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Decomposing Poverty Changes into Vertical and Horizontal Components

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Abstract: Variations in aggregate poverty indices ca be due to differences in average poverty intensity, to changes in the welfare distances between those poor of initially unequal welfare status, and/or to emerging disparities in welfare among those poor of initially similar welfare status. This note uses a general cost-of-inequality approach that decomposes the total change in poverty into a sum of indices of each of these three components. This decomposition can serve *inter alia* to integrate horizontal and vertical equity criteria in the poverty alleviation assessment of social and economic programs. The use of these measures is briefly illustrated using Tunisian data.

Keywords: Poverty, Vertical Equity, Horizontal Equity, Targeting, Tunisia

JEL Classification: D12, D63, H53, I32, I38

1 Introduction

This note presents a method that decomposes aggregate poverty differences into movements in average poverty intensity and into changes in the vertical and horizontal locations of the poor. Such aggregate poverty differences can be due, for instance, to the effect migration, socio-economic mobility or growth. They can also arise from the impact of a policy or when comparing the impact of two policies. In decomposing such poverty differences, we will account for the role of three summary measures:

1. A measure of the differences in average poverty intensity, which captures by how much the average poverty gap is affected by a distributional change. This measure is distribution-insensitive across the poor. In a policy context, it can be linked to popular targeting-accuracy indicators and related to rates of benefit leakages.

2. A measure of the vertical impact of the change. This captures the extent to which vertical inequality in the distribution of poverty gaps is reduced by a distributional change. For policy purposes, it can serve to assess the respect of the vertical equity (VE) criterion, which demands a search for a reduction in the welfare gaps that separate unequal individuals.

3. A measure of the horizontal impact of the distributional change. In a policy context, this can be linked to the horizontal equity (HE) of the program. The ”classical” definition of HE indeed defines HE as ”the equal treatment of equals” (see Musgrave (1959)), and there is horizontal inequity (HI) when HE is violated.

The paper mainly shows how a simple combination of the above measures can capture the trade-offs as well as the differences between indicators of average poverty intensity and indicators of vertical and horizontal distances. This can be useful for descriptive as well as for policy design purposes.

Section 2 outlines the basic methodology, Section 3 shows how to decompose the total cost of inequality into vertical and horizontal contributions, and Section 4 illustrates briefly the methodology using 1990 Tunisian household data.

2 The basic methodology

2.1 Poverty and inequality

Consider a vector $\mathbf{y} = (y_1, y_2, ..., y_N; n_1, n_2, ..., n_N)$ of living standards y_h (incomes, for short) for a population of $n = \sum_{h=1}^{N} n_h$ individuals. Let the poverty line be denoted as z. Many of the common poverty measures can be expressed in terms of poverty gaps, $g_h(z) = \max(z - y_h, 0)$, with $\mathbf{g}(z)$ the vector of these poverty gaps.[1] An important subset of these measures is the class of the FGT (Foster, Greer and Thorbecke (1984)) additively decomposable indices, which are defined as:

$$P_\alpha(\mathbf{g}(z)) = n^{-1} \sum_{h=1}^{N} n_h g_h(z)^\alpha \tag{1}$$

where α may be considered as a measure of aversion to inequality of poverty gaps.

In the manner of Atkinson (1970) for the measurement of social welfare and inequality, let $\Gamma_\alpha(\mathbf{g}(z))$ be the "equally-distributed equivalent (EDE) poverty gap", *viz*, that poverty gap which, if assigned equally to all individuals, would produce the same poverty measure as that generated by the actual distribution of poverty gaps. Using (1), $\Gamma_\alpha(\mathbf{g}(z))$ is given implicitly for $\alpha > 0$ as

$$\Gamma_\alpha(\mathbf{g}(z)) = P_\alpha(\mathbf{g}(z))^{\frac{1}{\alpha}} \text{ for } \alpha > 0.$$

Note that $\Gamma_1(\mathbf{g}(z))$ is the average poverty gap. For $\alpha > 1$, the more important the difference between $\Gamma_\alpha(\mathbf{g}(z))$ and $\Gamma_1(\mathbf{g}(z))$, the more unequal is the distribution of poverty gaps. A natural measure of the cost of inequality is then given by:

$$C_\alpha(z) = \Gamma_\alpha(\mathbf{g}(z)) - \Gamma_1(\mathbf{g}(z)) \text{ for } \alpha \geq 1. \tag{2}$$

Because $C_\alpha(z)$ is given in *per capita* money-metric terms, it can be compared directly to $\Gamma_1(\mathbf{g}(z))$. By (2), total poverty can be expressed as:

$$\Gamma_\alpha(\mathbf{g}(z)) = \Gamma_1(\mathbf{g}(z)) + C_\alpha(z),\ \alpha \geq 1. \tag{3}$$

Note that it is only when the poverty gaps are equally distributed across the *total* population that the cost of inequality becomes zero.

[1] On this, see for instance Jenkins and Lambert (1997).

2.2 Poverty and targeting

Now consider a distributional change i which leads to an income distribution $\mathbf{y}^i$ with respective y_h^i, $g_h^i(z)$, $\mathbf{g}^i(z)$, and $C_\alpha^i(z)$. Assume that the *per capita* change in income is given by ρ^i. The leakage of that change away from the poor is then given for a change i by

$$L^i(z) = \rho^i - (\Gamma_1(\mathbf{g}(z)) - \Gamma_1(\mathbf{g}^i(z)). \tag{4}$$

The overall poverty impact is given by:

$$E_\alpha^i(z) = \Gamma_\alpha(\mathbf{g}(z)) - \Gamma_\alpha(\mathbf{g}^i(z)). \tag{5}$$

$E_\alpha^i(z)$ can be thought of as a poverty-effectiveness measure of the change i. Using (2), we can rewrite (5) as:

$$E_\alpha^i(z) = \rho - L^i(z) + C_\alpha(z) - C_\alpha^i(z). \tag{6}$$

The poverty effectiveness of the change is thus a function of the average change ρ^i, the leakage to the non-poor $L^i(z)$, and the redistributive impact $C_\alpha(z) - C_\alpha^i(z)$.

3 Horizontal and vertical effects

3.1 Horizontal effects

For any fixed y_h in pre-change $\mathbf{y}$, let $\Omega(y_h)$ denote the group of n_h equals located at point y_h. Let $\gamma_\alpha^i(g_h(z))$ then be the post-change EDE poverty gap at y_h,

$$\gamma_\alpha^i(g_h(z)) = \left(n_h^{-1} \sum_{\Omega(y_h)} g_h^i(z)^\alpha \right)^{1/\alpha}. \tag{7}$$

Using the cost-of-inequality approach developed in Section 2, a natural measure of the local cost of horizontal variability at y_h is then given by:

$$\eta_\alpha^i(g_h(z)) = \gamma_\alpha^i(g_h(z)) - \gamma_1^i(g_h(z)) \geq 0. \tag{8}$$

In a policy context this can be interpreted as a local cost of HI at y_h, generated by post-policy inequality within the members of $\Omega(y_h)$. An obvious next step is to aggregate the $\eta^i_\alpha(g_h(z))$ across the y_h. Using population shares to do this[2], an aggregate index of horizontal variability (and thus of HI) is obtained as:

$$H^i_\alpha(z) = n^{-1}\sum_{h=1}^{N} n_h \eta^i_\alpha(g_h(z)). \tag{9}$$

3.2 Vertical effect

Focus now on the distribution of the local EDE poverty gaps $\gamma^i_\alpha(g_h(z))$. Denote this distribution as $\gamma^i_\alpha(z) = \left(\gamma^i_\alpha\left(g_1(z)\right), ..., \gamma^i_\alpha\left(g_N(z)\right); n_1, ..., n_N\right)$. The cost of inequality with $\gamma^i_\alpha(z)$ is then given by:

$$C^{*i}_\alpha(z) = \Gamma_\alpha(\gamma^i_\alpha(z)) - \Gamma_1(\gamma^i_\alpha(z)). \tag{10}$$

$C^{*i}_\alpha(z)$ can then be interpreted as the cost of inequality of a post-change distribution in which everyone is attributed his group-equivalent poverty gap. The vertical effectiveness (or vertical equity VE) of that change can then be assessed through a comparison of (10) with the cost of inequality in the initial distribution of poverty gaps:

$$V^i_\alpha(z) = C_\alpha(z) - C^{*i}_\alpha(z). \tag{11}$$

3.3 Overall poverty effectiveness

We then have:

Theorem 1 *The poverty effectiveness of a distributional change i is given by*

$$E^i_\alpha(z) = \rho^i - L^i(z) + V^i_\alpha(z) - H^i_\alpha(z). \tag{12}$$

Proof of Theorem 1. See appendix. ■

If we assume identical the *per capita* impact of two distributional changes, 1 and 2, such that $\rho^1 = \rho^2$, and if we denote $\Delta F = F^2 - F^1$, the difference in poverty effectiveness between two distributional changes is given by:

$$\Delta E_\alpha(z) = -\Delta L(z) + \Delta V_\alpha(z) - \Delta H_\alpha(z). \tag{13}$$

Note that the formulation of (13) shows clearly the nature of the trade-off that can emerge between leakage and vertical and horizontal effects. A change

[2] See for instance Blackorby, Donaldson and Auersperg (1981), Musgrave (1990), Lambert and Ramos (1997), Duclos and Lambert (2000), and Bibi (2002).

can dominate another even with a higher leakage and a lower degree of vertical effectiveness if it introduces less horizontal variability. When $\alpha = 1$, however, $V_1^i(z) = H_1^i(z) = 0$, which says that differences in poverty effectiveness depend solely on differences in leakages away from the poor.

4 An application to Tunisia

We illustrate the use of the methodology presented above using a 1990 Tunisian survey, "*Enquête Nationale sur le Budget et la Consommation des Ménages* 1990" (National Household Budget and Expenditure Survey). This household survey is multipurpose and provides information on consumption expenditures for various items as well as extensive socio-demographic information on 7734 households. The main anti-poverty program currently in force in Tunisia is based on the subsidization of food consumption and thus on "commodity targeting".[3] Government expenditures on that program have been substantial throughout the 1980's and the 1990's, amounting to 4.1% of GDP in 1984, 2.9% in 1990, and 2% in 1995. We compare the outcome of this program with that of an alternative one based on regional targeting – involving the same overall budgetary outlay for the government – in the manner of Kanbur (1987).[4] For expositional simplicity, we ignore the extent of deadweight losses under commodity targeting. A real *per capita* poverty line z of 360 *Tunisian Dinars* per year (roughly equal to the often-used US$1-a-day line) is used. As in Duclos and Lambert (2000), we identify the post-policy distribution of pre-policy equals using a non-parametric estimation of the joint distribution of pre-policy and post-policy incomes.

Table 1 shows the estimates of the poverty effectiveness measures following this hypothetical reform. Briefly, the impact of regional targeting of transfers would be more variable horizontally than that of the current system of commodity targeting, as shown here by $H_\alpha(z)$ for $\alpha = 2, 3$. But although the HE violations which would arise with this hypothetical reform would certainly reduce its poverty impact, they would not be considered enough here to offset its higher

[3] Details about this program can be found in Tuck and Lindert (1996).

[4] When the minimization of $P_\alpha(\mathbf{g}(z))$ at the national level is the policymaker's objective, the available budget should be allocated such as to equalize the $P_{\alpha-1}(\mathbf{g}_j(z))$ of each region j to a common value. Our regional targeting scheme thus works as follows. Transfers are first awarded to everyone living in the poorest region such as to equalize the region's $P_{\alpha-1}(\mathbf{g}_j(z))$ to that of the next poorest region. Transfers are then awarded to each person living in these two poorest regions such as to equalize their $P_{\alpha-1}(\mathbf{g}_j(z))$ to that of the third poorest region. This pattern is repeated until the entire available budget is spent.

vertical effect (as shown by $V_\alpha(z)$) and lower rate of leakage (shown by $L(z)$)). Overall, therefore, $E_\alpha(z)$ is larger for regional targeting. This also serves to show how this paper's decomposition methodology can be useful for understanding and optimizing the poverty impact of poverty alleviation schemes.

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5 Appendix

Proof of Theorem 1. Recall that by (10) we have

$$C_\alpha^{*i}(z) = \Gamma_\alpha(\gamma_\alpha^i(z)) - \Gamma_1(\gamma_\alpha^i(z)). \tag{14}$$

Noting that $\Gamma_\alpha(\gamma_\alpha^i(z)) = \Gamma_\alpha(\mathbf{g}^i(z))$ and adding and subtracting $\Gamma_1(\mathbf{g}^i(z))$ on the right-hand side of (14), we find

$$C_\alpha^{*i}(z) = C_\alpha^i(z) - H_\alpha^i(z). \tag{15}$$

Since the VE of change i is given by

$$V_\alpha^i(z) = C_\alpha(z) - C_\alpha^{*i}(z), \tag{16}$$

subtracting $C_\alpha(z)$ from the two sides of (15) and using (16), it follows that

$$C_\alpha(z) - C_\alpha^i(z) = V_\alpha^i(z) - H_\alpha^i(z). \tag{17}$$

The proof of Theorem 1 follows from substituting (17) into (6). ■

Table 1: Poverty effectiveness of two types of targeting in Tunisia (in 1990 Tunisian Dinars)

	Benchmark	Commodity targeting	Regional targeting
ρ^i	0	34.79	34.79
$\Gamma_1(\mathbf{g}^i(z))$	35.64	28.33	23.41
$L^i(z)$	-	27.48	22.56
$E_1^i(z)$	-	7.31	12.23
$\Gamma_2(\mathbf{g}^i(z))$	75.55	65.36	55.19
$H_2^i(z)$	0	0.28	1.53
$V_2^i(z)$	-	3.16	9.65
$E_2^i(z)$	-	10.19	20.35
$\Gamma_3(\mathbf{g}^i(z))$	102.37	91.47	78.13
$H_3^i(z)$	0	0.51	2.70
$V_3^i(z)$	-	4.10	14.71
$E_3^i(z)$	-	10.89	24.23