffs. Our starting point is the concept of 'self-centered inequity aversion' as defined in Fehr and Schmidt (1999). Although agents care about their absolute payoff as in the economist's standard model, they are also interested in the 'fairness' of their payoff relative to that of others. An individual experiences dissatisfaction if he is worse off than his reference group (disadvantageous inequity) and may perhaps also suffer dissatisfaction if better off (advantageous inequity). He nevertheless suffers more from inequity that is to his disadvantage than from one to his advantage. We qualify the notion of inequity aversion by also allowing for the possibility that agents obtain satisfaction from outperforming rivals. Our agents nevertheless remain 'inequality averse' in the sense that they suffer more from being outdone than they would gain from outdoing their rivals by the same margin.

Intuitively, the principal cannot profit from a distaste for advantageous inequity on the part of the contestants — empathy with the losers. The reason is that this introduces a wedge between what is paid to the winning agent and his subjective benefit, thereby reducing the utility gain from winning. But what about a distaste for disadvantageous inequity, which may be interpreted as envy or frustration from losing? For a given prize structure, this increases the utility difference between winning and losing and therefore increases incentives. The principal can then reduce the spread between money prizes and still induce the same effort. On the other hand, the disutility of losing the tournament is now greater, which presumably reduces the agents' willingness to participate (as when someone refuses to play a game because he 'hates to lose'), thus requiring compensation from the principal. As far as the principal's costs are concerned, the consequences of dealing with more envious agents therefore appear to be ambiguous.

As noted by Fehr and Schmidt (2002), there is hardly any theoretical literature on the interaction between the agents' concern for equity and the provision of incentives.[3]

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[3]But see the recent paper by Englmaier and Wambach (2002).

In the context of tournament models, Kräkel (2000) examined the influence of 'relative deprivation' on the contestants' effort decision. He shows that, for given tournament prizes, agents concerned only about relative income exert more effort than if they maximized their expected absolute payoff. However, this contribution does not study the optimal prize structure from the principal's point of view. In a paper written concurrently with ours, Grund and Sliwka (2002) develop a model similar to the one presented here. We differ by assuming that agents face a liability limit (the workers' wage cannot be negative), which introduces the possibility that they earn rent in the optimal tournament. Moreover, our principal is able at a cost to increase the extent to which the ranking of contestants provides information about relative effort. The idea is that a more elaborate albeit more costly tournament design (e.g. a more thorough ranking procedure) reduces the influence of luck on the outcome. Thus, a tournament is characterized here both by its prize structure and its ranking procedure.[4]

We determine the cost function of effort from the principal's point of view; that is, we consider the cost to the principal of inducing arbitrary levels of effort. When the agents' liability limit is not binding in the optimal tournament, our conclusion is that the principal would not benefit from more envious contestants, as in Grund and Sliwka (2002). As suggested above, the intuition is then that contestants must be compensated for the expected frustration of turning out losers. This conclusion is reversed if the agents' liability limit is binding and they earn rent. Such a case necessarily arises when organizing informative tournaments is sufficiently costly. The intuition is now that, because they are more 'aggressive', more envious contestants allow the principal both to reduce the winning prize (thus reducing the agents' rent) and to economize on the costs of the ranking procedure. With binding liability limits, more envious contestants can also benefit the principal even when there is no rent. Although such agents need to be paid more in expected value, the principal benefits

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[4]We also assume that agents assess equity in terms of income net of effort costs. They would therefore experience inequity if they earned the same income, but had exerted different effort levels (see also Akerlof and Yellen (1990) or Levine (1991) for instance).

from using a coarser and therefore less costly ranking procedure.

The next two sections describe our basic framework, assuming a two-contestant tournament for simplicity. Section 4 derives the optimal wage structure when the informational properties of the tournament are taken as given. Section 5 analyzes the optimal tournament design with endogenous precision and presents our main results. Section 6 discusses the implications and concludes.

2. PREFERENCES

A contest is used to create incentives in a moral hazard situation with two identical risk neutral workers $i = 1, 2$. The workers dislike inequities relative to their reference group which in this case is limited to the other contestant. Specifically, following Fehr and Schmidt (1999), we write the utility of payoff π_i when the co-worker earns π_j as

$$U(\pi_i, \pi_j) = \pi_i - [\alpha \max(\pi_j - \pi_i, 0) + \beta \max(\pi_i - \pi_j, 0)], \quad i, j = 1, 2 \qquad (1)$$

The terms in the square bracket are the utility effects of disadvantageous and advantageous inequality respectively. Fehr and Schmidt assume $\alpha > \beta$ and $0 \leq \beta < 1$. The implication is that a worse off individual is willing to trade-off some of his personal gain against a decrease in his peer's payoff. This may denote envy or the disutility of being outdone. When $\beta > 0$, a better off individual is fair-minded (or suffers from the envy of others) since he would trade-off some decrease in his personal gain against an increase in the peer's payoff. $\beta < 1$ implies that a worker always benefits from an increase in his own payoff, while $\alpha > \beta$ means that he dislikes a difference against him more than one in his favor.

In this formulation inequities between workers are always a source of disutility. This can be relaxed somewhat by allowing β to be negative, i.e. by considering the possibility that an individual obtains satisfaction from being better off than others. We impose $\alpha > |\beta|$ so that the disutility from being outperformed is greater than the benefit (or disutility as the case may be) from being better off.
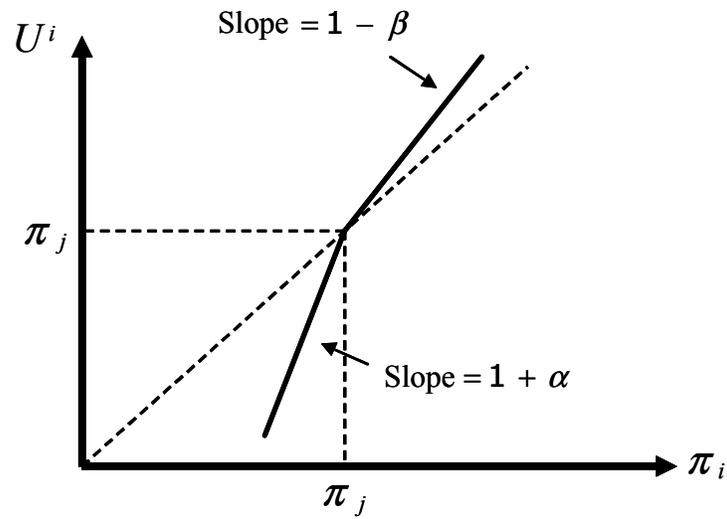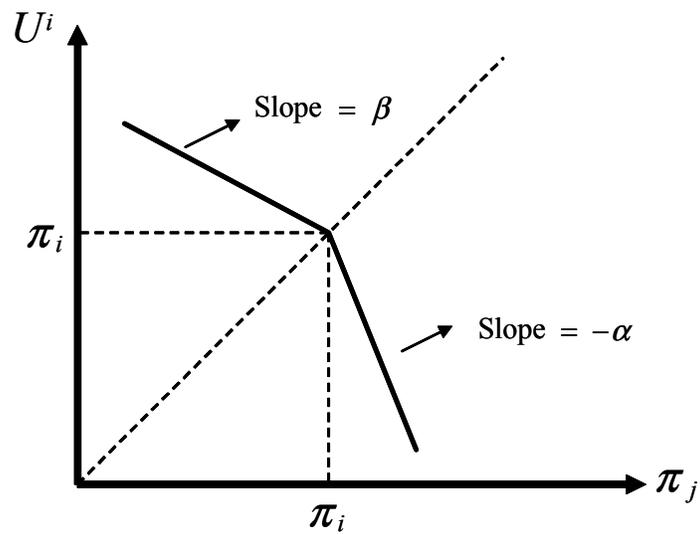
Figure 1a

Figure 1b

Utility as a function of own payoff is represented in figure 1a for the case where β is negative. $\alpha > |\beta|$ implies concavity, which amounts to risk aversion with respect

to gambles with the possibility of turning out ahead or behind one's peer. This holds irrespective of the sign of β but a positive value would lead to greater risk aversion. In figure 1b, the individual's utility is drawn as a function of the peer's payoff. Utility is everywhere decreasing if β is negative as in the figure, while with a positive β a maximum is reached at the point where payoffs are equalized. In either case $\alpha > |\beta|$ implies concavity in figure 1b.

The preceding assumptions characterize inequality aversion, even though one may possibly obtain satisfaction from outperforming one's peer — and although in some cases an inequality averse individual also exhibits risk aversion in the usual sense. To make this clear, suppose individual i faces equal chances of getting $\pi + \varepsilon$ or $\pi - \varepsilon$, while his peer gets π for sure. With ε positive, individual i's expected utility is then

$$U^i = \pi - \tfrac{1}{2}(\alpha + \beta)\varepsilon \qquad (2)$$

Since $\alpha > |\beta|$ implies $\alpha + \beta > 0$, individual i is risk averse in the sense that he would prefer π for sure. Suppose now that i gets π for sure while his peer faces equal chances of getting $\pi + \varepsilon$ or $\pi - \varepsilon$. Individual i's utility is again as in (2), i.e. he would prefer that his peer also gets π for sure. The second term in (2) will be referred to as the inequality premium.

3. The tournament contract

The contest is defined by its prize structure and the performance evaluation process. We write l and w for the wages of the losing and the winning party respectively. The net payoff of a worker undertaking effort e is then either $\pi = l - c(e)$ or $\pi = w - c(e)$ where $c(e)$ is the cost of undertaking effort, an increasing and strictly convex function. Though e is not contractible, workers are assumed to see each other's effort and are therefore able, at the end of the contest, to compare their net payoff. Given the tournament design, the principal observes for each worker a performance measure

$$x_i = e_i + \varepsilon_i, \ \ i = 1, 2 \qquad (3)$$

where the ε_i's are i.i.d. error terms. The x_i's can generally be thought as aggregating information from various sources. Some measures may be readily available like sales data pertaining to a salesperson, while others are costly to gather or may reflect soft information like the principal's more or less subjective appreciation. The care or thoroughness with which performance is assessed is part of the tournament design and is a decision variable from the principal's point of view. Greater precision in relative performance assessment requires a more costly tournament. We assume this is contractible, i.e. the principal can commit to some level of precision.[5]

The party with the best outcome wins. Thus, agent i prevails if $x_i > x_j$ or equivalently $\varepsilon_j - \varepsilon_i < e_i - e_j$. Denoting with H the c.d.f. of the difference in error terms for a given tournament design, worker i wins the contest with probability $H(e_i - e_j)$. The agent's optimization problem is therefore

$$\max_{\widehat{e}} \; E\left[U|\widehat{e}, e\right] = H(\widehat{e} - e)u_w(\widehat{e}, e) + \left[1 - H(\widehat{e} - e)\right]u_l(\widehat{e}, e) \qquad \text{(I)}$$

where e denotes the effort of the other contestant at the Nash equilibrium and where u_w and u_l are short-hand for the worker's utility upon winning or losing.

We assume the existence of a Nash equilibrium in pure strategies. Since both contestants have the same preferences, the equilibrium is symmetric and we derive the cost to the principal of implementing some arbitrary effort level e. The principal's problem is to minimize the sum of wages and performance assessment costs, subject to

$$e \in \arg\max_{\widehat{e}} \; E\left[U|\widehat{e}, e\right] \qquad \text{(IC)}$$

$$E\left[U|e, e\right] \geq u_A \qquad \text{(PC)}$$

$$l \geq 0 \qquad \text{(NC)}$$

The first equation is the worker's incentive compatibility constraint at the Nash equilibrium. The next condition guarantees that it is in each workers' interest to par-

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[5]For instance, in many sports contests 'precision' is verifiable as it depends on the number of games that must be played before picking a winner.

ticipate in the contest, given the utility $u_A \geq 0$ in an alternative occupation. The last inequality is a limited liability condition reflecting the capital-market constraints faced by workers and which restrict the use of bonding or entrance fees. For simplicity, the floor wage is required here to be non-negative.

4.   The wage structure

We first determine the wage structure to induce effort e, taking as given the informational properties of the tournament design. At the symmetric Nash equilibrium, the probability that either party wins is $H(0) = \frac{1}{2}$. From the workers' optimization problem and denoting $H'(0) \equiv h$, the incentive compatibility constraint can be rewritten as the first order condition[6]

$$h \cdot (u_w - u_l) + \frac{1}{2}\left[\frac{\partial u_w}{\partial \widehat{e}} + \frac{\partial u_l}{\partial \widehat{e}}\right] = 0, \text{ for } \widehat{e} = e \tag{4}$$

The first term is the marginal benefit from greater effort through the increased probability of winning. The second term is the expected marginal disutility of effort. The utility of winning or losing when one expends effort $\widehat{e}$ while the other exerts e is respectively

$$u_w(\widehat{e}, e) = w - c(\widehat{e}) - \beta\left[(w - c(\widehat{e})) - (l - c(e))\right] \tag{5}$$

$$u_l(\widehat{e}, e) = l - c(\widehat{e}) - \alpha\left[(w - c(e)) - (l - c(\widehat{e}))\right] \tag{6}$$

Since $w > l$ and for $\widehat{e}$ not too different from e, the payoff upon winning is greater than the payoff to the other party, hence the value of u_w in (5). Similarly, the payoff upon losing is less than that of the other party, leading to u_l as defined in (6).

Denoting the wage spread by $\Delta w = w - l$, the difference in utility between winning and losing in the first-order condition (4) is

$$(u_w - u_l)|_{e=\widehat{e}} = (1 + \alpha - \beta)\Delta w \tag{7}$$

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[6]A pure strategy equilibrium exists, with the agents' behavior described by the first-order condition, only if chance is a significant factor in the outcome of the contest (see Lazear and Rosen (1981) and Nalebuff and Stiglitz (1983)). This is assumed throughout.

Under inequality aversion, the utility spread is larger than the wage spread. The second term in the first-order condition is

$$\frac{1}{2}\left[\frac{\partial u_w}{\partial \widehat{e}} + \frac{\partial u_l}{\partial \widehat{e}}\right]\Bigg|_{e=\widehat{e}} = -\left[1 + \tfrac{1}{2}(\alpha - \beta)\right]c'(e) \tag{8}$$

The marginal disutility of effort depends on whether the worker wins or loses the contest. If he wins, the utility loss from greater effort is $(1-\beta)c'(e)$. For a fair-minded individual $(\beta > 0)$ this is less than the marginal effort cost as working more makes winning appear less unfair. Conversely, it is greater if the individual is greedy $(\beta < 0)$ since his payoff advantage is then reduced. If the worker loses after marginally raising effort, the utility loss from greater effort is $(1 + \alpha)c'(e)$. The marginal disutility is then greater than the marginal effort cost because it now appears all the more unjust to lose. Taking expected values, inequality aversion has the same effect as raising the marginal cost of effort by the factor $\frac{1}{2}(\alpha - \beta)$.

Inequality aversion magnifies both the utility effect of the wage spread and the marginal cost of effort. Substituting from (7) and (8) in the first-order condition (4), the wage spread needed to induce effort level e is

$$\Delta w = \lambda \frac{c'(e)}{h} \quad \text{where } \lambda = \frac{1 + \frac{1}{2}(\alpha - \beta)}{1 + \alpha - \beta} \tag{9}$$

Overall the first effect dominates since $\lambda < 1$. Moreover, λ is decreasing in α and increasing in β.

Proposition 1. *For given performance measures, the wage spread required for inducing a given effort level is decreasing in α and increasing in β.*

Alternatively, given the wage spread, effort is increasing in α and decreasing in β. The result suggests that hiring more envious or less fair-minded workers may lower the costs to the principal by reducing the wage spread needed to provide the adequate incentives. However, the wage structure must also induce participation and be compatible with the workers' financial limits.

The expected per worker wage is

$$\tfrac{1}{2}w + \tfrac{1}{2}l = l + \tfrac{1}{2}\Delta w \tag{10}$$

The wage spread being determined by the incentive requirement, the principal minimizes wage costs by setting the floor wage as small as possible, subject to the limited liability condition $l \geq 0$ and the participation constraint

$$\left(\tfrac{1}{2}u_w + \tfrac{1}{2}u_l\right)\Big|_{e=\widehat{e}} = l + \tfrac{1}{2}(1 - \alpha - \beta)\Delta w - c(e) \geq u_A \tag{PC}$$

The two constraints cannot simultaneously be slack, otherwise the principal could lower his wage costs by reducing the floor wage. There are therefore two possibilities: either

$$l = u_A + c(e) - \tfrac{1}{2}(1 - \alpha - \beta)\Delta w \geq 0 \tag{11}$$

and the participation constraint is binding or the workers earn rent and $l = 0$.

NO RENT. Substituting for l from the binding participation constraint and writing $W(e)$ for the expected wage required to induce effort level e, we have

$$
\begin{aligned}
W(e) &= u_A + c(e) + \tfrac{1}{2}(\alpha + \beta)\Delta w \\
&= u_A + c(e) + \mu \frac{c'(e)}{2h} \quad \text{where } \mu = \lambda(\alpha + \beta)
\end{aligned}
\tag{12}
$$

The third term on the right hand side is the inequality premium.[7] It is easily checked that μ is increasing in both α and β. Hence, when no rent is earned, inequality aversion is undesirable since it increases wage costs. Moreover, wage costs are increasing with the extent of the worker's concern with relative payoffs.

RENT. The no rent case occurs only when the previous floor wage is compatible with the workers' financial constraint. The inequality in (11) always holds if the workers' concern with relative payoffs is strong in the sense that $\alpha + \beta > 1$. Otherwise,

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[7]The expression for the inequality premium is the same as in (2), given that the wage gap between the workers is now Δw and may be advantageous or disadvantageous with probability one half.

whether or not the inequality holds depends on the size of the required wage spread relative to the reservation utility and the cost of effort. In the remaining, we assume inequality aversion is not too large, i.e. $\alpha + \beta < 1$. From (11), when the required wage spread is sufficiently large, the floor wage consistent with no rent becomes negative, thus violating the workers' liability limit. Hence the principal must set $l = 0$ and leave a rent. The per worker expected wage is then

$$W(e) = \tfrac{1}{2}\Delta w = \lambda \frac{c'(e)}{2h} \tag{13}$$

In this case, the effect on wage costs of changes in the externality parameters follows directly from the results in Proposition 1. Combining the rent and no-rent situations, we get the following characterization.

Proposition 2. *For given performance measures, wage costs are increasing in β. They are increasing in α in no-rent cases and decreasing if workers earn rent.*

Empathy with the loser is therefore bad from the point of view of wage costs, while greater satisfaction from outperforming one's rival is beneficial. The effect of envy depends on the situation. When workers earn rent, a principal who could choose the workers' types would prefer more envious workers.

5. TOURNAMENT DESIGN

We now turn to the informational properties of the tournament, which until now have been taken as given. As is well known, $h = H'(0)$ reflects the importance of luck in the outcome of the contest, which in turn depends on how carefully relative performance is assessed. To see this, note that $2h = \partial \log H(\hat{e} - e)/\partial \hat{e}$ evaluated at $\hat{e} = e$. That is, h reflects the extent to which a change in effort affects the probability of winning, which is a natural measure of the precision of the underlying information structure.[8]

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[8] See O'Keefe et al. (1984) or Hvide (2002) for similar observations and Demougin and Fluet (2001) for an analysis along these lines in the context of individualistic wage schemes. Alternatively,

Everything else equal, whether or not workers earn rent depends on the importance of luck in tournament outcomes. Making explicit the role of h, per worker wage costs are

$$W(e, h) = \max\left[\lambda\frac{c'(e)}{2h}, u_A + c(e) + \mu\frac{c'(e)}{2h}\right] \tag{14}$$

The first expression on the right hand side is for the situation with rent, when per worker wage costs equal the expected wage spread. The second expression is for the no rent case, with wage costs equal to sum of the worker's reservation utility and effort cost, plus the inequality premium. Since $\alpha + \beta < 1$ implies $\lambda > \mu$, the expression for the case with rent is the relevant one for small values of h.

The wage cost function is decreasing in h with a kink at

$$h_c = \frac{1}{2}(\lambda - \mu)\frac{c'(e)}{u_A + c(e)} \tag{15}$$

where both the rent and no-rent expressions are equal. The intuition is that better information reduces the wage spread required to induce a given effort level. In the rent case, this translates directly into lower wage costs. In the region with positive rent, a smaller wage spread means a smaller inequality premium, so that greater precision again leads to lower wage costs.

Wage costs are also convex in h. That is, the marginal benefit from greater precision, $-W_h(e, h)$, is decreasing in h. In particular,

$$-W_h^-(e, h_c) = \lambda\frac{c'(e)}{2h_c^2} > \mu\frac{c'(e)}{2h_c^2} = -W_h^+(e, h) \tag{16}$$

where the notation refers to the left and right derivatives at $h = h_c$. Convexity with respect to h means that a marginal increase in precision has a greater effect on wage costs the lower the initial precision. In particular, the benefits to the principal from an increase in h are greater when the workers extract rent than under no rent (see figure 2).

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it is easily seen that the density H' is symmetric around zero with variance equal to $2Var(\varepsilon_i)$. Thus, a larger variance in the error terms implies a smaller h (see Lazear, 1995).

Allowing the principal to design the information content of the contest, we now assume the principal can initially commit to some h, incurring a per worker cost $\gamma(h)$ with $\gamma' > 0$ and $\gamma'' \geq 0$. When more resources are invested in the tournament procedure, the outcome of the contest provides better information about potential differences between the contestants' effort. Of course, in equilibrium both contestants choose the same effort level. Nevertheless, precision matters since for a given wage structure it determines the incentives to supply effort. The principal's problem is now to choose the least cost tournament subject to the incentive, participation and limited liability constraints. Building on the foregoing results, the overall cost to the principal is therefore

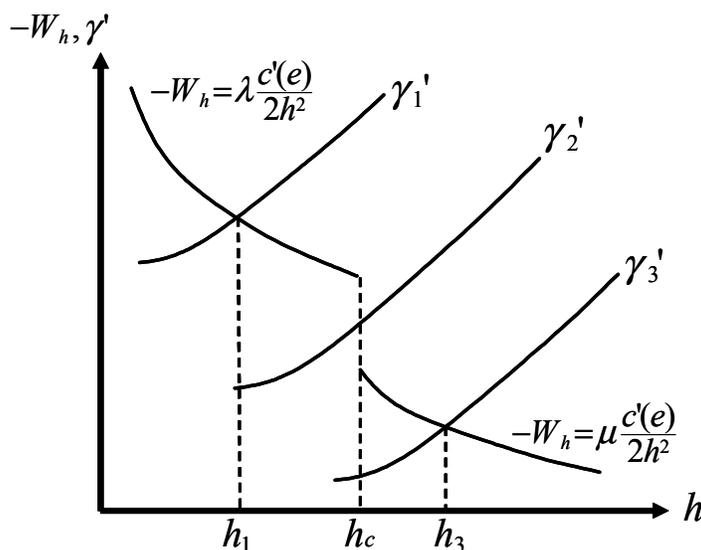$$C^P(e) = \min_h \ W(e, h) + \gamma(h) \tag{17}$$

Figure 2

The determination of the optimal precision is illustrated in figure 2. With precision costs such as γ_1' or γ_3', the solution is an interior one equating marginal precision costs and marginal benefits in terms of wage reduction. For intermediate costs such as γ_2',

we have the corner solution $h^* = h_c$ where both the participation and limited liability constraints are binding. Such situations are now no longer non generic, by contrast with the preceding section where precision was taken as given. Small changes in precision costs around γ_2' do not affect the solution $h^* = h_c$. Changes in the other parameters will shift h_c to the right or to the left, but $h^* = h_c$ remains optimal if such changes are not too large.

We now examine how the externality parameters affect the principal's total costs and the optimal tournament design. When the solution is an interior one, the cost effects are qualitatively the same as in the foregoing section (and follow from the envelope theorem). For corner solutions, total costs are

$$C^P(e) = \tfrac{1}{2}\Delta w + \gamma(h_c) = \lambda\frac{c'(e)}{2h_c} + \gamma(h_c) \tag{18}$$

and the change in costs is the sum of the direct effect on λ and of the effect on h_c as defined in (15). A larger β increases costs, while the effect of α can be shown to be given by

$$\frac{dC^P}{d\alpha} = \frac{h_c}{\lambda - \mu}\left\{\left[W_h^+(e, h_c) + \gamma'(h_c)\right]\frac{\partial\lambda}{\partial\alpha} - \left[W_h^-(e, h_c) + \gamma'(h_c)\right]\frac{\partial\mu}{\partial\alpha}\right\} \tag{19}$$

As can be seen from the figure, the expression in the first square bracket is positive, that in the second is negative. Recalling that λ is decreasing in α while μ is increasing, whether a larger α increases or decreases costs therefore depends on the marginal cost of precision. When this is large, i.e. near the upper benefit curve in the figure, the term in the second square bracket is negligible and consequently $dC^P/d\alpha < 0$. The sign is reversed when $\gamma'(h_c)$ is near the lower benefit curve. The proof of the next proposition and of the foregoing statements is in the appendix.

Proposition 3. *The principal's total costs are increasing in β. They are respectively decreasing in α when precision is relatively costly and increasing when it is relatively cheap.*

Everything else equal, the principal never benefits from more compassionate workers and whether he benefits from more envious ones depends on how costly it is to provide a careful assessment of relative performance. The effects on wage spread, total wage costs and optimal precision are summarized in the tables 1 and 2 (see the appendix for the proofs).

Table 1: effect of increases in α

	rent > 0, $l = 0$	rent $= 0$, $l = 0$	rent $= 0$, $l > 0$
C^P	$-$	$-$ if γ' is large $+$ if γ' is small	$+$
h	$-$	$-$	$+$
Δw	$-$	$+$	$-$
W	$-$	$+$	$+$

Table 2: effect of increases in β

	rent > 0, $l = 0$	rent $= 0$, $l = 0$	rent $= 0$, $l > 0$
C^P	$+$	$+$	$+$
h	$+$	$-$	$+$
Δw	$+$	$+$	$?$
W	$+$	$+$	$+$

In the first column, precision costs are large and workers extract rent. An increase in α then leads to a smaller wage spread, since envy provides more incentives, and also allows the principal to economize on precision costs (an increase in β has the opposite effects). In the last column, precision costs are small and there is no rent. An increase in either α or β then leads the principal to choose a more informative

tournament design in order to reduce the inequality premium. The increase in h following an increase in α unambiguously reduces the wage spread. By contrast, the effect of a larger β is ambiguous. By itself it would reduce incentives and require a larger spread. However, the ensuing increase in h may be such as to allow a smaller wage spread.

The effects for corner solutions shown in the middle column may be non intuitive. Consider an increase in α. If the tournament design remained unchanged, this would reduce the workers' expected utility because of the larger inequality premium. To maintain participation, the principal must therefore increase the expected wage. Both the increase in α and in the wage spread result in too strong incentives. Precision is consequently reduced to reestablish the equilibrium effort at the required level. A similar argument applies for increases in β, noting that the increase in wage spread needed to maintain participation results in too much effort even though β has increased.

6.   Discussion and concluding comments

Remarking that actual pay systems appear more egalitarian than would seem to be predicted by the economics of incentives, Baker et al. (1988) argue that 'economic explanations' should be provided rather than drawing on the notions of 'fairness', 'equity' or 'morale' often stressed by practitioners. Still, supposing that equity considerations are relevant, how far do they go in accounting for wage compression?

In the foregoing model a greater propensity for envy translates into a smaller wage spread between winner and loser (at least in the interior solutions). Two factors are at work. On the one hand more envy increases incentives, thereby allowing a smaller spread to induce the required effort. When workers extract rent, the spread is smaller not because the principal cares about the workers' dislike for inequities, but because their inequity aversion makes smaller spreads feasible. On the other hand greater envy also imposes a cost on workers. When participation constraints are binding,

this increases wage costs. The principal then seeks to reduce the inequality premium through better performance assessment so as to reduce the wage spread even further. By contrast, fairness considerations in the sense of more fair-minded workers do not generally lead to wage compression. The reason is that they reduce incentives and must be compensated by other means. In the model, the principal compensates by more careful performance assessments and generally also by larger spreads.

While these results were derived in the context of tournaments, similar conclusions can be obtained with individualistic pay systems. To illustrate, consider a bonus scheme in a firm with two workers with a propensity for envy (i.e. $\alpha > 0$ and $\beta = 0$). Denote the base wage by l, the bonus by Δw_B and let $p(e)$ be the probability of meeting the fixed standard for earning the bonus, where $p' > 0$ and $p'' < 0$. Assuming that meeting the standard constitutes independent events, a worker exerting effort $\widehat{e}$ when his co-worker exerts e has expected utility

$$l + p(\widehat{e})\Delta w_B - c(\widehat{e}) - p(e)(1 - p(\widehat{e}))\alpha\left[\Delta w_B + c(\widehat{e}) - c(e)\right]$$

The last term is the expected dissatisfaction from turning out worse off than the co-worker.

From the agent's first-order condition, the bonus needed to implement effort e is

$$\Delta w_B = \frac{\lambda_B c'(e)}{p'(e)} \quad \text{where } \lambda_B = \frac{1 + p(e)(1 - p(e))\alpha}{1 + p(e)\alpha} \tag{20}$$

Wage costs per worker are $W(e) = l + p(e)\Delta w$. When workers earn rent, $l = 0$ and wage costs equal the expected bonus. In no-rent solutions,

$$W_B(e) = u_A + c(e) + p(e)(1 - p(e))\alpha\Delta w_B \tag{21}$$

where the last term is the inequality premium.[9] It is easily checked that the bonus and therefore the potential for wage inequality is decreasing in α, while the inequality

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[9]Disadvantageous inequality is experienced when only the co-worker gets the bonus. In equilibrium this occurs with probability $p(e)(1 - p(e))$, hence the expression for the inequality premium in (21).

premium is increasing. The effects of a greater propensity for envy is thus the same as in the tournament model. That is, the principal would like to employ more envious workers when rent needs to be paid out and less envious ones otherwise.[10]

Suppose now the principal can use either a tournament or a bonus scheme. How do these compare given inequality aversion? In a tournament the outcome is always unequal. Does this mean that tournaments are therefore characterized by a greater inequality premium? If so, tournaments would be disadvantageous when participation constraints are binding. Conversely, if tournaments give more scope to envy as a motivator, they could have lower wage costs when workers earn rent.

For the sake of comparison, assume the probability of meeting the standard under the bonus scheme satisfies

$$\frac{p'(e)}{p(e)} = \left.\frac{\partial \log H(\widehat{e} - e)}{\partial \widehat{e}}\right|_{\widehat{e}=e} \equiv 2h \tag{22}$$

That is, the probability of good performance is equally sensitive to effort under either scheme. In the bonus system we now have

$$p(e)\Delta w_B = \frac{\lambda_B c'(e)}{2h} \quad \text{where } \lambda_B = \frac{1 + p(e)(1 - p(e))\alpha}{1 + p(e)\alpha} \tag{23}$$

This must be compared to the expected prize difference under the tournament. From the preceding sections (letting $\beta = 0$) this is given by

$$\tfrac{1}{2}\Delta w_T = \frac{\lambda_T c'(e)}{2h} \quad \text{where } \lambda_T = \frac{1 + \tfrac{1}{2}\alpha}{1 + \alpha} \tag{24}$$

Finally, in no-rent solutions, wages costs under the tournament are

$$W_T(e) = u_A + c(e) + \tfrac{1}{2}\alpha\Delta w_T \tag{25}$$

which must be compared to (21) for the bonus system.

Obviously, both schemes have the same wage costs if there is no envy (the inequality premia disappear and $\lambda_B = \lambda_T = 1$). But what if workers are inequity averse?

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[10]The effects are similar if the information structure on which the bonus scheme is based is made endogenous.

Comparing the inequality premia in (21) and (25), it is clear that $p(e)(1 - p(e)) < \frac{1}{2}$, i.e. the probability of experiencing disadvantageous inequity is always smaller with the bonus system. However, this could be more than compensated by Δw_B being much larger than Δw_T. As it turns out, one can indeed show that the inequality premium is always smaller with the bonus system than with the tournament. Thus, an individualistic bonus is more advantageous when the required effort can be implemented without paying out rent.

Whether a tournament should be used when workers earn rent is less clear cut. A tournament has lower wage costs when $\lambda_T < \lambda_B$. The latter holds only if the probability $p(e)$ characterizing the bonus scheme is not too large[11]. The intuition is as follows. In the tournament the marginal benefit from more effort is proportional to $(1 + \alpha)$, in the bonus system it is proportional to $(1 + p(e)\alpha)$. The reason is that, if a worker loses the tournament, he experiences disadvantageous inequality with certainty. By contrast, if he does not get the bonus, disadvantageous inequality is experienced only with probability $p(e)$, which is the probability that the co-worker independently gets the bonus. Thus, the marginal benefit from more effort is greater in the tournament, although the discrepancy is smaller the larger the probability $p(e)$. On the other hand, envy also affects the disutility of effort. As discussed in the preceding sections, the disutility of effort is increased by the expected frustration from being outperformed. In the tournament, the marginal disutility of effort is $(1 + \frac{1}{2}\alpha)c'(e)$. In the bonus scheme, it is $[1 + p(e)(1 - p(e))\alpha]c'(e)$. This is smaller than in the tournament and the more so when the probability of getting the bonus is large. Overall, for $p(e)$ sufficiently large, the bonus scheme provides more incentives per dollar of expected bonus and is therefore cheaper when workers earn rent.

Several other issues could be examined. For instance, Lazear (1989) showed that wage compression is useful in reducing uncooperative behavior when rewards are

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[11]The condition is always satisfied if $p(e)$ is not greater than one half. For reasonable values of α, say less than unity, the condition holds if $p(e)$ is less than two thirds.

based on relative comparisons and workers can adversely affect each other's performance. If workers are inequity averse, sabotage could be a problem even in purely individualistic bonus schemes and could lead to wage compression in such schemes as well.[12] Another possible extension is to consider the merit of group bonuses under inequity aversion. There is no inequality premium if a bonus is paid only when workers perform well as a group. This is presumably optimal if it can be achieved without paying rent. By contrast, when rent cannot be avoided, an individual bonus scheme or a tournament could possibly lead to lower wages because it relies more on envy as a motivator.

<center>APPENDIX</center>

Proof of proposition 3: We limit the proof to the corner solution where both constraints are binding. From (18),

$$\frac{dC^P}{d\theta} = \frac{\lambda_\theta c'(e)}{2h_c} + \left[\gamma'(h_c) - \frac{\lambda c'(e)}{2h_c^2}\right]\frac{\partial h_c}{\partial \theta}, \quad \theta = \alpha, \beta \tag{26}$$

where λ_θ denotes the derivative with respect to θ. From (15),

$$\frac{\partial h_c}{\partial \theta} = \frac{1}{2}\left(\frac{c'(e)}{u_A + c(e)}\right)(\lambda_\theta - \mu_\theta) \tag{27}$$

The sign is negative for changes in α since λ_α and μ_α are respectively negative and positive. For changes in β, both λ_β and μ_β are positive. However, under the assumption $\alpha > |\beta|$,

$$\lambda_\beta - \mu_\beta = \frac{1}{1+\alpha-\beta}\left[\frac{1}{2}\left(\frac{1-\alpha-\beta}{1+\alpha-\beta}\right) - \left(1 + \frac{\alpha-\beta}{2}\right)\right] < 0 \tag{28}$$

Noting that the expression in brackets in (26) is negative, C^P is increasing in β. For changes in α, substitute for $\partial h_c/\partial \alpha$ from (27) and again from (15) to obtain

$$\begin{aligned}\frac{dC^P}{d\alpha} &= \frac{\lambda_\alpha c'(e)}{2h_c} + \left[\gamma'(h_c) - \frac{\lambda c'(e)}{2h_c^2}\right]\frac{1}{2}\left(\frac{c'(e)}{u_A + c(e)}\right)(\lambda_\alpha - \mu_\alpha) \\ &= \frac{\lambda_\alpha c'(e)}{2h_c} + \left[\gamma'(h_c) - \frac{\lambda c'(e)}{2h_c^2}\right]\left(\frac{h_c}{\lambda - \mu}\right)(\lambda_\alpha - \mu_\alpha)\end{aligned} \tag{29}$$

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[12]See Mui (1995) for an interesting analysis of sabotaging behavior due to envy.

This leads to $dC^P/d\alpha$ as written in the text. ∎

Tables 1 and 2: For corner solutions, h_c is decreasing in both α and β as shown in the proof of proposition 3. For interior solutions the effects on h are obvious from figure 2. The wage spread is given by

$$\Delta w = \frac{\lambda c'(e)}{h} \tag{30}$$

When there is positive rent, the first-order condition can be rewritten as

$$\frac{\lambda c'(e)}{h} = 2h\gamma'(h) \tag{31}$$

The effects on Δw then follow directly from the effects on h. Using (30), Δw decreases with α in the no rent interior solution since λ_α is negative and h increases; the effect of a change in β is ambiguous. For corner solutions, Δw increase with β since λ_β is positive and h_c decreases. For changes in α, substituting from (27) and (15), we have

$$
\begin{aligned}
\frac{\partial \Delta w}{\partial \alpha} &= \frac{\lambda_\alpha c'(e)}{h_c} - \frac{\lambda c'(e)}{h_c^2}\frac{\partial h_c}{\partial \theta} \\
&= \frac{c'(e)\lambda^2}{h_c(\lambda - \mu)} > 0
\end{aligned}
\tag{32}
$$

In the case with rent or in the no-rent corner solution, $W = \Delta w/2$ and the effects on W follow from those on Δw. In the no-rent interior solution,

$$W = u_A + c(e) + \frac{\mu c'(e)}{2h} \tag{33}$$

Noting that the first-order condition can now be rewritten as

$$\frac{\mu c'(e)}{2h} = h\gamma'(h) \tag{34}$$

the effects on W follow directly from the changes in h. ∎

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