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Nominal Wage Rigidity as a Nash Equilibrium

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Résumé: Les modèles des microfondements des rigidités nominales montrent qu'en présence de rigidités réelles, les firmes ont une incitation très forte à ajuster leurs prix même si les autres firmes ne le font pas : la rigidité des prix n'est pas un équilibre de Nash à moins que le coût fixe d'ajuster les prix soit trop élevé pour être plausible. Nous montrons que la rigidité des salaires nominaux peut être un équilibre de Nash même sans rigidités réelles et lorsque le coût fixe d'ajuster le salaire nominal est relativement faible. La taille du coût d'ajustement nécessaire pour supporter la rigidité des salaires nominaux décroît au fur et à mesure que l'élasticité de l'offre de travail augmente, mais elle reste très faible pour des valeurs empiriquement plausibles de cette élasticité. La taille nécessaire du coût d'ajustement n'est pas sensible au degré de substituabilité entre les types de travail dans la fonction de production.

Abstract : Models of the microfoundations of nominal price rigidities show that in the absence of real rigidities, individual firms have strong incentives to adjust prices even if other firms do not: price rigidity is not a Nash equilibrium unless the fixed cost of adjusting prices is implausibly high. This paper shows that nominal wage rigidity can be supported as a Nash equilibrium with relatively small adjustment costs and without real rigidities. The size of the necessary adjustment costs decreases labor supply elasticity increases, but is quite small for empirically plausible values of the latter. The minimum adjustment cost is relatively insensitive to the degree of substitutability between types of labor in production.

Keywords: Nominal wage rigidity, Nash equilibrium

JEL Classification: E1, E3

1 Introduction

An important goal of macroeconomic theory is to provide secure microfoundations for the behavioral equations used in general equilibrium models. The behavior of all agents should be based on optimization subject only to tastes and to technological constraints. This goal extends to price and wage-setting behavior. If agents fix prices (or wages) ahead of time and if they do not revise these prices in response to macroeconomic shocks, this should also be an optimal decision based on technological constraints that make revising prices costly.

The goal of completely endogenizing price and wage-setting behavior has been put on hold by the "new neoclassical synthesis" in macroeconomics[1] (henceforth NNS). The NNS approach involves incorporating *exogenous* nominal rigidities into dynamic general equilibrium models. Agents are simply constrained exogenously to set prices or wages in advance and not to revise these prices: only the levels at which prices or wages are optimal. The goal of this research program is to explain, by introducing a plausible degree of exogenous nominal rigidity, the observed rigidity of prices and the highly persistent fluctuations of macroeconomic aggregates in response to monetary shocks.

Recently, Chari, Kehoe and McGrattan (2000) established that nominal *price* rigidities are not sufficient by themselves to explain the observed large and persistent response of output to monetary shocks. In their model, firms that are allowed to adjust their prices make large adjustments that rapidly neutralize the effects of monetary shocks. In earlier work, Ball and Romer (1990) showed that nominal price rigidities cannot be explained as an equilibrium outcome without the presence of large *real rigidities*, which they define as firms' marginal costs being insensitive to aggregate output: in standard neoclassical models with constant returns to scale and intertemporal substitution in labor supply, firms' marginal costs are highly procyclical, so that they have a strong incentive to adjust their prices in response to a change in aggregate demand, even if other firms keep their prices constant. The loss in profits from not adjusting prices outweighs the fixed costs of adjusting prices (such as menu costs) unless the latter are implausibly high, and nominal price rigidity is not a Nash equilibrium.[2] In a related paper, Jeanne (1998) showed that nominal price rigidities combined with real rigidities could generate substantial persistence in dynamic general equilibrium models. The re-

[1]The phrase was coined by Goodfriend and King (1997).

[2]Possible sources of real rigidities include increasing returns to scale, intermediate inputs that represent a significant fraction of production costs, and countercyclical markups due to the elasticity of output demand being procyclical.

sults of these three papers suggest that the same model features (real rigidities) lead both to supporting nominal price rigidities as an equilibrium outcome and to generating a high degree of persistence.[3]

Huang and Liu (2002) showed recently that a standard neoclassical model with monopolistically competitive households that rent differentiated labor to firms and that set their nominal wages in advance in staggered fashion can generate much more persistence than an equivalent model with nominal price rigidities. It is not necessary to introduce substantial real rigidities to generate persistence in models based on nominal wage rigidities: this is important, given that the empirical plausibility of real rigidities is controversial. Given the relationship in models with price rigidities between persistence and supporting nominal price rigidity as an equilibrium outcome, Huang and Liu's results beg the following question. Can nominal wage rigidities more easily be supported as an equilibrium outcome than nominal price rigidities?

This paper answers the question in the affirmative by developing a simple model of wage determination by monopolistically competitive households. It shows that nominal wage rigidities can be supported as a Nash equilibrium with relatively small fixed costs of adjusting wages, and without imposing real rigidities that reduce the incentives of firms to adjust prices in response to fluctuations in aggregate output. It also shows that the size of adjustment costs necessary to support nominal wage rigidities decreases as the substitutability between different types of labor increases, but remains small for empirically plausible values of the latter. The model also has the property that the minimum adjustment cost required to support nominal wage rigidity as a Nash equilibrium is relatively insensitive to the degree of substitutability between types of labor in production.

The paper is structured as follows. The following (second) section surveys some recent papers that address the related questions of endogenous persistence, of supporting nominal rigidities as an equilibrium outcome, and of indeterminacy and sunspot equilibria. The third section sets up a simple model of wage determination by monopolistically competitive households that is closely related to the model of Ball and Romer (1990) and the textbook model of Romer (2000). The fourth section addresses the model's calibration, calculates the size of transaction costs necessary to sustain nominal wage rigidity as a Nash equilibrium, and investigates the robustness of the results to changes in the model's parameter values.

[3]Kiley (1997) relates models with real rigidities to yet another phenomenon. He shows that the same model features that contribute to endogenous price rigidity and persistence also lead to indeterminacy and the presence of sunspot equilibria under flexible prices.

Table 1: Principal Results from the Literature

Type of model	Nash	Persistence	Multiple
Nominal Price rigidity (standard model)	no (BR)	no (CKM)	no (Kiley)
Nominal Price rigidity (w/ real rigidities)	yes (BR)	yes (Jeanne, Alexopoulos)	yes (Kiley)
Nominal Wage rigidity (standard model)	yes (this paper)	yes (HL, AGP, Andersen, Erceg)	no (Kiley)
AGP: Ambler, Guay and Phaneuf (2001) Alexopoulos: Alexopoulos (2001) Andersen: Andersen (1998) BR: Ball and Romer (1990) CKM: Chari, Kehoe and McGrattan (2000) Erceg: Erceg (1997) HL: Huang and Liu (2002) Jeanne: Jeanne (1998) Kiley (1997)			

The fifth section addresses the question of the optimal length of wage contracts as a function of the size of the fixed cost of adjusting the nominal wage. The sixth section concludes.

2 Literature

Table 1 summarizes the main conclusions of recent research on nominal rigidities as it relates to persistence, multiple equilibria, and the sustainability of nominal rigidities as an equilibrium outcome. The table indicates whether the type of model indicated at the left of a given row is compatible with the phenomenon indicated in a given column. "Nash" refers to whether or not nominal wage or price rigidities can be supported as a Nash equilibrium. "Persistence" refers to whether a temporary shock has a large and prolonged impact on real variables in the type of model indicated. "Multiple" refers to whether the particular type of model can lead to multiple or sunspot equilibria in versions where both wages and prices are flexible. For each type of model and result, the principal sources of the result in the literature are indicated.

Akerlof and Yellen (1985) were pioneers in attempting to provide micro-foundations for nominal price rigidities. Rather than introducing menu costs into a model with rational price-setting firms, they investigated how near-rational behavior by firms could affect macroeconomic equilibrium. They set up a model in which firms, by not adjusting prices to their new optimal levels in the face of an aggregate demand shock, suffer only a second-order loss in profits, while the loss in terms of welfare for the economy as a whole is first-order. Mankiw (1985) reinterpreted Akerlof and Yellen's near-rationality result as rational behavior in the face of small fixed costs (menu costs) of changing prices.

These results seemed to provide a theoretical underpinning for nominal rigidities, but Ball and Romer (1990) showed that this is not the case if the nominal wage adjusts in response to aggregate demand shocks. They developed a model with a competitive labor market and plausible labor supply behavior. In order for nominal price rigidity to be a Nash equilibrium, firms must be willing to vary the quantities that they supply without changing their prices. This is not the case for plausible values of workers' labor supply elasticity. As aggregate output and employment adjust in response to a demand shock, the nominal wage responds to maintain equilibrium in the labor market. A change in the market-clearing nominal wage shifts the marginal costs curves of firms and makes their marginal costs highly procyclical.

Notwithstanding the results of Ball and Romer, researchers in the 1990s continued to develop dynamic general equilibrium models that imposed exogenous nominal price rigidities in order to try to explain the persistent effects of demand shocks on macroeconomic aggregates, leaving aside the question of whether the nominal price rigidity can be explained as an equilibrium outcome.[4] Chari, Kehoe and McGrattan (2000) showed that dynamic general equilibrium models cannot explain a large amount of endogenous persistence. They set up a model with a "small" amount of exogenous price stickiness, that is to say with firms that fix their prices for empirically plausible lengths of time. They showed that in order to generate persistent output fluctuations from monetary shocks, there must be a high degree of *endogenous* price stickiness. This means that firms, when allowed to ad-

[4] An exception is Danziger (1999), who built a general equilibrium model in which state-contingent price adjustment is an equilibrium outcome.

just their prices, do not change their prices by very much. For plausible parameter values in their model, this is not the case. Price stickiness is not a significant propagation mechanism for temporary demand shocks.

The paper by Kiley (1997) completes the first row of Table 1. He showed that incorporating real rigidities in dynamic general equilibrium models leads to two parallel results. First, confirming the results of Ball and Romer (1990), this can lead to nominal price rigidities being a Nash equilibrium for price-setting firms subject to menu costs. Second, when exogenous price stickiness is not imposed, the same models with real rigidities lead to multiple dynamic equilibria and the possibility of sunspot fluctuations. Models with sunspot equilibria can also produce sizeable and persistent economic fluctuations in the response to demand shocks and even in response to extrinsic uncertainty or sunspots.[5] Without real rigidities, however, standard models with plausible parameter values have unique steady states and unique saddle-point dynamics in the neighborhood of their steady states.

In addition to analyzing a standard model with a competitive labor market, Ball and Romer (1990) also constructed a model with a smaller elasticity of the equilibrium real wage with respect to variations in aggregate output. This "real wage function" means that the response of aggregate employment no longer depends on workers' labor supply elasticities. They showed that if the real wage is sufficiently inelastic to changes in aggregate output, nominal price rigidity can be sustained as a Nash equilibrium. This gives us the first entry in the second row of the table. Jeanne (1998) took this analysis a step further by showing that models with nominal price rigidities combined with real rigidities can explain a substantial amount of persistence. In a related paper, Alexopoulos (2001) built a dynamic general equilibrium model of the business cycle with efficiency wages in the labor market and limited participation in financial markets. Efficiency wages in her model dampen the response of real wages to output fluctuations. She obtained large and persistent responses of real aggregates to demand shocks. The last column in the second row of Table 1 brings us back to the results of Kiley (1997), summarized in the preceding paragraph.

To summarize, models with substantial real rigidities can explain endogenous nominal price rigidity, persistent responses of output and other real variables to monetary shocks, and multiple or sunspot equilibria. The issue of the empirical plausibility of the degree of real rigidity that is necessary to lead to substantial persistence and to endogenous nominal price rigidity is still open.

[5] See Farmer (1999, chapters 7, 8) for a survey of the literature on indeterminacy and sunspots.

Huang and Liu (2002) extended a standard dynamic general equilibrium business cycle model to include both monopolistically competitive firms selling differentiated goods and monopolistically competitive households selling differentiated labor services in the labor market. They contrasted versions of their model with nominal wage rigidities and nominal price rigidities and showed that, with plausible parameter values, the staggered price mechanism cannot generate persistence while the staggered wage mechanism can, given a small degree of (exogenously imposed) nominal wage rigidity.[6] The papers by Erceg (1997) and Andersen (1998) are complementary to Huang and Liu (2002), and show that nominal wage rigidities can lead to substantial persistence. Ambler, Guay and Phaneuf (2001) built a dynamic general equilibrium model with nominal wage rigidities and labor adjustment costs, and estimated its parameters using the generalized method of moments. The model's overidentifying restrictions were accepted by the data, and it generated persistent fluctuations in output and employment in response to temporary aggregate demand shocks. In the case of models with nominal wage rigidity, the analogy between endogenous stickiness and multiple equilibria breaks down. A corollary of Kiley's (1997) results is that the standard model cannot generate indeterminacy and multiple equilibria if exogenous wage stickiness is removed.

This paper is the first to analyze the possibility of supporting nominal wage rigidity as an equilibrium outcome. The relationship between endogenous persistence and endogenous price stickiness suggests that there may be a similar relationship in the case of nominal wage rigidities. A closely related paper on the welfare effects of nominal wage contracts also suggests that this is a promising avenue to explore. Cho, Cooley and Phaneuf (1997) built a dynamic general equilibrium business cycle model in which staggered wage contracts à la Calvo (1983) were exogenously imposed. In other words, not only the frequency of adjustment of wages but also the level to which they adjust was exogenous, instead of the level being determined optimally by monopolistically competitive wage setters. In their model, the the welfare loss due to the presence of nominal wage contracts was quite small (about 0.05% of GDP) for one-year contracts and for degrees of wage indexation up to 60%.

[6]Edge (2000) argues that Huang and Liu's conclusions depend on the assumption that all firms use identical inputs. She sets up a model in which there are firm-specific factor inputs and in which nominal price rigidities can generate as much endogenous persistence as nominal wage rigidities.

3 Basic Model

The model is closely related to Ball and Romer (1990) and Romer (2000, chapter 6). It is a "standard" model in the sense that there are no increasing returns to scale, no intermediate inputs in production, and monopolistically competitive firms face constant-elasticity demand curves for their output, so that their markups are constant in response to changes in aggregate demand. The only new feature is the inclusion, following Huang and Liu (2002), of monopolistic competition in the labor market, with households that sell differentiated types of labor to firms.

3.1 Production

There is a continuum of monopolistically competitive firms on the unit interval. Goods prices are flexible.[7] Firms have identical production functions, which depend on an aggregate labor input. Firm i produces its output subject to the following production function:

$$Y_{it} = N_{it} \tag{1}$$

where Y_{it} is the output of firm i and N_{it} is the quantity of aggregate labor employed by firm i. Aggregate labor is a composite of different labor types. A given type of labor is supplied by household s. There is a continuum of households on the unit interval. We have:

$$N_t = \left(\int_0^1 N(s)_t{}^{(\theta-1)/\theta} ds \right)^{\theta/(\theta-1)}, \tag{2}$$

where N_t is aggregate employment and $N(s)_t$ is the quantity of labor of type s supplied by household s. In order to distinguish between labor supplied by a particular type of household and labor demanded by individual firms, we follow the convention that the employment of composite labor at individual firms is indexed by an i subscript and individual households' labor supplies are indexed by the argument s. Aggregate employment is just the sum of employment at the different firms in the economy:

$$N_t \equiv \int_0^1 N_{it} di. \tag{3}$$

[7]Because firms have flat marginal cost curves in the model and because each firm's markup over marginal cost is constant, goods prices will be fixed insofar as nominal wages remain fixed.

Profit maximization leads to the following conditional demand for labor of type s:[8]

$$N(s)_t = \left(\frac{W(s)_t}{W_t}\right)^{-\theta} N_t, \tag{4}$$

where $W(s)_t$ is the nominal wage set by the household of type s and where W_t is the average wage index defined by:

$$W_t \equiv \left(\int_0^1 W(s)_t{}^{(1-\theta)} ds\right)^{1/(1-\theta)}.$$

3.2 Households

Households supply labor to firms. Each household s is a monopoly supplier of a particular type of labor, and households take into account the conditional demand for their type of labor when setting their nominal wage. A household of type s maximizes the following intertemporal utility function:

$$U(s) = E_t \sum_{i=0}^{\infty} \beta^i u\left(C(s)_{t+i}, N(s)_{t+i}\right), \tag{5}$$

where $C(s)_t$ is consumption and $N(s)_t$ denotes hours worked. Period utility of household s is given by:

$$u\left(C(s)_t, N(s)_t\right) = \frac{1}{1-\gamma_1} C(s)_t{}^{(1-\gamma_1)} - \frac{\phi}{1+\gamma_2} N(s)_t{}^{(1+\gamma_2)}. \tag{6}$$

The aggregate consumption good is a bundle of the goods produced the different firms in the economy, given by:

$$C_t = \left(\int_0^1 Y_{it}{}^{(\eta-1)/\eta} di\right)^{(\eta-1)/\eta} = \int_0^1 C(s)_t ds. \tag{7}$$

It follows that the exact price index for aggregate consumption is given by:

$$P_t \equiv \left(\int_0^1 P_{it}{}^{(1-\eta)} ds\right)^{1/(1-\eta)}, \tag{8}$$

[8] Two interpretations of the aggregation of different types of labor services are possible. First, it would be possible to introduce a competitive "bundler" (see Canzoneri, Cumby and Diba, 2001) or "broker" who rents labor services from individual households and then rents aggregate labor services to firms. The second interpretation is that individual firms themselves hire different types of labor from individual households. Nothing substantive hinges on the interpretation.

and that the conditional demand for the output of firm i is given by:

$$Y_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\eta} C_t. \tag{9}$$

With constant-elasticity demand curves, firms set markups which are a fixed proportion of their marginal cost of production. Households derive income from wages, from dividend payments by firms, and from government transfers. The household's period budget constraint can be written as:

$$\frac{W(s)_t}{P_t} N(s)_t + \pi_t + \frac{M(s)_t}{P_t} + T(s)_t = C(s)_t + \frac{M(s)_{t+1}}{P_t}, \tag{10}$$

where $M(s)_t$ denotes the household's holdings of nominal money balances, π_t denotes dividend payments from firms (assumed to be equal across households), and $T(s)_t$ denotes the real value of transfers from the government. The household's maximization problem leads to the following rule for setting its nominal wage:

$$\frac{\partial U}{\partial N(s)_t} = \frac{W(s)_t}{P_t} \frac{\theta - 1}{\theta} \frac{\partial U}{\partial C(s)_t}. \tag{11}$$

This condition is standard. It says that the marginal disutility of working an extra hour must be equal to the marginal utility of consumption from working an extra hour, taking into account the fact that because the household is a monopolistic supplier of labor services, working more has a negative effect on the household's real wage.

3.3 Symmetrical Flexible-Wage Equilibrium

The model is closed with the following cash-in-advance constraint:

$$C_t = \frac{M_t}{P_t} \tag{12}$$

where M_t is the aggregate money stock. All households in the model are identical, so all will choose the same nominal wage and consumption level. Imposing a symmetrical equilibrium and aggregating, the model can be reduced to the cash-in-advance constraint plus the following four equations:

$$P_t = \frac{\eta}{\eta - 1} W_t, \tag{13}$$

$$Y_t = N_t, \tag{14}$$

$$Y_t = C_t, \tag{15}$$

$$\frac{W_t}{P_t}\frac{\theta - 1}{\theta}{C_t}^{-\gamma_1} = {N_t}^{\gamma_2}. \tag{16}$$

Equation (13) follows from profit maximization by firms. The price of output is proportional to the marginal cost of production, which is equal to the nominal wage. Equation (14) is the aggregate production function. Equation (15) reflects the fact that there is no public spending and no investment in the model: all of aggregate output is consumed. Equation (16) follows directly from aggregating households' first order conditions for the choice of the nominal wage. Equations (12) through (16) can be solved for the equilibrium values of W_t, P_t, Y_t, N_t and C_t. From equation (16), after substituting out C_t and N_t using (15) and (14), the solution for equilibrium output under flexible wages and prices is given by:

$$Y_t = \left(\frac{\theta - 1}{\theta}\frac{\eta - 1}{\eta}\right)^{1/(\gamma_1+\gamma_2)}. \tag{17}$$

It can easily be shown that the optimal level of output for a social planner who maximizes the utility of a representative household (imposing the same number of hours worked and consumption across all households) is equal to one. The equilibrium value of output is lower than this first-best optimum because of households' monopoly power in the labor market and firms' monopoly power in the goods market. As θ and η tend to infinity, this monopoly power disappears and the market equilibrium converges to the social optimum.

4 Fixed Nominal Wages as a Nash Equilibrium

In this section, we consider the impact on one household of a reduction in aggregate demand (a fall in the nominal money supply) if all other households maintain a constant nominal wage. The household will have an incentive to adjust its wage. It remains to be seen whether the welfare loss from not adjusting is small enough so that a small fixed cost of adjustment would support nominal wage rigidities as an equilibrium outcome.

We follow the same line of reasoning that Ball and Romer (1990) and Romer (2000) apply to price adjustment by firms. We assume that the economy is initially in an equilibrium compatible with full wage and price flexibility. Then, we posit a reduction in the money stock and suppose that all households but one maintain a constant nominal wage. For the remaining household, we calculate the level of utility it can attain if it maintains a constant nominal wage and if it adjusts its wage optimally in reaction to the shock. Finally, we express the utility loss from not adjusting its wage as a compensating variation, which is just the percentage of its level of consumption with a constant nominal wage that it would have to receive to be made as well off as if it adjusted its wage optimally.

Dropping time subscripts, call $u(s)_1$ the utility level of a household that does not adjust its nominal wage when aggregate demand decreases, assuming that other households also do not adjust their wages. This is given by:

$$u(s)_1 = u\left(C(s)_1, N(s)_1\right),$$

where the numbered subscripts are used to refer to consumption and labor supply in the case of no adjustment. Call $u(s)_2$ the utility level of a household that adjusts its nominal wage. This is defined by:

$$u(s)_2 = u\left(C(s)_2, N(s)_2\right).$$

It must be the case that $u(s)_2 \geq u(s)_1$. We are interested in calculating the level of consumption $C(s)_3$ such that:

$$u(s)_2 = u\left(C(s)_3, N(s)_1\right).$$

Given the functional form of the utility function, this is just equal to:

$$C(s)_3 = \left(\left(\frac{C(s)_2{}^{(1-\gamma_1)}}{1-\gamma_1} - \phi\frac{N(s)_2{}^{(1+\gamma_2)}}{1+\gamma_2} + \phi\frac{N(s)_1{}^{(1+\gamma_2)}}{1+\gamma_2}\right)^{(1-\gamma_1)}\right)^{1/(1-\gamma_1)}.$$

Finally, define the compensating variation as follows:

$$CV \equiv \frac{C(s)_3 - C(s)_1}{C(s)_1}. \tag{18}$$

This is just the amount of additional consumption that the household (that does not adjust its wage) must be given to make it as well off as if it did adjust its wage, expressed as a fraction of the level of consumption that it enjoys if it keeps a fixed nominal wage.

Table 2: Model Calibration

Parameter	Value
γ_1	2.0
γ_2	2.0
θ	5.0
η	5.0
ϕ	1.0

Table 2 gives parameter values for our base-case scenario. The values of γ_1 and γ_2 are standard in the literature. In particular, the value chosen for γ_2 gives an elasticity of labor supply with respect to the real wage equal to one half. The ϕ parameter affects the level of the real wage, but not the level of employment in the flexible wage/price equilibrium, which depends only on the parameters θ, γ_1 and γ_2 because of equations (17) and (14). The value of θ is taken from Ball and Romer (1990) and Romer (2000, chapter 6), and gives a markup of 25% over marginal cost. The value of η is close to the upper end of the range of values estimated by Griffin (1992, 1996). With these parameter values, the flexible-wage equilibrium gives a level of output, employment, and consumption equal to 0.894, less than the first-best level of output with flexible wages, which is equal to 1.0.

We consider, following Romer (2000, chapter 6) a three percent drop in the money supply. If all households but one maintain a constant nominal wage, output and employment fall to 97% of their flexible wage/price levels. With parameters equal to their base-case values, the compensating variation amounts to 0.13% of the initial level of consumption.[9] In other words, an individual household suffers a utility loss equivalent to a little over one tenth of one percent of its initial consumption level if it foregoes adjusting its nominal wage. This can be compared to the result in Romer (2000) using a similar model that a firm's loss from not adjusting its output price in response to a three percent drop in aggregate demand would

[9] In the model of Cho, Cooley and Phaneuf (1997), the welfare cost of labor contracts is equal to 0.05% of GDP with one-year contracts. Three main differences explain the differences in our results. First, their model contains capital. Second, they calculate the welfare loss for calibrated values of the variances of technology and monetary shocks, while here we consider the costs of a once-and-for-all drop in aggregate demand of 3%. Third, in their model, the amount by which households adjust their wage is determined by their ad hoc wage adjustment equation, while in the model of this paper it is determined by households' optimal behavior.

amount to approximately 25% of its revenue. The conclusion of this analysis is clear: the size of adjustment costs necessary to support nominal wage rigidity as a Nash equilibrium is an order of magnitude smaller than the adjustment costs needed to support nominal price rigidity as an equilibrium outcome.

4.1 Discussion

In the basic model developed above, nominal wage rigidity naturally gives rise to real rigidity in the following sense. Labor is the only factor of production, and there are constant returns to scale. Since firms are assumed to be price takers in the labor market, each individual's firm's marginal cost curve is flat. In response to variations in aggregate demand, if nominal wages are fixed then nominal prices (which are equal to marginal costs under perfect competition or to a constant markup over marginal costs under monopolistic competition in the goods market with constant-elasticity demand curves) remain constant as well. The real wage is completely rigid in response to changes in output. Contrast this to a model similar to the basic model but with only nominal price rigidities and with monopolistic competition only in the goods market. In order for aggregate output to change, employment must change, and furthermore employment must lie on the aggregate labor supply curve. If labor supply is relatively inelastic, the nominal wage is highly sensitive to variations in output and employment.

In Figure 1, we reinterpret the discussion in Romer (2000, chapter 6). The graph illustrates the incentives of an individual firm to reduce its output price in the face of a negative monetary shock which reduces aggregate demand, under the assumption that all other firms in the economy do not adjust their output prices. For a given nominal wage rate, the firm's marginal cost curve is completely flat in the model developed above, since labor is the only factor of production and there are constant returns to scale. If there is no adjustment of the nominal wage, the firm's optimal price does not change, since its optimal markup is a fixed percentage of its marginal cost. However, we know that if aggregate employment falls, this entails a downward movement along the aggregate labor supply curve. From the individual firm's point of view, there is an exogenous decrease in the nominal wage at which it can hire workers. Its marginal cost curve shifts down, and its incentive to adjust its price is captured by the shaded triangular area.

The analysis is very different in the case of a household considering whether or not to adjust its nominal wage. Figure 2 gives a graphical representation of this case. The first order condition for the household's choice of $W(s)$ can be

Table 3: Sensitivity to γ_2

Value of γ_2	Value of CV in %
0.25	0.08%
0.50	0.09%
1.00	0.10%
2.00	0.13%
10.00	0.33%

rewritten as follows:

$$\left(N(s)_t + W(s)_t \frac{\partial N(s)_t}{\partial W(s)_t}\right) = P_t \left(\frac{\partial N(s)_t}{\partial W(s)_t}\right) \left(\frac{\partial U}{\partial N(s)_t}\right) / \left(\frac{\partial U}{\partial C(s)_t}\right).$$

The left hand side gives the household's marginal revenue from decreasing its nominal wage by a small amount, measured in units of consumption, while the right hand side gives the marginal cost in terms of foregone leisure. The marginal cost is an increasing function of $N(s))_t$ because of the increasing disutility of work and downward-sloping demand curve for the household's labor services. The aggregate price level acts as a shift variable for the households marginal cost curve, just as the aggregate nominal wage rate acts as a shift variable for each firm's marginal cost curve in Figure 1.

In the basic model, with constant returns to labor in the production function, there is endogenous price rigidity. As long as other households maintain a constant nominal wage, firms maintain constant prices. The price level is a shift variable for the marginal cost curve depicted in Figure 2, but there is in fact no shift in the marginal cost curve. The incentive for an individual household to adjust its wage in response to a drop in aggregate demand is given by the shaded triangular area in Figure 2, much smaller than what its incentive would be with a downward shift in its marginal cost curve.

4.2 Sensitivity Analysis

Table 3 shows how the size of the compensating variation changes in response to changes in the value of γ_2, which measures the inverse of the elasticity of labor supply.

Table 4: Sensitivity to θ

Value of θ	Value of CV in %
1.25	0.04%
2.00	0.08%
5.00	0.13%
10.00	0.14%
20.00	0.15%
40.00	0.15%

The table shows that the costs of not adjusting the nominal wage increase as labor supply elasticity decreases. In this respect, the results are similar to those of Ball and Romer (1990). If we interpret a high labor supply elasticity to mean high real wage rigidity, then more real wage rigidity means that smaller costs of adjusting nominal wages can support nominal wage rigidity as an equilibrium outcome.

However, the results are quantitatively very different than the results for price adjustment in the Ball and Romer (1990) model. Even with a very low labor supply elasticity ($\gamma_2 = 10$ corresponds to a labor supply elasticity of 0.1), the costs of not adjusting the real wage amount to only one third of one percent of the household's initial consumption level.

Table 4 shows how the compensating variation changes with the value of θ, the elasticity of substitution between different types of labor. The table shows that the costs to the household of not adjusting its nominal wage are not very sensitive to the degree of substitutability between labor types in the model.

5 Optimal Contract Length

The results derived so far have been static, having to do with the cost of not adjusting the nominal wage in the same period as an aggregate demand shock hits. It is possible to calculate the socially optimal contract length that maximizes expected utility for a representative household given the stochastic process generating the money supply and a given fixed cost of adjusting the nominal wage.

In this section, we modify the model so that each household fixes its nominal wage and faces a fixed probability $(1 - d)$ with which the nominal wage contract "expires" at the beginning of each period. Households set their nominal wage optimally knowing that at the beginning of each period they will receive a signal (with probability d) to reoptimize their nominal wage. This formulation was originally applied to price setting by Calvo (1983). It can be shown that this setup leads to the following first-order difference equations for the economy's wage dynamics:[10]

$$\tilde{Z}_t = d\tilde{Z}_{t-1} + (1-d)\tilde{W}_t + \gamma\left(\tilde{N}_t - \tilde{N}_t^o\right), \tag{19}$$

$$\tilde{W}_t = d\tilde{W}_{t-1} + (1-d)\tilde{Z}_t, \tag{20}$$

where the " ˜ " notation indicates that variables are measured in proportional deviations from their steady state levels, $\tilde{Z}_t$ is the nominal wage set by households who are allowed to adjust their wages at time t, $\tilde{W}_t$ is the average nominal wage in the economy, and $\tilde{N}_t^o$ is notional labor supply.[11] The γ parameter is related to the structural parameters of the model in the following way:

$$\gamma = \frac{(1-\beta d)\frac{NV_{NN}}{V_N}}{1+\frac{NV_{NN}}{V_N}\theta}, \tag{21}$$

where N is the steady state level of employment and where

$$V_{NN} \equiv -\phi N^{\gamma_2}$$

and

$$V_{NN} \equiv -\gamma_2\phi N^{(\gamma_2-1)},$$

so that in fact

$$\frac{NV_{NN}}{V_N} = \gamma_2.$$

These two dynamical equations for wage dynamics are combined with the following equilibrium conditions:

$$\tilde{M}_t - \tilde{P}_t = \tilde{Y}_t, \tag{22}$$

$$\tilde{Y}_t = \tilde{N}_t, \tag{23}$$

$$\tilde{Y}_t = \tilde{C}_t, \tag{24}$$

[10]See Huang and Liu (2002) or Ambler, Guay and Phaneuf (2001) for a derivation.

[11]Notional labor supply is the number of hours that would satisfy households' first order condition for the choice of hours. Ex post, after fixing the nominal wage, households agree to supply the number of hours of work demanded by firms.

$$\tilde{P}_t = \tilde{W}_t. \tag{25}$$

The model is closed with the following autoregressive process for the money supply:

$$\tilde{M}_t = \omega \tilde{M}_{t-1} + \tilde{u}_t. \tag{26}$$

The model can be reduced to a system of three dynamical equations in the unknowns $\tilde{M}_t$, $\tilde{W}_t$ and $\tilde{Z}_t$.

The model was simulated for its base case parameters, using $\omega = 0.9$ and a standard deviation for the money supply shocks $\sigma_u = 0.01$, assuming that the fixed cost of adjusting the nominal wage is equal to 0.10% of the steady-state level of per-period consumption under flexible wages, assuming a subjective discount rate $\beta = 0.99$[12] and for different values of the probability that a household is allowed to revise its nominal wage. The sample period was set equal to 10,000.

The simulations yielded values for the proportional deviations of variables from their steady state levels. It was then possible to calculate the unconditional expected utility for a representative household that works at and owns shares in all firms, so that it pays just the per capita amount of adjustment costs, equal to the fraction of households adjusting their wage times the adjustment cost.[13]

The results, which are presented in Table 5, are quite intuitive. Unconditional expected utility follows an inverse "U" shape as a function of the probability that a given wage contract remains in force at the beginning of each period. Average contract length is equal to $1/(1-d)$. For small values of d (a very short average contract length), an increase in d improves unconditional expected utility because the savings in aggregate adjustment costs swamp the loss due to the increased volatility of consumption, which arises from the magnified impact of monetary shocks on output as contracts increase in length. As the average contract length increases, the marginal adjustment cost savings from further increases in contract length fall, while the marginal costs due to increased consumption volatility increase. With $d = 0.8$, the two effects offset each other, and unconditional expected utility is maximized. If given the opportunity to choose d once and for all given the stochastic process generating the money supply and given the size of the fixed

[12]The β parameter is almost the only parameter of the model whose value depends on the assumed length in calendar time of one period in the model. We assume that one period is equal to one quarter. The interpretation of the size of the fixed cost of adjusting the nominal wage as a fraction of annual output also depends on the length of the period.

[13]An alternative assumption that yields the same results is that there are complete asset markets, with transfers among households each period that ensure that each household pays only its per capita share of total adjustment costs.

Table 5: Unconditional Expected Utility as a Function of d

Value of d	Expected Utility
0.1	-1.3575
0.2	-1.3574
0.3	-1.3573
0.4	-1.3573
0.5	-1.3572
0.6	-1.3571
0.7	-1.3571
0.8	-1.3571*
0.9	-1.3574

*: maximum value as a function of d

cost to adjust the nominal wage, a social planner would choose $d = 0.80$, or an average contract length of five quarters. The consensus view (see Taylor, 1999) is that average contract length in the U.S. is equal to about four quarters. So, if a given household must pay an adjustment cost of 0.10% of its per-period consumption to adjust its nominal wage, the socially optimal contract length would be as long or longer than the average duration of wage contracts in the U.S. data. With 20% of households adjusting their wage each period, total per capita adjustment costs would amount in this economy to 0.02% of GDP. Given this figure, interpreting the observed average length of wage contracts as an equilibrium outcome in the face of fixed costs of renegotiating wage contracts is well within the realm of plausibility.

In addition, it is possible that the equilibrium contract length in a decentralized economy is greater than the social optimum due to the presence of externalities, as argued by Ball (1987). A decision by one household to increase its contract length implies that the average wage level responds more slowly to aggregate demand shocks, and hence the price level responds more slowly as well. This makes it easier for other households to forecast future prices when setting their nominal wage, which is a positive externality. On the other hand, the real money supply is more variable, which implies a greater variability of aggregate demand and therefore the demand for other firms' products. The second effect is a negative externality. Ball (1987) shows that if the net externality is negative, the contract length chosen in a symmetric Nash equilibrium will be greater than the social optimum.[14]

6 Conclusions

Akerlof and Yellen (1985) and Mankiw (1985) were among the first researchers to investigate the question of whether small macroeconomic frictions could explain large fluctuations in response to exogenous shocks. These frictions have been interpreted as "menu costs", literally the costs of printing menus with revised prices.[15] Ball and Romer (1990) showed that in the absence of important real rigidities, menu costs would have to be huge in order for firms to decide not to change prices in the face of modest fluctuations in aggregate demand, because of the endogenous adjustment of real wages in response to shifts in aggregate demand. In the textbook example of Romer (2000), when faced with a 3% drop in aggregate demand the costs in foregone profits to a firm of not adjusting its prices amount to over a quarter of its revenue.

This paper has shown that it is much more plausible to justify nominal *wage* rigidities as an equilibrium in the presence of small macroeconomic frictions. For plausible parameter values in the basic model, the costs of not adjusting wages in response to a fall in aggregate demand of 3% are 0.13% of initial consumption. Costs of this size may well be less than the costs of renegotiating and resigning a new labor contract. In an extension to the model, for plausible parameterization of the money supply process, if adjusting the nominal wage costs 0.1% of the value of per-period consumption under flexible wages, a contract length of five quarters maximizes the expected utility of the representative household. This corresponds to or exceeds the average length of wage contracts in the data, and entails adjustment costs that in aggregate amount to only 0.02% of GDP.

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[14]In Ball's model, it is firms rather than households that choose their optimal contract length, and wages are determined by an expected market clearing rule.

[15]Mankiw and Reis (2001) propose an alternative interpretation based on the costs of acquiring and processing the information required by a firm to optimally revise its price.

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Figure 1

Firm's Incentive to Adjust Price,
Negative Aggregate Demand Shock

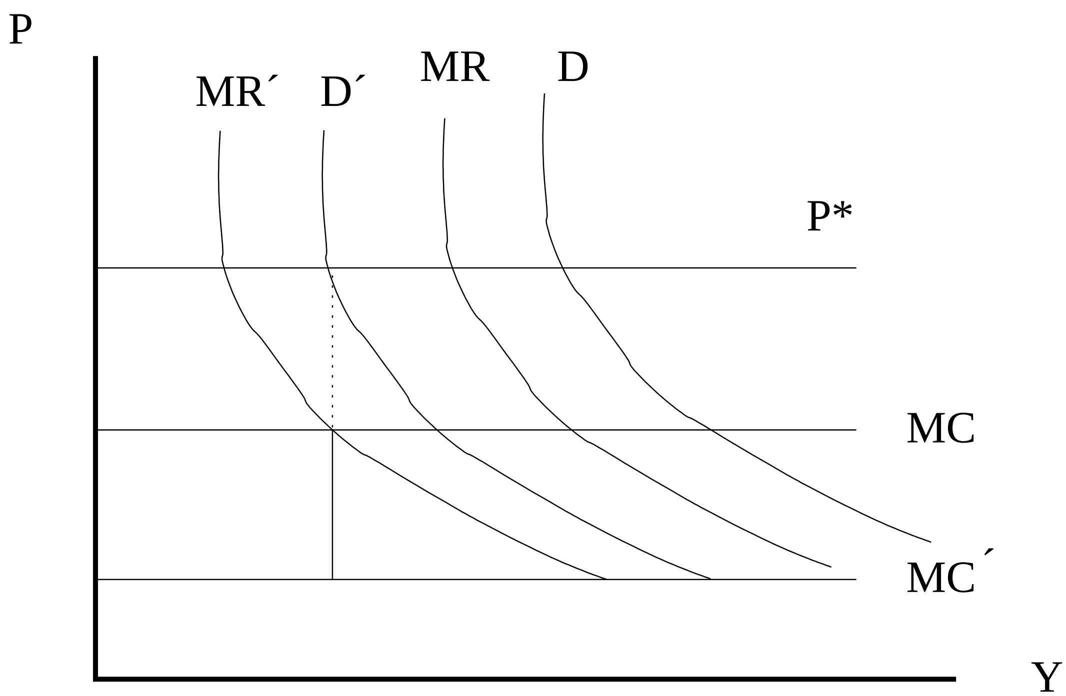

Figure 2

Household's Incentive to Adjust Wage,
Negative Aggregate Demand Shock