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Asset Returns and State-Dependent Risk Preferences

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Abstract: We propose a consumption-based capital asset pricing model in which the representative agent's preferences display state-dependent risk aversion. We obtain a valuation equation in which the vector of excess returns on equity includes both consumption risk as well as the risk associated with variations in preferences. We develop a simple model that can be estimated without specifying the functional form linking risk aversion with state variables. Our estimates are based on Markov chain Monte Carlo estimation of exact discrete-time parameterizations for linear diffusion processes. Since consumption risk is not forced to account for the entire risk premium, our results contrast sharply with estimates from models in which risk aversion is state-independent. We find that relaxing fixed risk preferences yields estimates for relative risk aversion that are (i) reasonable by usual standards, (ii) correlated with both consumption and returns and (iii) indicative of an additional preference risk of holding the assets.

Keywords: Asset pricing models, Bayesian analysis, continuous-time econometric models, data augmentation, equity premium puzzle, Markov chain Monte Carlo, risk aversion, state-dependent preferences, wealth

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1 Introduction

The consumption-based capital asset pricing model (C-CAPM) of Lucas (1978) and Breeden (1979) predicts a linear relationship between risk and return for risky assets. Under the assumption of separable, iso-elastic utility, the quantity of risk is measured by the consumption covariance, and the price of this risk is given by the Arrow-Pratt coefficient of relative risk aversion. Heuristically, an asset which pays low returns when consumption is also low (and marginal utility is high) is considered risky. A positive mean excess return is required to induce a risk-averse investor to hold it. Other things being equal, a higher level of risk aversion for the representative investor results in a higher equilibrium mean excess return. Although theoretically elegant, and intuitive, the empirical performance of this model has been found to be disappointing (Kocherlakota 1996, Campbell 2000, provide excellent surveys).

This paper attempts to resolve the empirical anomalies of the C-CAPM by replacing *un-conditional* iso-elastic utility by *conditional*, or state-dependent iso-elasticity. We derive a pricing equation where an additional preferences risk supplements the low observed consumption risk in justifying the high observed premia on equity. This second risk is related to the coincidence of low returns and high marginal utility obtained through changes in risk aversion. As the contribution of this second risk is found to be important, the estimated price of consumption risk – i.e. risk aversion – is much lower, thus addressing the *equity premium* puzzle. Second, this additional source of inter-temporal marginal rate of substitution (IMRS) risk justifies a larger precautionary demand for the risk-less asset, thereby potentially solving for the *low risk-free rate* puzzle. Finally, we show that the cyclicity of the prices of risks implied by the model has the potential to solve predictability puzzle.

State-contingent preferences have been used successfully to explain behavior characterized by apparently excessive, or changing risk aversion (Drèze and Rustichini 2000, provides a representation theorem for state-dependent utility, and a survey of applications). Robson (1992) reproduces the ‘concave-convex-concave’ behavior of Friedman and Savage (1948) by incorporating preference over status to the instantaneous utility function. When status is allowed to depend on the distribution of wealth, richer curvatures may be obtained. Karni, (1983, 1987) introduces state dependency to rationalize the purchase of flight insurance by agents already holding life insurance. Since implausibly high (and potentially changing) aversion to risk is also

found with standard asset pricing models, it seems reasonable to ask whether state-contingent preferences may be helpful in explaining the high returns on risky assets.

Early applications of state-dependent preferences to asset pricing are presented in Merton (1973) and in Cox, Ingersoll and Ross (1985). Both consider a representative agent whose current utility is affected by a vector of exogenous state variables. Although the particular functional form relating states and utility is left implicit, they use Itô processes to characterize the states, and analyze the equilibrium pricing implications. They obtain a $k + 1$ -factor linear premium where the individual risks are given by the covariances of returns with the return on total wealth (i.e. the market), and the k state variables that condition the agent's utility. Under state independence, and/or if the states are uncorrelated with returns, the model simplifies to the standard consumption covariance C-CAPM. With state-dependencies, these additional covariances represent important sources of IMRS risk that could justify larger premia.

More recently, Melino and Yiang (2001) explain why state-dependent preferences have the potential to resolve the equity premium puzzle. In a two-state Mehra and Prescott (1985) world, they show that the stochastic discount factor (SDF) should be (i) highly sensitive to the current state, and (ii) consistent with a counter-cyclical pattern to risk aversion, in order to match the first two moments of returns. Since observed consumption is relatively smooth and state-insensitive, state-contingent risk aversion appears to be a natural candidate towards achieving that aim. Along these lines, observing that mean equity premia increase during recessions, Campbell and Cochrane (1999) consider a time-non-separable Hyperbolic Absolute Risk Aversion (HARA) specification in which the subsistence level is time-varying, a parameterization which can also be associated with a 'slow-moving' habit. As consumption falls toward subsistence during downturns, relative risk aversion increases, thus implying counter-cyclical attitudes toward risk. Building from Robson (1992), Bakshi and Chen (1996) also allow for status preference in the VNM utility function. To the extent that wealth-dependent self-perception of status influences marginal utility of consumption, the equilibrium relative risk aversion is a decreasing function of the individual's wealth, and therefore also displays a counter-cyclical pattern.

Gordon and St-Amour (2000) discuss how state-dependent curvature leads to rotations of the marginal utility schedule, which increase risks to the IMRS. If initial consumption is low and risk aversion is counter-

cyclical, an unanticipated fall in consumption raises risk aversion and translates into a larger increase in marginal utility than under fixed preferences. These additional sources of risks affect the agent’s portfolio decisions, and consequently the equilibrium returns on assets. As in Cox et al. (1985), excess returns then incorporate a second risk reflecting co-movements in the state – and thus in risk aversion – and returns, reducing the emphasis placed on consumption risk in explaining the high premia. In the special case in which risk aversion and asset returns are not correlated, the model simplifies to one similar to that studied by Chou, Engle and Kane (1992). The difference is not innocuous; we shall see that this preference risk plays a substantial role in explaining observed premia.¹

We choose to address the issue of state-dependencies from a somewhat more general perspective compared to the previous literature: instead of specifying a functional form relating risk aversion to proxies for the state of the world, we model risk aversion as a latent variable. Nonetheless, our framework imposes more structure compared to unrestricted latent-variable models of conditional mean returns.² Our parameterization of conditional iso-elasticity is both parsimonious and general: the concavity index is assumed to follow an Itô process. This assumption is useful as the joint process of curvature, log consumption and cumulative excess returns can be written as a multivariate arithmetic Brownian motion. One advantage of this setup is that exact likelihood representations exist, and can be used to control for time aggregation, rather than using discrete-time approximation. We use Bayesian Markov chain Monte Carlo (MCMC) and data augmentation techniques to estimate the state-dependent model using excess returns data.

The main findings of this study are the posterior moments for time-varying risk aversion. These estimates are (i) moderately volatile, (ii) correlated with returns and consumption and (iii) well within the range of values that many consider to be reasonable (e.g. between 0 and 10). We find that the magnitude of the additional concavity risk is much larger than that implied by consumption-returns covariances. Furthermore, the estimated contribution of this risk is positive, so that the model can justify the high observed premia

¹Others have been more critical in the application of state-dependent preferences to asset pricing puzzles. Danthine, Donaldson, Giannikos and Guirguis (2002) argue that, although able to match the observed equity premium and HJ bounds, the state-dependent model produces an excessive variance for equity and a predicted risk-free rate that remains too high.

²See Genotte (1986) for a separation result between estimation and optimization when conditional returns are time-varying, but not observable. Brandt and Kang (2001) estimate a latent-variable model of conditional mean returns and volatility. They find strong support for the presence of a counter-cyclical pattern in excess returns. Our model restricts conditional means to be defined by a C-CAPM, and focuses only on first moments. Our results are also consistent with a strong counter-cyclical pattern.

on risky assets without requiring high levels of risk aversion. Our results also suggest that risk aversion is negatively correlated with unanticipated consumption shocks as well as with other pro-cyclical business conditions indicators. This is consistent with a counter-cyclical pattern to risk aversion.

The paper is organized as follows. After the model is outlined in Section 2, our next objective is to identify the time series path of risk aversion indices consistent with post-war monthly US data. In Section 3, we estimate the state-dependent model by means of data-augmentation techniques, with results discussed in Section 4. Finally, a conclusion reviews the main findings, and offers suggestions for future research.

2 Theoretical Framework

As in most empirical asset pricing studies, we focus on a pure exchange economy populated by a large number of homogeneous agents, with a single perishable consumption good, and perfect markets. This framework is convenient since it simplifies the computation of equilibrium quantities. Agents who are identically endowed, and who have identical preferences do not trade, and consume their endowment. Equilibrium prices can then be characterized through the Euler equations for an interior optimum, evaluated at the aggregate consumption levels. This section first describes the environment, followed by a discussion of the state-dependent preferences of a representative agent. We subsequently discuss the implications for equilibrium returns.

2.1 Economic environment

2.1.1 Uncertainty and opportunities

Our setup is entirely standard with the exception of the instantaneous utility function. For completeness, we characterize the economic environment closely following the description in Duffie (1992). We assume that the stochastic environment is described by a standard independent Brownian motion $\mathbf{B} \in \mathbb{R}^d$ on a probability space (Ω, \mathcal{F}, P) , with filtration $\mathbb{F} = \{\mathcal{F}_t : t \geq 0\}$ and infinite time horizon. The investment set consists of N securities, in zero net supply, with (cum-dividends) prices $\mathbf{X} \in \mathbb{R}^N$, which are assumed to be adapted Itô processes.

Next, consider the trading strategy $\boldsymbol{\alpha} \in \mathbb{R}^N$, consisting of the values invested in the various securities. Denote by $\mathcal{H}^2(\mathbf{X})$ the set of trading strategies satisfying the integrability constraint $E(\int_0^t \boldsymbol{\alpha}_s^2 ds) < \infty$ for $t > 0$. Moreover, let C and e denote the consumption and endowment processes respectively, and let \mathcal{L}_+^2 denote the non-negative set of Itô processes also satisfying the corresponding integrability constraint e.g. $E(\int_0^t C_s^2 ds) < \infty$ for $t > 0$. The trading strategy $\boldsymbol{\alpha}$ finances the consumption process $C - e$ if:

$$\boldsymbol{\alpha}'_t \mathbf{X}_t = \int_0^t \boldsymbol{\alpha}'_s d\mathbf{X}_s - \int_0^t (C_s - e_s) ds \geq 0, \forall t. \quad (2.1)$$

Then, the choice set of a representative agent is given by:

$$\Lambda = \{(C, \alpha) \in \mathcal{L}_+^2 \times \mathcal{H}^2(\mathbf{X}) : \alpha \text{ finances } C - e\}. \quad (2.2)$$

2.1.2 Preferences

Next, we assume that the representative agent has separable VNM preferences given by:

$$U(C, \gamma) = E \int_0^\infty u(C_t, \gamma_t, t) dt \quad (2.3)$$

where γ_t is a non-negative time-varying curvature index. We make the following assumption.

Assumption 1 $\gamma_t \in \mathcal{L}_+^2$ is an exogenous adapted Itô process.

The agent's problem is therefore:

$$\sup_{(C, \alpha) \in \Lambda} U(C, \gamma). \quad (2.4)$$

We now characterize further the agent's preferences. More specifically, we assume that within-period utility is *conditionally iso-elastic*, i.e.:

$$u(C_t, \gamma_t, t) = \exp(-\delta t) \frac{\Theta}{(1 - \gamma_t)} \left(\frac{C_t}{\Theta} \right)^{1 - \gamma_t}, \gamma_t > 0, \forall t, \quad (2.5)$$

where $\delta > 0$ is a constant subjective discount rate and $\Theta > 0$ is a constant subjective scaling factor. Since, conditional on the realization of an event \mathbf{B}_t , γ_t is also the Arrow-Pratt measure of relative risk aversion for a-temporal risk, we refer to the model in (2.3) and (2.5) as *state-dependent risk aversion* (SRA) preferences. Note that since expected utility is unique up to an affine transformation, *unconditional* iso-elastic (CRRA) preferences are obtained by imposing that γ_t , is a constant $\forall t$.

Preferences (2.5) are a special case of Merton (1973) and Cox et al. (1985) who consider a model where instantaneous utility is an implicit function of the state of the world. These state variables are taken as having an impact on production – and thus on equilibrium returns – but can also include “... state variables that do not affect production opportunities but are nevertheless of interest to individuals.” (Cox et al. 1985,

p. 366). In particular, the state could be related to individual decisions, such as past consumption, or accumulated wealth. Assumption 1 rules out such cases, with the preference state being unaffected by the agent's choices. Merton (1990) also distinguishes between *systematic*, i.e. states that are correlated with returns, and *idiosyncratic* state variables, i.e. states that are uncorrelated with returns. Our setup implicitly allows for both systematic and idiosyncratic states.

Utility (2.5) is a state-dependent case of the Hyperbolic Absolute Risk Aversion (HARA) utility function

$$u(C, t) = \exp(-\delta t) \frac{\Theta}{(1 - \gamma)} \left(\frac{C}{\Theta} - \eta \right)^{1-\gamma},$$

where η is a bliss parameter (Merton 1971, Rubinstein 1974). In our framework, we impose $\eta = 0$, and allow for a state-contingent concavity index γ_t , while we remain agnostic concerning the relevant state. Alternatively, Campbell and Cochrane (1999) consider the case where concavity γ is constant, while the bliss parameter η_t is a function of the state that is related to aggregate consumption. Bakshi and Chen (1996) instead impose that $\eta = 0$, and γ is a constant, while they allow for a state-dependent subjective metric Θ_t , where the state is associated to total wealth.

The instantaneous utility function (2.5) captures the effects of pro- and counter-cyclical risk aversion on marginal utility risk. As outlined in Gordon and St-Amour (2000), changes in the curvature index γ_t cause rotations of the marginal utility schedule about the pivot point $C_t = \Theta$. Hence, the level of consumption with respect to the subjective scaling parameter Θ determines how a change in risk aversion affects marginal utility. To see this, consider the case of an unanticipated decline in consumption resulting in an increase in marginal utility. For the case of low consumption ($C_t < \Theta$), counter-cyclical risk aversion implies an increase in risk aversion, causing a clockwise rotation in the marginal utility schedule. As a result, marginal utility increases more than in the state-independent case. Conversely, for the case of high consumption ($C_t > \Theta$), pro-cyclical risk aversion implies a decrease in risk aversion, causing a counter-clockwise rotation in the marginal utility schedule. Again, marginal utility increases more than in the standard case. Finally, when consumption levels are close to the subjective scaling factor ($C_t \approx \Theta$), the SRA effect is minimized, i.e. fluctuations in curvature cause only limited movements in the marginal utility schedule.

An important feature of the SRA framework is that it is flexible enough to incorporate either counter- or pro-cyclical risk aversion to have an effect on the the volatility of marginal utility. In what follows, we let this issue be determined empirically. This additional contributor to marginal utility risk will be shown to have important implications for the pricing of securities.

2.2 Security market equilibrium

A security-market equilibrium for this economy is a collection $\{X; (C, \alpha)\}$ such that (C, α) solves (2.4) and markets clear:

$$\alpha = \mathbf{0}, \text{ and } C - e = 0. \quad (2.6)$$

We now derive the equilibrium risk premium for risky assets, along with the risk-free rate consistent with SRA.

Under the maintained assumption that γ_t is exogenous, a standard argument establishes that equilibrium is characterized by a unique, non-negative *state-price deflator* π_t defined by:

$$\pi_t = u_c(e_t, \gamma_t, t), \quad (2.7)$$

where $u_c(e_t, \gamma_t, t)$ is the instantaneous marginal utility of consumption evaluated at equilibrium consumption. This state-price deflator has the property that the deflated security price process $\mathbf{X}\pi$ is a martingale (Duffie 1992, p. 98). Denote by $d\mathbf{R}_t \equiv d\mathbf{X}_t/\mathbf{X}_t$ the return process. Let $\boldsymbol{\mu}_t^z \in \mathbb{R}^k$ and $\boldsymbol{\sigma}_t^z \in \mathbb{R}^{k \times d}$ denote the drift and diffusion on any arbitrary Itô process $\mathbf{z} \in \mathbb{R}^k$. Then, no arbitrage implies that:

$$\boldsymbol{\mu}_t^R - r_t = \frac{-1}{\pi_t} \boldsymbol{\sigma}_t^R \boldsymbol{\sigma}_t^{\pi \top}, \quad (2.8)$$

$$r_t = \frac{-\mu_t^\pi}{\pi_t}, \quad (2.9)$$

where r_t is the risk-free short rate.

Clearly, since the endowment e and the curvature index γ are Itô processes, then so is π . A straightforward application of Itô's lemma in Appendix A reveals the following main results:

Proposition 1 Let $c_t \equiv C_t/\Theta$ be the scaled consumption level. The equity premia and risk-free rate are given by:

$$\mu_t^R - r_t = \gamma_t \sigma_t^R (\sigma_t^{cT}/c_t) + \log(c_t) \sigma_t^R \sigma_t^{\gamma T}; \quad (2.10)$$

$$\begin{aligned} r_t = & \delta + \gamma_t (\mu_t^c/c_t) + \log(c_t) \mu_t^\gamma - 0.5 \gamma_t (\gamma_t + 1) (\sigma_t^c/c_t) (\sigma_t^{cT}/c_t) \\ & + [1 - \gamma_t \log(c_t)] (\sigma_t^c/c_t) \sigma_t^{\gamma T} - 0.5 [\log(c_t)]^2 \sigma_t^\gamma \sigma_t^{\gamma T}. \end{aligned} \quad (2.11)$$

Proof. See Appendix A. ■

Equations (2.7) and (2.8) establish the standard C-CAPM result that the source of risk is the covariance of returns with changes in marginal utility. However, under SRA (2.5), unless C_t is close to Θ (i.e. c_t is close to 1) and/or γ_t is constant, changes in marginal utility are ascribed to changes in γ_t as well as changes in consumption c_t . Since both can co-vary with returns, the risk premium (2.10) captures both sources of IMRS risk.

The price of the consumption risk $\sigma_t^R (\sigma_t^{cT}/c_t)$ is given by the state-dependent risk aversion index γ_t . The price of the preferences risk $\sigma_t^R \sigma_t^{\gamma T}$ is given by the log of weighted consumption c_t . For low (high) consumption, i.e. $c_t < (>)1$, if returns and risk aversion are negatively (positively) correlated, then the contribution of that second risk to the premia is positive. Hence, holding the risky asset entails a larger risk to marginal utility due to the coincidence of low returns, and larger marginal utility obtained through low consumption and high risk aversion. This second risk can in theory rationalize the equity premium puzzle without having to inflate risk aversion to unrealistic levels.

The presence of this second source of risk is common to all models of state-dependent preferences where the state is systematic. Merton (1973) shows that state dependence induces additional hedging portfolios in the agent's demand for assets that supplement the standard mean-variance efficient portfolio.³ As a result, the ratio of asset demands is not preferences-free, and the 2-fund (risk-free + market) separation generalizes to a $(k + 2)$ -funds separation theorem to take into account shifts in the k systematic preference states.

³This result parallels the one obtained under a time-varying investment opportunity set, with state-independent preferences (Merton 1990). See also Wachter (2001) for closed-form solutions under state-independent preferences, when returns display mean reversion, and markets are complete.

Consequently, the security market line extends to a security market hyperplane with excess returns being linear in the excess return on the market, and the k replicating funds.

Similarly, a multi-factor pricing equation is obtained by Cox et al. (1985) in a general equilibrium context whereby the k additional risks are given by the covariances with wealth and the k state variables that condition utility. In both cases, if preferences are state-independent, and/or if the state variables are idiosyncratic (i.e., uncorrelated with returns), the model simplifies to the usual single consumption beta. The same reasoning applies here, except that the incidence of the state variables is completely summarized by the assumption that γ is an Itô process. Hence, only a single additional state-related beta is obtained in the pricing equation.

Note also that SRA has the potential for successfully addressing the other C-CAPM pricing anomalies. In particular, even if the innovations are all conditionally homoscedastic, i.e. $\sigma_t^i = \sigma^i, \forall i, t$, the risk premium will in general be time-varying, through the presence of time-varying prices of consumption $[\gamma_t]$ and preferences $[\log(c_t)]$ risks. For example, if risk aversion is counter-cyclical, periods of low consumption ($c_t < 1$) would witness an increase in the equity premia, which would subsequently be reduced as consumption increases ($c_t > 1$) during recoveries. Moreover, if consumption is low and risk aversion co-varies negatively with consumption, a lower risk-free rate obtains in (2.11). Note also that, regardless of the consumption level, the volatility of risk aversion contributes to reducing the short rate, thereby potentially addressing the low risk-free rate puzzle. Whether SRA actually solves these additional pricing anomalies remains an open question which we leave for further research. The rest of this analysis focuses on the equity premia.

3 Empirical Application

The discussion in Section 2 suggests that the SRA model may offer a potential explanation for the equity premium puzzle. Our aim in this section is to estimate a simple SRA model and see whether or not it can fit observed risk premia with plausible levels of risk aversion.

The risk premium in (2.10) was developed for the case in which the data follow arbitrary Itô processes. More structure must be added on that process in order to apply the model to the data. A convenient feature of the model is the linearity of the risk premium (2.10) in $\log(c_t)$ and γ_t . We adopt an econometric model in which excess returns, consumption, and risk aversion are generated by a multivariate arithmetic Brownian motion process. The restricted form of the model – i.e., the model with (2.10) imposed – will also have this form. This continuous-time joint process is particularly useful, since there exists a closed-form expression for the corresponding likelihood function for time-averaged data. More importantly, this structure allows us to identify and estimate the realized risk aversion process. This section outlines the econometric model, as well as the estimation procedure.

3.1 The Parameterized Model

3.1.1 Multivariate Arithmetic Brownian Motion

Suppose for now that γ_t is an observable process. The theoretical treatment in Section 2 is made operational in three steps. First, we consider the case of a single risky portfolio (i.e. $N = 1$) with *excess* return dR_t^e and cumulative excess return process R_t^e . We then postulate a tri-variate process for excess returns, consumption growth, and changes in risk aversion. Second we assume conditional homoscedasticity, and replace the initial independent Brownian motion \mathbf{B}_t by the correlated process $\tilde{\mathbf{B}}_t \in \mathbb{R}^3$, where we denote by $\tilde{\boldsymbol{\sigma}} \in \mathbb{R}^{3 \times 3}$ the diagonal diffusion matrix, and by $\boldsymbol{\rho} \in \mathbb{R}^{3 \times 3}$ the matrix of correlation coefficients.⁴ Finally, we impose the theoretical restrictions (2.10) on excess returns, but allow for unrestricted processes for $\log(c_t)$ and γ_t . The

⁴Hence, in terms of the previous notation, we have that $\tilde{\boldsymbol{\sigma}}^i \tilde{\boldsymbol{\sigma}}^j \boldsymbol{\rho}^{ij} \equiv \boldsymbol{\sigma}^i \boldsymbol{\sigma}^{j\top}$. The homoscedasticity assumption is required to make use of closed-form likelihood functions for time-averaged data. Second moments under SRA could be modelled under a stochastic volatility framework, exploiting the theoretical restrictions. We plan to pursue this approach in subsequent research.

resulting model is:

$$\begin{aligned}
\begin{pmatrix} dR_t^e \\ d\log(c_t) \\ d\gamma_t \end{pmatrix} &= \left\{ \begin{pmatrix} 0 & \tilde{\sigma}^\gamma \tilde{\sigma}^e \rho^{e\gamma} & \tilde{\sigma}^{\log(c)} \tilde{\sigma}^e \rho^{e\log(c)} \\ a^{\log(c)e} & a^{\log(c)\log(c)} & a^{\log(c)\gamma} \\ a^{\gamma e} & a^{\gamma\log(c)} & a^{\gamma\gamma} \end{pmatrix} \begin{pmatrix} R_t^e \\ \log(c_t) \\ \gamma_t \end{pmatrix} + \begin{pmatrix} 0 \\ b^{\log(c)} \\ b^\gamma \end{pmatrix} \right\} dt \\
&+ \begin{pmatrix} \tilde{\sigma}^e \\ \tilde{\sigma}^{\log(c)} \\ \tilde{\sigma}^\gamma \end{pmatrix} \begin{pmatrix} d\tilde{B}_t^e \\ d\tilde{B}_t^{\log(c)} \\ d\tilde{B}_t^\gamma \end{pmatrix} \tag{3.1}
\end{aligned}$$

or, more compactly as:

$$d\mathbf{Y}_t = \{\mathbf{A}\mathbf{Y}_t + \mathbf{b}\} dt + \tilde{\boldsymbol{\sigma}} d\tilde{\mathbf{B}}_t, \tag{3.2}$$

where $\mathbf{Y}_t \equiv [R_t^e, \log(c_t), \gamma_t]'$, and the other parameters and Brownian motion are implicitly defined.

Since by Itô's lemma $\tilde{\sigma}^{\log(c)} = \tilde{\sigma}^e/c$, these restrictions on \mathbf{A} and \mathbf{b} ensure that the expected risk premium in (3.1) is analogous to (2.10):

$$\begin{aligned}
\mu_t^e &= \log(c_t) \tilde{\sigma}^\gamma \tilde{\sigma}^e \rho^{e\gamma} + \gamma_t \tilde{\sigma}^{\log(c)} \tilde{\sigma}^e \rho^{e\log(c)} \\
&= \log(c_t) \tilde{\sigma}^{e\gamma} + \gamma_t \tilde{\sigma}^{e\log(c)} \\
&= \log(c_t) \boldsymbol{\sigma}_t^R \boldsymbol{\sigma}_t^{\gamma\top} + \gamma_t \boldsymbol{\sigma}_t^R (\boldsymbol{\sigma}_t^{c\top}/c_t) \\
&= \mu_t^R - r_t \tag{3.3}
\end{aligned}$$

The other parameters $\{a^{\log(c)e}, a^{\log(c)\log(c)}, a^{\log(c)\gamma}, a^{\gamma e}, a^{\gamma\log(c)}, a^{\gamma\gamma}\}$ in (3.1) are unrestricted feedback effects, while $\{b^{\log(c)}, b^\gamma\}$ are unrestricted constants. That is, no restrictions are placed on how the cumulative returns, consumption and risk aversion affect changes in consumption and risk aversion. The mean excess return, however, is restricted by the theoretical model. Finally, the standard deviation matrix $\tilde{\boldsymbol{\sigma}}$ is also unrestricted.

An important feature of (3.2) is that it defines a multivariate arithmetic Brownian motion. As is shown next, this characteristic allows us to take into account the temporal aggregation bias using discrete data in the estimation, without resorting to discrete-time approximations to the diffusion process.

3.1.2 The Augmented Likelihood Function

Even if risk aversion were an observable process, we do not have data available in a continuous-time format consistent with the model. In particular, monthly consumption data is an average of continuous flow measures. Fortunately, the simple nature of (3.2) allows us to make use of well-known results in the application of continuous-time models to time-averaged data in order to obtain an expression for the “augmented” likelihood function (i.e., the likelihood function for the case in which risk aversion is observed).

Suppose that \mathbf{Y}_t satisfies (3.2) and define $\mathbf{Y}_t \equiv \int_{t-1}^t \mathbf{Y}_\tau d\tau$ as the time-averaged measure. It can then be shown that the discrete time series process \mathbf{Y} satisfies

$$\mathbf{Y}_t = \lambda \mathbf{Y}_{t-1} + \mathbf{g} + \boldsymbol{\eta}_t, \quad (3.4)$$

where $\lambda \equiv e^{\mathbf{A}}$, $\mathbf{g} \equiv (e^{\mathbf{A}} - \mathbf{I})\mathbf{A}^{-1}\mathbf{b}$, where $e^{\mathbf{A}}$ is the matrix exponential of \mathbf{A} and where the error vector $\boldsymbol{\eta}_t$ is a multivariate MA(1) process (Phillips 1976, Bergstrom 1984, Grossman, Melino and Shiller 1987, Melino 1996). The likelihood function is based on the likelihood function for that VARMA(1,1) model described by (3.4).

Let $\boldsymbol{\gamma} \equiv \{\gamma_t\}_{t=1}^T$ represent the time series vector of time-averaged risk aversion coefficients from a sample of T observations. The initial value γ_0 is treated as a parameter. Let $\boldsymbol{\phi} \equiv \{\mathbf{A}, \mathbf{b}, \boldsymbol{\sigma}, \boldsymbol{\rho}\}$ denote the set of structural parameters. Given values for $\boldsymbol{\eta}_0$, $\boldsymbol{\phi}$, $\boldsymbol{\gamma}$ and γ_0 , we can then evaluate the augmented likelihood function $L(\boldsymbol{\phi}|\boldsymbol{\gamma}, \gamma_0, \boldsymbol{\eta}_0, data)$ according to the procedure outlined in the Appendix B.

3.1.3 Identification Issues

An important feature of the restricted model (3.1) is that it reflects the restrictions necessary to identify the latent risk aversion series. Firstly, by modelling the process as a diffusion, we assume that γ_t is smooth. Secondly, it is well known that, for the generic iso-elastic C-CAPM, positive risk aversion is a sufficient

condition for the Euler equations to characterize the optimum. In our context, interior solutions at t are guaranteed if γ_t is nonnegative. Since we follow the tradition in the asset pricing literature of assuming that the Euler equations are valid throughout the sample, we impose this restriction on the *realized* values for risk aversion. Given the equivalence in the way that our Bayesian estimation algorithm treats both the parameters and the latent variables, this “sample selection” assumption can be interpreted as imposing a prior belief that risk aversion is positive throughout the sample, i.e., that we believe that the data are consistent with the Euler equations. This restriction is incorporated into the likelihood function by including a correction in which negative values of γ_t are ruled out. Thirdly, the SRA restrictions mean that there are no free parameters that can be adjusted so that an arbitrary process $\tilde{\gamma}_t$ can be made to satisfy (3.3). It is perhaps useful to elaborate on this point.

Suppose for now that Θ is known; we will return to this point shortly. We observe consumption and excess returns, so $\log(c_t), \tilde{\sigma}^e, \tilde{\sigma}^{\log(c)}, \rho^{e \log(c)}$ can be identified independently. For a given γ_t , we can obtain the remaining terms $\tilde{\sigma}^\gamma, \rho^{e\gamma}$. Suppose that this process γ_t satisfies (3.3), and consider the linear transformation $\gamma'_t = \phi_0 + \phi_1 \gamma_t$. Note that $\tilde{\sigma}^{\gamma'} = \phi_1 \tilde{\sigma}^\gamma$ whereas the correlation term remains unchanged, so there are no other free parameters that need to be adjusted. If we substitute the new values of γ'_t and $\tilde{\sigma}^{\gamma'}$ into (3.3), it is straightforward to demonstrate that the new risk aversion series can reproduce the expected risk premium for any t only if $\phi_0 = 0$ and $\phi_1 = 1$.

On the other hand, the information contained in excess returns data is not sufficient to identify both Θ and γ_t . In other applications of models of time-varying relative risk aversion (Campbell and Cochrane 1999, Gordon and St-Amour 2000), the risk-free rate is used to identify the scale of the consumption series. Unfortunately, we are not able to incorporate (2.11) into the econometric model. Convenient expressions for the likelihood function of time-averaged data are only available for a limited class of models, and including the quadratic terms in $\log(c_t)$ and γ_t would greatly complicate estimation of the model. In the rounds of estimation below, we provide results using various calibrated values for the subjective scaling factor. We find that varying Θ has the effect of changing estimates for certain parameters. Different values for the scaling metric on the other hand do not appear to affect our estimates for risk aversion.

3.2 Estimation Algorithm

We make use of Bayesian analysis to estimate our model. Much has been written (Zellner 1971, Leamer 1978, Poirier 1988, Poirier 1995) on the theoretical justifications for doing so, and the recent development of Markov chain Monte Carlo techniques has greatly increased the feasibility of Bayesian methods of inference. In the current context, the fact that Bayesian methods are not obliged to make use of asymptotic theory makes them far preferable to the use of classical methods. The reason is that (3.3) implies that expected excess returns will depend on log consumption. If consumption is assumed to grow without bound, and if the processes $\log(c)$ and γ are not co-integrated, then predictions for excess returns could be dominated by the $\log(c_t)\tilde{\sigma}^\gamma\tilde{\sigma}^e\rho^{e\gamma}$ term. If this were the case, classical inference based on large-sample approximations would be problematic.⁵ On the other hand, results from a Bayesian estimation are valid for any finite sample. The empirical results below suggest that our simple diffusion model is a reasonable approximation for our data; the model produces plausible estimates that have no detectable trend.⁶

If the risk aversion series were observed along with the vector $\boldsymbol{\eta}_0$, inferences about $\boldsymbol{\phi}$ would be based on the posterior distribution:

$$\mathcal{P}(\boldsymbol{\phi}|\gamma, \gamma_0, \boldsymbol{\eta}_0, data) = \frac{L(\boldsymbol{\phi}|\gamma, \gamma_0, \boldsymbol{\eta}_0, data)\mathcal{P}(\gamma_0)\mathcal{P}(\boldsymbol{\phi})}{\int L(\boldsymbol{\phi}|\gamma, \boldsymbol{\eta}_0, data)\mathcal{P}(\gamma_0)\mathcal{P}(\boldsymbol{\phi}) d\gamma_0 d\boldsymbol{\phi}}. \quad (3.5)$$

Two practical difficulties are posed by (3.5): γ , γ_0 and $\boldsymbol{\eta}_0$ are not observed, and even if these values were observed, the non-standard form of the likelihood function suggests that evaluating the integral in the denominator of (3.5) looks to be a particularly daunting task. Two recent developments in the Bayesian statistical literature prove to be extremely useful in addressing these problems.

Tanner and Wong (1987) note that in many latent variable models, the estimation of the parameter vector is straightforward if the latent variables were observed; this is the case in the current setting. Their “data augmentation” approach is based on simulating values for the missing data from the model. The augmented data can then be used to estimate the fixed parameters. The applicability of data augmentation techniques

⁵A simple solution to this problem would be to let $\Theta = \Theta_t$, a deterministic function of time, so that $\log(c_t) \equiv \log(C_t/\Theta_t)$ is stationary.

⁶Keeping in mind that the log of real per-capita consumption ranges from 8.6 to 9.4 in our sample, this assumption is not too farfetched.

– and Bayesian methods in general – has become markedly easier with the development of Markov chain Monte Carlo techniques. It is often the case that using MCMC with data augmentation is easier than trying to estimate latent variable models using classical techniques (McCulloch and Rossi 1994, Jacquier, Polson and Rossi 1994).

In this application, the estimation is based on an iterative algorithm in which draws are made according to:

$$\begin{aligned}
\gamma^{(m)} &\sim \mathcal{P}(\gamma|\gamma_0^{(m-1)}, \boldsymbol{\eta}_0^{(m-1)}, \boldsymbol{\phi}^{(m-1)}, data); \\
\boldsymbol{\eta}_0^{(m)} &\sim \mathcal{P}(\boldsymbol{\eta}_0|\gamma^{(m)}, \gamma_0^{(m-1)}, \boldsymbol{\phi}^{(m-1)}, data); \\
\gamma_0^{(m)} &\sim \mathcal{P}(\gamma_0|\gamma^{(m)}, \boldsymbol{\eta}_0^{(m)}, \boldsymbol{\phi}^{(m-1)}, data); \\
\boldsymbol{\phi}^{(m)} &\sim \mathcal{P}(\boldsymbol{\phi}|\gamma^{(m)}, \gamma_0^{(m)}, \boldsymbol{\eta}_0^{(m)}, data).
\end{aligned}
\tag{3.6}$$

Under fairly weak conditions that are known to be satisfied in this application,⁷ the sequence of draws $\{\boldsymbol{\phi}^{(m)}, \gamma^{(m)}, \gamma_0^{(m)}, \boldsymbol{\eta}_0^{(m)}\}$ forms an aperiodic and irreducible Markov chain whose stable distribution is the joint posterior distribution $\mathcal{P}(\boldsymbol{\phi}, \gamma, \gamma_0, \boldsymbol{\eta}_0|data)$ (Gelfand and Smith 1990). Given a sample of N draws from the posterior, we can consistently estimate the posterior moments of the parameters of interest. We make use of data augmentation in the first two steps in (3.6), and in both cases, the simulation is straightforward. Implementation details are outlined in Appendix C.

3.3 Prior Specification

The fixed parameters of (3.2) are only instrumental to our analysis. Our main interest lies in estimating the coefficients of relative risk aversion. Because our focus is on determining what levels of risk aversion are consistent with the data, we prefer to use priors that are sufficiently diffuse so as to allow the form of the posterior distribution to be dominated by the likelihood function.

Since the parameters describe the diffusion process of a set of variables that are fairly smooth, we believe that the absolute values of some of the elements of $\boldsymbol{\phi}$ will be relatively small. In specifying the priors for the unrestricted elements of \mathbf{A} and \mathbf{b} , we use normal priors centered around zero. Prior beliefs about the elements of $\bar{\boldsymbol{\sigma}}$ are also described by a diffuse normal distribution. The prior means are chosen to be consistent

⁷See for instance Roberts and Smith (1994). The algorithm used in this study satisfies the sufficient condition that the full conditional densities used to generate new values for the chain have positive mass everywhere in the admissible region.

with $\tilde{\sigma}^{\log(c)} = 0.01$, $\tilde{\sigma}^e = 0.02$, $\rho^{e \log(c)} = 0.1$, $\rho^{\log(c)\gamma} = -0.5$ and $\rho^{e\gamma} = 0.5$. The choices for the prior means for $\tilde{\sigma}^{\log(c)}$, and $\tilde{\sigma}^e$ are roughly consistent with the levels of observed volatility for consumption growth and for excess returns. We expect that movements in risk aversion will tend to be negatively correlated with consumption growth. From (3.3), the usual CRRA model will provide estimates of γ that are biased upwards if the term $\log(c_t)\tilde{\sigma}^\gamma\tilde{\sigma}^e\rho^{e\gamma}$ is positive. In our data set, $\log(C)$ is always positive, and, in the absence of strong priors on the subjective scaling factor, we may start by setting $\Theta = 1$, so $\log(c)$ is positive. We believe that the estimates for risk aversion generated by the CRRA models are implausibly high, which suggests a positive correlation between excess returns and risk aversion.

In choosing priors for the parameters governing the diffusion process for γ_t , we incorporate our belief that fluctuations in levels of risk aversion will be fairly small. In the two-state model developed in Gordon and St-Amour (2000) it was found that changes in relative risk aversion are small and infrequent. In our main results, we use an inverse-gamma distribution for $(\tilde{\sigma}^\gamma)^2$ that is equivalent to the information contained in a fictitious sample of 100 observations of innovations in risk aversion with a mean of zero and standard deviation of 0.05. Lastly, the mean parameter for the prior distribution for γ_0 is set equal to unity. We believe that these values are reasonably close to the region of the parameter space where the likelihood function has mass. In order to attain local uniformity in this region, the prior standard deviations for parameters other than $(\tilde{\sigma}^\gamma)^2$ are set at the relatively large value of 10. Our results are robust to this choice of prior. In another round of estimation, we used a diffuse prior for the variance of the risk aversion term, centered around 0.05, but with a standard deviation of 10. We obtained results that are qualitatively similar to those in Section 4.

4 Results

4.1 Estimation Details

Data and estimation Our results are based on a standard US monthly data set that has been used extensively in assessing the performance of various asset pricing models. The consumption measure is per-capita expenditures on nondurable goods and services measured in constant 1992 dollars, the risk-free rate is proxied by the real rate of return on 3-month T-Bills, and the S&P500 composite index serves as our risky asset. Our sampling period is 1962:8 to 2000:12, for a total of 461 observations.

A well-known problem of modelling this data set with continuous-time models is that consumption is measured as a time-averaged series, whereas holding-period returns are not. Therefore, to make all series conformable, we follow the procedure outlined in Grossman et al. (1987) and Hansen and Singleton (1996) which involves using the time-averaged returns. In particular, we calculate the holding-period return over the previous month for each day of the month. We then take the average of those monthly returns over the days of the month. Excess returns are obtained by subtracting the risk-free return from the risky return.⁸

Although the Markov chain (3.6) will in principle converge from any starting value and using any proposing distribution for the Metropolis-Hastings algorithm, it is more efficient to make use of a few trial runs to identify the region in which the posterior has mass in order to select the appropriate spread for the proposing distributions. Our results are based on a sample of 2000 draws from the posterior; technical details are described in Appendix C. The numerical standard errors for the posterior means for the parameters ϕ and for γ_t were calculated using the method in Geweke (1992). They are generally around 0.5% of their estimated posterior means.

Presentation of results Our main results for the model (3.1) are presented in Tables 1 and 2 in Appendix D. Panel A of Table 1 presents the estimated unrestricted feedback and constant parameters (\mathbf{A}, \mathbf{b}) in the drift terms. Panel B of Table 1 reports the estimated diffusion ($\tilde{\sigma}^i$) and correlation terms (ρ^{ij}). Table 2 presents the sample moments for the derived risk aversion series (γ_t).

⁸To compute the excess price variable R_t^e we simply accumulated excess returns, from a base observation of 0.

We mentioned earlier that the subjective scaling factor Θ is not identifiable from excess returns alone. We therefore calibrate this parameter to three values: $\log(\Theta) = 0$ (column 1), $\log(\Theta) = 9.06$ (column 2), and $\log(\Theta) = 16.00$ (column 3) and estimate all the remaining parameters. In the first round of estimation, we impose the restriction that the agent's scaled consumption is consumption itself. Since, in our sample, log consumption ranges between 8.63 and 9.43, this implies that consumption is always larger than the subjective metric (i.e. consumption is always 'high' relative to the metric, and $\log(c_t) > 0$ throughout the sample period). In the second round of estimation, we set the subjective metric equal to the unconditional mean value for consumption, i.e. $\Theta = E(C_t)$. As mentioned earlier, since the level of scaled consumption is centered on the pivotal point ($c = 1$) on the marginal utility schedule, the effect of SRA should be minimal. In the third round of estimation, we set the subjective metric to a relatively high value of 16. This implies that consumption is always smaller than the subjective metric (i.e. consumption is always 'low', and $\log(c_t) < 0$ throughout the sample period).

4.2 Estimated parameters

A first observation from the estimated drift parameters in panel A of Table 1 is that the magnitude of instantaneous changes in consumption falls in the level of consumption, but increases in the level of risk aversion. Moreover, this last effect is larger in absolute value compared to the other. Secondly, the magnitude of instantaneous changes in risk aversion falls in the levels of consumption and of risk aversion. Again, the feedback effect of the level of risk aversion is more important compared to that of consumption. We therefore find strong feedback effects between levels of and changes in consumption and risk aversion, with risk aversion levels having particularly strong effects on changes in consumption and risk aversion. These effects are robust to the calibration of the subjective metric Θ . Conversely, we find that the value of the metric has an incidence on the effect of the cumulative returns level on changes in consumption and risk aversion. First, a high returns level reduces anticipated changes in consumption and increases anticipated changes in risk aversion when the SRA effect is larger (i.e. when $\log(\Theta) \neq 9.06$). Hence, the feedback effect of returns on consumption and risk aversion depends on the level of scaled consumption.

Turning to the diffusion parameters in panel B, we find again that most parameters are robust to the choice of Θ , with the exception of the correlations involving returns. In particular, positive innovations in returns tend to be associated with positive innovation in consumption, and negative innovations in risk aversion. Moreover, these effects are re-enforced when the subjective scaling metric increases. This suggests a counter-cyclical pattern to unanticipated changes in risk aversion which is confirmed by the strong negative correlation between innovations in consumption and risk aversion.

4.3 Risk aversion estimates

Panel A of Table 2 presents the sample moments for the time series of γ_t . These results reveal that the risk aversion series is (i) moderate by any usual standard and (ii) moderately volatile. Indeed, the coefficient of variation for risk aversion is only 0.35, compared to 14.60 for excess returns. The SRA model therefore points to a representative agent that is neither excessively risk averse, nor subject to high volatility in his level of risk aversion. Interestingly, the estimation results regarding risk aversion are robust the calibrated value for the subjective metric Θ .

A third feature again confirms counter-cyclical attitudes toward risk. In order to characterize further the ‘state of the world’, we use the experimental indices of Stock and Watson (2003).⁹ Correlations between changes in γ_t and the various indices are presented in panel B. They indicate that increases in risk aversion are associated with falls in the business conditions index and/or increases in the recession index, whether leading or coincident, and whether financial variables are included or not. Again, those results are robust to the calibration of Θ .

This interpretation is confirmed when we look at the levels of risk aversion for $\log(\Theta) = 0$ which are displayed in Figure 1 (thick line, RHS scale), along with the coincident business conditions index (thin line, LHS scale).¹⁰ Indeed, we see that an improvement in the state of the world tends to be associated with a decrease in risk aversion. In addition to being intuitively appealing, the three findings concerning risk aversion have important consequences for asset pricing, an issue to which we now turn.

⁹These continuous indices are available on a monthly basis, as coincident/leading, recession/conditions and with/without financial variables. To avoid potential non-stationarity, we use first-differences for the indices and then compute correlations.

¹⁰Figure 1 plots the deviations from a polynomial deterministic trend in the log of the XCI indicator.

4.4 Implications for asset returns

To summarize our findings thus far, we have estimated a risk aversion index for the SRA model of equity premium that is (i) reasonable, (ii) moderately volatile, and (iii) counter-cyclical. The implication of (i) in particular are that the additional contribution of curvature risk to IMRS risk must be important in order to reproduce the high observed premium on equity.

Table 3 presents the sample moments for the components of predicted premium, as well as those of the observed equity premium. To illustrate the contribution of each sources to IMRS risk, we distinguish between consumption, $\gamma_t \tilde{\sigma}^{e \log(c)}$, and curvature risk terms, $\log(c_t) \tilde{\sigma}^{e \gamma}$, as well as the total predicted excess returns. A first observation is that the calibrated value for Θ has a more important effects on the results. This finding is consistent with our earlier result that the calibration of the subjective metric has an important incidence for feedback parameters as well as correlations between returns on one hand, and consumption and risk aversion on the other, whereas the other parameters remain unaffected. Unsurprisingly, the predicted premia, which involves mainly returns correlations, is similarly dependent on the value of Θ .

Panel A presents the sample means. Recall that when $\log(\Theta) = 9.06$, scaled consumption is centered on the pivot point and the SRA effect is minimized. Indeed, in this case, curvature risk becomes negligible and the predicted premia is only 0.15% of the observed one. Relaxing this restriction increases the SRA effect. In particular, we can see that curvature risk is by far the most important contributor to IMRS risk compared to consumption risk when $\log(\Theta) \neq 9.06$. Consequently, the predicted premium increases substantially. This being said, the additional IMRS risk generated by SRA is insufficient to reproduce the observed premium when $\log(\Theta) = 0$. Nonetheless for $\log(\Theta) = 16$, the predicted premium matches well the observed one. We therefore conclude that the SRA model is able to reproduce the high observed premium on equity at reasonable levels of risk aversion when the subjective metric is calibrated to a high level.

Panel B presents the volatility of the predicted and observed premia. We can see that increasing the subjective metric generally increases the volatility of the predicted premium. However, this volatility remains very low compared to that of the observed premium. This result is a direct consequence of our earlier finding that risk aversion is only moderately volatile. Since neither risk aversion nor consumption are subject to high variance, and because the innovations are assumed to be conditional homoscedastic, the theoretical premium

remains relatively smooth. This suggests that further research should focus on more complex heteroscedastic structures in order to model second moments.

Finally, we analyze the correlation of the predicted and observed excess returns with the state variables in panel C. It appears that SRA is unable to match the pro-cyclical pattern found in observed premia. Indeed, recall that risk aversion was found to be counter-cyclical, especially with respect to coincident business indicators. This translates into a counter-cyclical predicted premia with respect to that same variable. Correlations with other indicators are negligible.

To conclude, our estimation results show that, for high levels of the subjective metric Θ , the SRA model is able to reproduce the high observed mean excess return on equity at reasonable levels of risk aversion, thereby successfully addressing the equity premium puzzle. Although the focus of this study is on equity, it is interesting to ask how the model performs with respect to the risk-free rate. For that purpose, we calibrate the subjective discount rate $\delta = 0.03/12$, a realistic value, and compute the predicted short rate using (2.11). We find that the predicted rate is *too low*, and even negative for $\log(\Theta) = 0$, or 16. However, the theoretical risk-free rate approaches its empirical counterpart (0.0030 vs 0.0017) for $\log(\Theta) = 9.06$. These elements suggest that a joint estimation of risk aversion and the subjective metric using the short rate and the premium might yield realistic results. In a similar context, Gordon and St-Amour (2000) find that a SRA model that uses stock and bond prices to estimate both γ_t and Θ fits the risk-free rate fairly well. We plan to address this issue in future research.

5 Conclusion

The application of state-dependent preference specifications to the C-CAPM is motivated by a long-standing paradox of conventional macro-economic asset pricing models: since asset returns and consumption are weakly correlated, the observed equity premia can only be generated by an unreasonably high prices of consumption risk, i.e. excessive risk aversion. In the SRA framework, observed risk premia can be replicated with plausible levels of relative risk aversion indices. The reason for this is the presence of an additional concavity risk that supplements the usual consumption risk within the valuation equation. Moreover, the price of this risk is a function of log consumption, providing a rationale for time-varying and cyclical excess returns. Finally, the increment in IMRS risk obtained under SRA justifies a low risk-free rate of return through a precautionary savings argument.

Our empirical implementation concentrates on identifying both the characteristics of the stochastic process for risk aversion consistent with US data as well as the predicted values of this process. After specifying a simple linear multivariate Brownian motion, we use data augmentation techniques to provide estimates for their realized values at each data point. In resorting to Bayesian methods, we provide finite-sample results based on the exact likelihood function for time-averaged data. The estimated posterior moments point to a sequence of relative risk aversion indices that are:

- Well within the plausible range;
- Moderately volatile compared to returns;
- Counter-cyclical;
- Able to justify the high observed equity premium when the subjective metric is calibrated to a high value.

Overall, our results indicate that the additional curvature risk associated with the covariance between risk aversion and returns is a strong contributor to the IMRS risk.

Our estimates should be interpreted as an unrestricted reduced form that can be used as a guide in developing other specifications. For example, the finding that innovations in consumption and risk aversion are strongly correlated suggests that both processes could be modeled as functions of a single state variable.

A natural candidate would be wealth: it seems plausible to suppose that unanticipated increases in wealth would increase consumption and reduce risk aversion. Theoretically, incorporating wealth into the current model does not pose any great difficulties (the Euler equations for equilibrium would now include terms that take into account the fact that the agent would now have partial control over the evolution of the state variable), but empirical implementation of the model would be greatly hampered by the lack of reliable high-frequency wealth data. On the other hand, data augmentation approaches similar to that used here may prove to be useful research tools toward that aim.

A Proof of Proposition 1

First, from (2.7) and instantaneous utility (2.5) we have that:

$$\begin{aligned}\pi_t &= e^{-\delta t} c_t^{-\gamma t} \\ &= \pi(\mathbf{Z}_t, t), \text{ where } \mathbf{Z}_t \equiv [c_t, \gamma t]^\top, \text{ and} \\ d\mathbf{Z}_t &= \boldsymbol{\mu}_t^Z dt + \boldsymbol{\sigma}_t^Z d\mathbf{B}_t.\end{aligned}$$

By Itô's lemma, we have that the drift and diffusion of π_t are respectively:

$$\boldsymbol{\mu}_t^\pi = \boldsymbol{\pi}_Z(\mathbf{Z}_t, t) \boldsymbol{\mu}_t^Z + \pi_t(\mathbf{Z}_t, t) + 0.5 \text{tr}[\boldsymbol{\sigma}_t^Z \boldsymbol{\sigma}_t^{Z\top} \boldsymbol{\pi}_{ZZ}(\mathbf{Z}_t, t)], \quad (\text{A.1})$$

$$\boldsymbol{\sigma}_t^\pi = \boldsymbol{\pi}_Z(\mathbf{Z}_t, t) \boldsymbol{\sigma}_t^Z \quad (\text{A.2})$$

where $\boldsymbol{\pi}_Z(\mathbf{Z}_t, t)$ denotes the gradient row vector, $\pi_t(\mathbf{Z}_t, t)$ is the derivative with respect to time, and $\boldsymbol{\pi}_{ZZ}(\mathbf{Z}_t, t)$ is the Hessian matrix. The relevant elements are:

$$\pi_c(\mathbf{Z}_t, t) = \frac{-\gamma t}{c_t} \pi_t, \quad \pi_\gamma(\mathbf{Z}_t, t) = -\log(c_t) \pi_t, \quad \pi_t(\mathbf{Z}_t, t) = -\delta \pi_t; \quad (\text{A.3})$$

and

$$\pi_{cc}(\mathbf{Z}_t, t) = \frac{\gamma t(\gamma t + 1)}{c_t^2} \pi_t, \quad \pi_{c\gamma}(\mathbf{Z}_t, t) = \frac{-[1 - \gamma t \log(c_t)]}{c_t} \pi_t, \quad \pi_{\gamma\gamma}(\mathbf{Z}_t, t) = [\log(c_t)]^2 \pi_t. \quad (\text{A.4})$$

Substitute in (A.1) and (A.2), and use the resulting expressions in (2.8) and (2.9) to obtain the equity premia (2.10) and the risk-free rate (2.9). \blacksquare

B Derivation of the augmented likelihood function

The error vector $\boldsymbol{\eta}_t$ in (3.4) is normally distributed, but these errors are not *iid*. They follow a MA(1) process such that $E[\boldsymbol{\eta}_t \boldsymbol{\eta}_t'] \equiv \boldsymbol{\Omega}_0$, $E[\boldsymbol{\eta}_t \boldsymbol{\eta}_{t-1}'] \equiv \boldsymbol{\Omega}_1$, and $E[\boldsymbol{\eta}_t \boldsymbol{\eta}_{t-s}'] = 0$, for $|s| > 1$. Grossman et al. (1987) provide a straightforward way of calculating $\boldsymbol{\Omega}_0$ and $\boldsymbol{\Omega}_1$ as functions of $\boldsymbol{\phi}$.

It is well known (Hamilton 1994, Chib and Greenberg 1994, Melino 1996) that estimation of the parameters of moving average processes is greatly simplified when the model is characterized in terms of its state-space representation. Our approach is based on the Bayesian procedure outlined by Chib and Greenberg (1994).

Since the error terms are not independent, the joint density of the sample cannot be expressed as the product of the marginal densities for $\boldsymbol{\eta}_t$. However, we can make use of the fact that $\boldsymbol{\eta}_t$ and $\boldsymbol{\eta}_{t-s}$ are uncorrelated for values of $s \geq 2$. Since the errors are normally distributed, the fact that they are uncorrelated means that $\mathcal{P}(\boldsymbol{\eta}_t | \boldsymbol{\eta}_{t-1}, \boldsymbol{\eta}_{t-2}, \dots) = \mathcal{P}(\boldsymbol{\eta}_t | \boldsymbol{\eta}_{t-1})$. These conditional distributions are also normally distributed, with conditional mean $\hat{\boldsymbol{\eta}}_{t|t-1} \equiv \boldsymbol{\Omega}_1 \boldsymbol{\Omega}_0^{-1} \boldsymbol{\eta}_{t-1}$ and with conditional variance $\mathbf{G}^{-1} \equiv \boldsymbol{\Omega}_0 - \boldsymbol{\Omega}_1 \boldsymbol{\Omega}_0^{-1} \boldsymbol{\Omega}_1'$. Consider a sample of T observations generated by (3.4). The conditional distribution of $\boldsymbol{\eta}_1, \boldsymbol{\eta}_2, \dots, \boldsymbol{\eta}_T$ given $\boldsymbol{\eta}_0$ is therefore

$$\mathcal{P}(\boldsymbol{\eta}_1, \boldsymbol{\eta}_2, \dots, \boldsymbol{\eta}_T | \boldsymbol{\eta}_0) = \prod_{t=1}^T \mathcal{P}(\boldsymbol{\eta}_t | \boldsymbol{\eta}_{t-1}). \quad (\text{B.1})$$

A problem is posed in the first period of the sample. Since data for \mathbf{Y}_{-1} are unavailable, there is no way to retrieve $\boldsymbol{\eta}_0$. In order to facilitate the derivation of the likelihood function we suppose that $\boldsymbol{\eta}_0$ is observed.

Given a value for the parameter vector $\boldsymbol{\phi}$, we can compute the reduced-form parameters $\boldsymbol{\lambda}$, \mathbf{g} and \mathbf{G} according to the procedures outlined above. From these parameters, we can then retrieve the sequence $\{\boldsymbol{\eta}_t\}_{t=1}^{T-1}$ from (3.4). These values - along with $\boldsymbol{\eta}_0$ - can then be used to compute the sequence $\{\hat{\boldsymbol{\eta}}_{t|t-1}\}_{t=1}^T$. Define $\boldsymbol{\varepsilon}_t \equiv \mathbf{Y}_t - \boldsymbol{\phi} \mathbf{Y}_{t-1} - \mathbf{g} - \hat{\boldsymbol{\eta}}_{t|t-1}$ and let $\boldsymbol{\gamma} \equiv \{\gamma_t\}_{t=1}^T$ represent the time series vector of time-averaged

risk aversion coefficients. The unconditional joint density for the augmented data set is therefore

$$\mathcal{P}(\text{data}, \gamma | \boldsymbol{\eta}_0, \gamma_0, \boldsymbol{\phi}) = (2\pi)^{-(n+2)T/2} |\mathbf{G}|^{\frac{T}{2}} \exp\{-0.5 \sum_{t=1}^T \boldsymbol{\varepsilon}_t' \mathbf{G} \boldsymbol{\varepsilon}_t\}. \quad (\text{B.2})$$

The sample selection correction associated with imposing positivity for simulated values of γ_t at each data point is taken into account by dividing (B.2) by the joint probability that a realized sequence of γ is positive:

$$\mathcal{P}(\text{data}, \gamma | \boldsymbol{\eta}_0, \gamma_0, \boldsymbol{\phi}, \gamma > 0) = \frac{(2\pi)^{-(n+2)T/2} |\mathbf{G}|^{\frac{T}{2}} \exp\{-0.5 \sum_{t=1}^T \boldsymbol{\varepsilon}_t' \mathbf{G} \boldsymbol{\varepsilon}_t\}}{\prod_{t=1}^T \mathcal{P}(\gamma_t > 0 | \mathbf{Y}_{t-1}, \boldsymbol{\phi})}. \quad (\text{B.3})$$

The augmented likelihood function is simply (B.3) interpreted as a function of the unknown parameters given the augmented data set.

C The MCMC Algorithm

Since (3.4) is linear in γ_t and $\boldsymbol{\eta}_t$, the full conditional distributions for each element of γ and of $\boldsymbol{\eta}_0$ are also normal. Consider the full conditional distribution $\mathcal{P}(\gamma_t | \gamma_{-t}, \boldsymbol{\eta}_0, \boldsymbol{\phi}, \text{data})$, where γ_{-t} denotes the elements of γ other than γ_t . Isolating the contribution of γ_t to the likelihood, we note that $\mathcal{P}(\gamma_t | \gamma_{-t}, \boldsymbol{\eta}_0, \boldsymbol{\phi}, \text{data})$ is proportional to the kernel of a normal density whose mean and variance are given by the usual Kalman filter expressions. The first step of (3.6) therefore reduces to a sequence of draws from a univariate normal distribution.¹¹

The second step of (3.6) is done in a similar manner. We note that we can isolate the contribution to the likelihood of each element of $\boldsymbol{\eta}_0$, and that its full conditional distribution also has the form of a univariate normal. The values of $\boldsymbol{\eta}_0$ are generated using the appropriate normal distributions.¹² Similarly, if the prior distribution for γ_0 is normal, then the full conditional distribution is also normal, with mean and variance given by well-known formulae (Poirier 1995, p. 293).

Given the output of the previous steps, the structural parameters can be dealt with in a straightforward fashion. Since the conditional distribution $\mathcal{P}(\boldsymbol{\phi} | \gamma, \gamma_0, \boldsymbol{\eta}_0, \text{data})$ is non-standard, we use the ‘‘Metropolis-within-Gibbs’’ technique of simulating draws for each element of $\boldsymbol{\phi}$ using the random-walk version of Metropolis-Hastings acceptance-rejection algorithm.¹³ The random walk candidate generating densities were mixtures of two normal distributions, each with mean 0. Candidates had a 90% probability of being drawn from a distribution whose standard deviation was approximately equal to the posterior standard deviation (calibrating these standard deviations required several preliminary rounds of estimation), and a 10% chance of being drawn from a distribution with a standard deviation 10 times as big. Giving the chain regular exposure to ‘extreme’ candidates attenuates (but can never entirely eliminate) the probability that it will get stuck in a ‘bad’ region of the state space. Typical acceptance rates for a given parameter were around 30%. In order to reduce autocorrelations in the chain, the sample of 2500 draws was generated by keeping every fifth draw of chain of 12,500 iterations. The first 500 draws of this sample were discarded, leaving the sample of 2000 draws used in calculating our estimates.

¹¹Kong, Liu and Wong (1994) note that there may be efficiency gains to be had by simulating the γ_t terms in blocks, instead of one at a time. This doesn’t seem to be the case here. Since the risk aversion series does not appear to be strongly autocorrelated (the new estimates have an autocorrelation coefficient of around 0.1) we found that there were essentially no efficiency gains to be had there; when we re-estimated the model using algorithms in which the risk aversion terms are simulated in blocks of $k = 2$ and $k = 3$, the numerical standard errors were more or less the same as the single-move case. It’s also worth noting that reducing the number of risk aversion simulation steps by a factor of k provides no computational gains, either: since we impose the restriction $\gamma_t > 0$, simulating a k -vector from a multivariate truncated normal requires k simulations from a truncated univariate normal.

¹²See Chib and Greenberg (1994) for a detailed treatment.

¹³See Tierney (1994) or Chib and Greenberg (1995) for a description.

D Tables

Table 1: Estimated parameters

parameter	interpretation	$\log(\Theta) = 0$	$\log(\Theta) = 9.06$	$\log(\Theta) = 16$
A. Unrestricted drift parameters				
$a^{\log(c)\log(c)}$	consumption level	-1.00e-4	-0.0012	-0.0089
	on consumption growth	(0.0003)	(2.4e-4)	(1.9e-4)
$a^{\log(c)e}$	cumulative returns	-0.0021	0.0093	-0.0172
	on consumption growth	(0.0060)	(0.0054)	(0.0052)
$a^{\log(c)\gamma}$	RA level	0.5905	0.5209	0.5786
	on consumption growth	(0.0357)	(0.0434)	(0.0488)
$a^{\gamma\log(c)}$	consumption level	-0.0397	-0.0403	-0.0446
	on RA change	(0.0029)	(0.0031)	(0.0038)
$a^{\gamma e}$	cumulative returns	0.0222	-0.0513	0.1158
	on RA change	(0.0310)	(0.0387)	(0.0176)
$a^{\gamma\gamma}$	RA level	-3.7650	-3.9384	-3.4816
	on RA change	(0.6077)	(0.2065)	(0.5040)
$b^{\log(c)}$	consumption	-0.0234	-0.0114	-0.0149
		(0.0056)	(0.0032)	(0.0042)
b^{γ}	RA	0.5194	0.5404	0.5442
		(0.0326)	(0.0148)	(0.0287)
B. Diffusion and correlation parameters				
$\tilde{\sigma}^e$	standard error	0.0430	0.0430	0.0430
	returns	(0.0015)	(0.0015)	(0.0014)
$\rho^{e\log(c)}$	consumption correlation	0.1121	0.1830	0.2837
	with returns	(0.0557)	(0.0733)	(0.1105)
$\rho^{e\gamma}$	RA correlation	0.0260	-0.0972	-0.1283
	with returns	(0.0590)	(0.1060)	(0.0725)
$\tilde{\sigma}^{\log(c)}$	standard error	0.0109	0.0108	0.0104
	consumption	(6.4e-4)	(5.3e-4)	(9.7e-4)
$\rho^{\log(c)\gamma}$	consumption correlation	-0.9532	-0.9430	-0.9502
	with RA	(0.0157)	(0.0157)	(0.0163)
$\tilde{\sigma}^{\gamma}$	standard error	0.0638	0.0624	0.0644
	RA	(0.0058)	(0.0060)	(0.0064)

Note: The estimates are the posterior means of the various features of interest; the posterior standard deviations are in parentheses; $c_t \equiv C_t/\Theta$, where Θ is subjective scaling factor; a, b as well as σ, ρ parameters refer to the parameters in the restricted Brownian motion model (3.1).

Table 2: Sample estimates for risk aversion

parameter	interpretation	$\log(\Theta) = 0$	$\log(\Theta) = 9.06$	$\log(\Theta) = 16$
A. Sample moments				
$E(\gamma_t)$	mean	0.0452	0.0421	0.0497
	risk aversion	(0.0076)	(0.0063)	(0.0095)
$\text{std}(\gamma_t)$	standard error	0.0158	0.0156	0.0189
	risk aversion	(0.0028)	0.0018	(0.0034)
B. Correlation with state variables				
$\rho(\Delta\gamma, \Delta XRI)$	leading recession index	0.0507	0.0751	0.0471
$\rho(\Delta\gamma, \Delta XLI)$	leading business conditions index	-0.0435	-0.0621	-0.0397
$\rho(\Delta\gamma, \Delta XCI)$	coincident business conditions index	-0.1482	-0.1046	-0.1602
$\rho(\Delta\gamma, \Delta XRI - C)$	coincident recession index	0.1268	0.1126	0.1205
$\rho(\Delta\gamma, \Delta XLI - 2)$	lead. bus. cond. ind., no financial series	-0.1245	-0.1202	-0.1039
$\rho(\Delta\gamma, \Delta XRI - 2)$	lead. recess. ind., no financial series	0.0806	0.0947	0.0554

Note: The estimates are the posterior means of the various features of interest; the posterior standard deviations are in parentheses; Θ is subjective scaling factor.

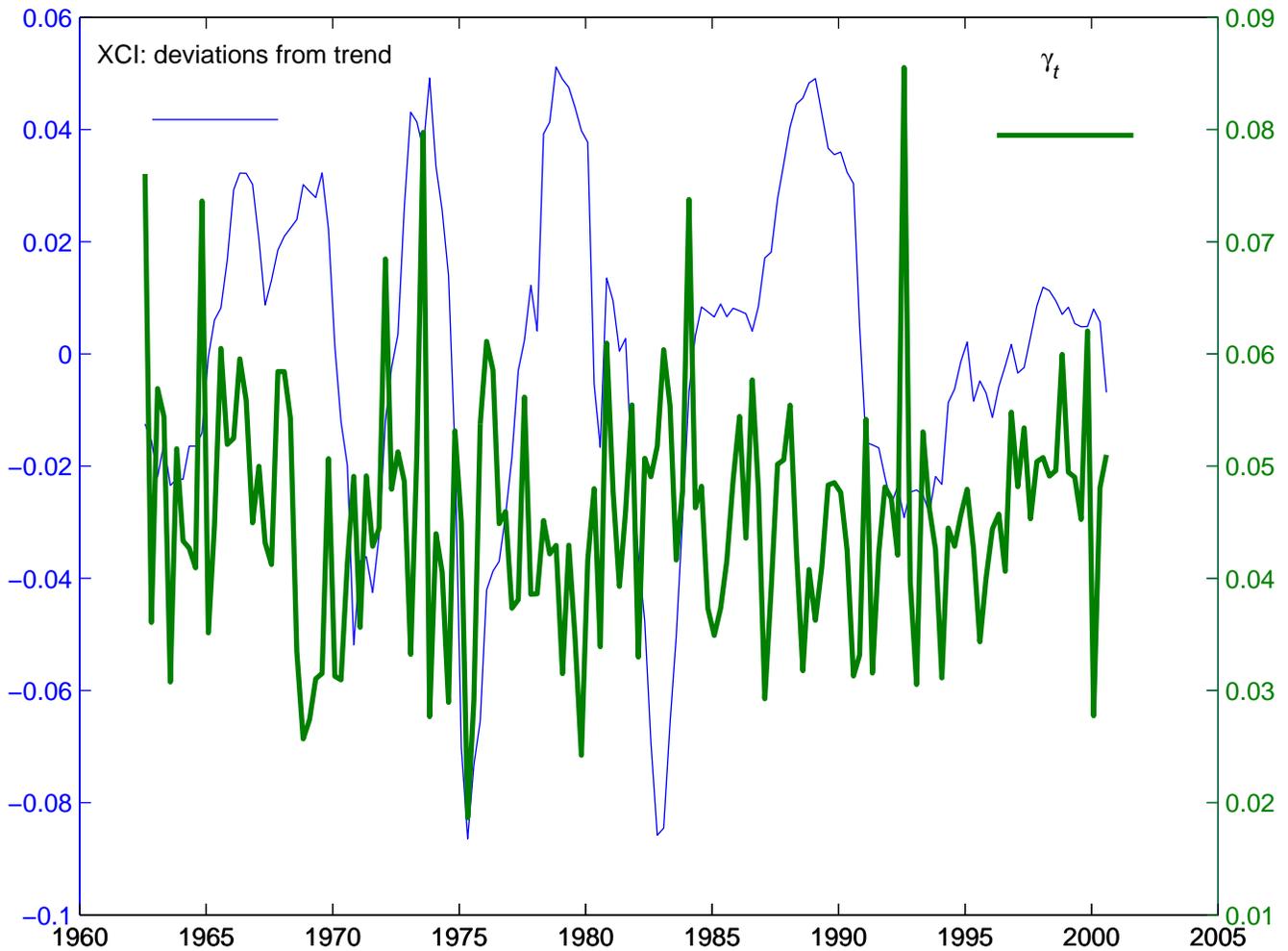
Table 3: Moments of excess returns

	$\log(\Theta) = 0$	$\log(\Theta) = 9.06$	$\log(\Theta) = 16$	data
A. Mean				
consumption risk: $\gamma_t \tilde{\sigma}^{e \log(c)}$	2.4e-6	3.6e-6	6.3e-6	
curvature risk: $\log(c_t) \tilde{\sigma}^{e\gamma}$	6.5e-4	-2.4e-9	2.5e-3	
total	6.5e-4	3.6e-6	2.5e-3	2.4e-3
B. Standard deviations				
consumption risk: $\gamma_t \tilde{\sigma}^{e \log(c)}$	5.5e-7	9.7e-7	1.9e-6	
curvature risk: $\log(c_t) \tilde{\sigma}^{e\gamma}$	1.5e-5	5.5e-5	7.4e-5	
total	1.5e-5	5.5e-5	7.4e-5	3.5e-2
C. Correlation with state variables				
$\rho(R_e, \Delta XRI)$	0.0060	-0.0058	-0.0049	-0.0582
$\rho(R_e, \Delta XLI)$	-0.0167	0.0164	0.0159	0.1163
$\rho(R_e, \Delta XCI)$	0.1186	-0.1275	-0.1216	0.0495
$\rho(R_e, \Delta XRI - C)$	0.0054	0.0008	-0.0006	-0.1114
$\rho(R_e, \Delta XLI - 2)$	-0.0084	0.0039	0.0051	0.0853
$\rho(R_e, \Delta XRI - 2)$	0.0004	0.0012	0.0000	-0.1280

Note: See Table 2 for explanation of state variable indices.

E Figure

Figure 1: ESTIMATES FOR RISK AVERSION AND COINCIDENT INDEX



Note: Quarterly estimates for risk aversion γ_t (thick line, RHS scale) for first model ($\log(\Theta) = 0$), and deviations from deterministic trend in the log of coincident Stock-Watson (XCI) index (thin line, LHS scale).

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