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A Simple Model of Collective Consumption

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Résumé : Dans cet article, nous présentons un modèle collectif de comportement du ménage basé sur l'efficacité parétienne. Nous supposons également que (i) chaque membre du ménage est égoiste et la consommation est privée, (ii) il existe un ensemble de facteurs de distribution et (iii) il y a un bien exclusif. Ensuite, nous dérivons les restrictions testables impliquées par ces hypothèses.

Abstract : In this paper, we present a collective model of household demand based on Pareto-efficiency. In addition, we suppose that (i) each household member is egoistic and consumption is purely private, (ii) there is a set of distribution factors and (iii) there is one exclusive good. Then we derive the testable restrictions which are implied by this theoretical setting.

Mots clés : modèle collectif, demande du ménage, facteurs de distribution, matrice de Slutsky

Keywords : Collective Models, Household Demands, Distribution Factors, Slutsky Matrix

JEL Classification : D11

1 Introduction

In demand analysis, it is generally assumed that the many-person household can be treated as though it maximizes a single utility function under a budget constraint. Methodologically, however, this approach stands on weak grounds: it is clear that utility theory applies to individuals and not to households. See Pollak and Lundberg (1996) for a discussion of this issue.

During the last decade, several authors have aimed at explicitly taking into account the individualistic elements of the situation. In an unpublished paper, Bourguignon, Browning and Chiappori (1995) explore what a collective representation of household behavior implies for the properties of demands in the case where prices are constant. They adopt the theoretical framework previously used by Chiappori (1988, 1992) for studying labor supply: each household member is characterized by his (her) own preferences and the decision process results in Pareto efficient outcome. They next consider various models depending on the nature of goods (private or public) and the form of preferences (altruistic or egoistic). The underlying idea of these authors is to use a set of exogenous variables, called 'distribution factors', which influence the decision process without affecting the budget set or preferences. Then, the specific form taken by the influence of these variables implies restrictions on household behavior; see Browning, Bourguignon, Chiappori and Lechene (1994) for an empirical application. Moreover, if agents are egoistic and consumption is purely private, some structural components, such as the internal rule which determines the distribution of welfare within the household, can be recovered from observed behavior — an essential requirement when considering policy issues which involve individual welfare (see Alderman et al. (1995) for a discussion). This theoretical result is actually valid in a quite general context; in particular, observability of individual consumption is not required. However, things are much simpler when there is at least one exclusive good, i.e., a good which is consumed by only one person in the household.[1] More recently, Chiappori and Ekeland (2002a, 2002b) have extended this setting to incorporate variable prices. They consider a wide range of collective models (including a model with egoistic agents and private consumption) and demonstrate that household demands must always satisfy a symmetry plus rank one condition (SR1) on the Slutsky matrix; moreover, this condition is sufficient when there is no distribution factor. Finally, Browning and Chiappori (1998) test the SR1 condition on data drawn from a series of surveys of household expenditures from Canada.

The present paper completes this growing literature. In what follows, we study the properties of a collective model of household demand with variable prices under a triplet of distinctive assumptions. To be precise, we suppose the following:

a. Each household member is egoistic (or altruistic in a 'caring' sense) and consumption is purely private.

[1] If a good is consumed by only one person in the household, the individual consumption of this good is observed and, of course, coincides with the household consumption.

b. There is one good which is exclusively consumed by one member.

c. There is at least one distribution factor.

It is fair to say that the implications of these assumptions are well-known for collective models with constant prices. They are investigated by Bourguignon, Browning and Chiappori (1995) and, in a slightly different context, by Donni (2001). Still, there does not exist, to the best of our knowledge, a comprehensive study of the collective model of household demand based on (a)–(c) when prices vary. The main contribution of the present paper is to fill this gap. Scientific curiosity, however, is not the only motivation here. As it will be clear in the remainder of the text, these assumptions have a set of consequences which are very attractive, especially when identification issues are involved. At this stage, we may remark that (a) is necessary to identify some structural components of the decision process[2] whereas the conjunction of (b) and (c) permits to greatly simplify the theoretical developments (moreover, it is actually impossible to identify useful structural components if neither (b) nor (c) is supposed; see Chiappori and Ekeland (2002b)). By comparison, Bourguignon, Browning and Chiappori (1995) consider a model with constant prices based only on (a) and (c), but the proof of identification turns out to be particularly complicated.[3]

In considering the derivation of the main results, we have found it profitable to use a concept of 'conditional' functions, previously developed by Bourguignon, Browning and Chiappori (1995), whereby the demand for one good is expressed as a function of the demand for another good. Our results can then be summarized as follows. Firstly, we demonstrate that household demands have to satisfy a symmetry and negativity condition, analogue to the Slutsky condition, and more tractable than the SR1 condition of Browning, Chiappori and Ekeland. Secondly, we derive a set of simple restrictions resulting from both the structure of preferences and the presence of distribution factors. These are different from those presented in the literature until now. Thirdly, we show that some elements of the decision process can be identified from observation of household demands. Precisely, the individual Marshallian demands and the internal rule which determines the distribution of welfare within the household can be partially recovered. In conclusion, it must be stressed that the derivation of all these results turns out to be particularly simple and elegant (simplicity is a relative guarantee of robustness in the case of empirical implementations).

The paper is structured as follows. In Section 2, we describe the theoretical model and its assumptions and, in Section 3, we present the main results. In Section 4, we briefly conclude.

[2]We must admit, to be precise, that recent studies by Donni (2002) and Chiappori and Ekeland (2002b) demonstrate that the privateness of consumption is not essential for identifying structural elements of the decision process. However, it is absolutely required that agents are egoistic or 'caring'.

[3]Other identification results, not mentioned here, come from the labor supply context (see Chiappori (1988, 1992) Chiappori, Fortin and Lacroix (2002) and Chiappori and Ekeland (2002b) for example). They are based on a pair of exclusive goods.

2 The Model

2.1 Basic Assumptions

We consider the case of two-member (say A and B) households. Individual demands for good n ($n = 1, \ldots, N$) are denoted by Z^{An} and Z^{Bn} with prices P_n. We only observe the aggregate consumption $Z^n = Z^{An} + Z^{Bn}$ at the household level; Z^{An} and Z^{Bn} are treated as unobservable. However, there is one good, denoted by X, which is exclusively consumed by member A (say) with price Q. Exogenous income is denoted by Y. Let $Z^A = (Z^{A1}, \ldots, Z^{AN})$ and $Z^B = (Z^{B1}, \ldots, Z^{BN})$ be the vectors of consumptions and $P = (P_1, \ldots, P_N)$ the vector of prices. We make the following assumption on preferences.

Assumption 1 Household members are characterized by 'egoistic' utility functions of the form $u^A(X, Z^A)$ and $u^B(Z^B)$ which are increasing and strongly concave in all their arguments.

Two points must be stressed. First, the agents are said 'egoistic' in the sense that their utility only depends on their own consumption. This is more restrictive than the case considered by Browning and Chiappori (1998) where the utility also depends on the partner's consumption.[4] Second, the concept of exclusive good is explained in greater depth by Bourguignon, Browning and Chiappori (1995). The most typical example is given by clothes inasmuch as some pieces of clothing in the household are always worn by the husband and others by the wife. This idea is at the heart of several empirical applications, for instance, by Browning, Bourguignon, Chiappori and Lechene (1994) and Donni (2002). Another important, but more questionable, example of exclusive good, which comes from collective models of labor supply, is leisure.

We adopt the collective approach where the household decisions result in Pareto-efficient outcomes and no additional assumptions is made about the process. This is common and does not deserve a justification.

Assumption 2 The outcome of the decision process is Pareto-efficient.

Efficiency essentially means that there exists a scalar μ such that the household behavior is a solution to the following program:

$$\max_{\{Z^A, Z^B, X\}} (1-\mu) \cdot u^A(X, Z^A) + \mu \cdot u^B(Z^B) \tag{$\bar{\text{P}}$}$$

with respect to

$$P' \cdot (Z^A + Z^B) + Q \cdot X = Y,$$

where $\mu \in]0, 1[$ has an obvious interpretation as a 'distribution of power' index. This index is assumed to be a single-valued and differentiable function of P,

[4] However, all the results immediately extend to the case of 'caring' agents, with utility functions represented by the form: $W_i(u^A(X, Z^A), u^B(Z^B))$, as usual.

Q, Y and S where $S = (S_1, \ldots, S_J)$ is a vector of distribution factors, such as defined in the introduction. The most useful example is given by variables that describe the respective contribution of each member to the total exogenous income (in standard theory, only total exogenous income should matter for explaining household behavior). Typical examples can also be found in family economics: the state of the market for marriage, the legislation on divorce or the respective male and female unemployment rates.

The next step is to introduce what we call the sharing rule. To do that, we present a classical result.

Proposition 1 *The demands $\bar{X}(P,Q,Y,S)$, $\bar{Z}^A(P,Q,Y,S)$ and $\bar{Z}^B(P,Q,Y,S)$ are solutions to ($\bar{P}$) if and only if there exists a function ρ such that $\bar{X}(P,Q,Y,S)$, $\bar{Z}^A(P,Q,Y,S)$ and $\bar{Z}^B(P,Q,Y,S)$ are solutions to*

$$\max_{\{X,Z^A\}} u^A(X,Z^A) \quad \textit{subject to} \quad P' \cdot Z^A + Q \cdot X = \rho \ , \tag{1}$$
$$\max_{\{Z^B\}} u^B(Z^B) \quad \textit{subject to} \quad P' \cdot Z^B = Y - \rho \ , \tag{2}$$

for any (P,Q,Y,S).

P roof. A straightforward application of the First and the Second Theorems of Welfare Economics; see also Bourguignon, Browning and Chiappori (1995). ■

The function $\rho(P,Q,Y,S)$, which denotes the part of exogenous income that person A receives, is the 'sharing rule'; the latter summarizes the decision process. Consequently, we can state that the household demands have the following functional structure:

$$\bar{X}(P,Q,Y,S) = \chi(P,Q,\rho(P,Q,Y,S)), \tag{3}$$
$$\bar{Z}^n(P,Q,Y,S) = \zeta^{An}(P,Q,\rho(P,Q,Y,S)) + \zeta^{Bn}(P,Y-\rho(P,Q,Y,S)), \tag{4}$$

where χ, ζ^{An}, ζ^{Bn} are traditional Marshallian demands. In particular, they satisfy the well-known restrictions of demand analysis in the single-utility framework (adding-up, homogeneity, symmetry and negativity). We assume here that the functions in the left-hand-side of (3) and (4) are observed.

2.2 Conditional Demands

Bourguignon, Browning and Chiappori (1995) define a type of conditional demands that turns out to be useful in the extended rational setting at stake here. Let us assume that the demand for the exclusive good is (locally) non constant on one distribution factor (say S_1, without loss of generality), i.e., $X_{S_1} \neq 0$, where F_V stands for the partial differential of function F with respect to variable V. Then $\bar{X}(P,Q,Y,S)$ can be inverted on this factor:

$$S_1 = S_1(P,Q,Y,S_{-1},X),$$

where S_{-1} is the vector of distribution factors without the first element. Let us substitute this into the demand for the nth good to obtain the conditional demands :

$$\tilde{Z}^n(P,Q,Y,X) = \zeta^{An}(P,Q,\kappa(P,Q,X)) + \zeta^{Bn}(P,Y-\kappa(P,Q,X)), \qquad (5)$$

where κ is the conditional sharing rule.[5] We use the fact here that the knowledge of the prices and the level of the exclusive good completely determines the share of A (this comes from the functional structure (3)). That is to say, the conditioning good is a 'sufficient statistics' for describing the vector of distribution factors and the exogenous income in the conditional sharing rule.

In the remainder of this paper, we assume that the conditional demands and the conditional sharing rule exist and are three times continuously differentiable in all their arguments. To begin with, we define $Z = (Z^1, \ldots, Z^N)'$ and give two trivial properties that any system of conditional demands has to satisfy. These do not require a formal proof.

Proposition 2 *The system of conditional demands $\tilde{Z}(Q,P,Y,X)$ satisfies the adding-up restriction $\tilde{Z}' \cdot P = Y - X \cdot Q$.*

Proposition 3 *The system of conditional demands $\tilde{Z}(Q,P,Y,X)$ satisfies the restriction $\tilde{Z}_{S'} = 0$.*

Another version of the second restriction can be found in Bourguignon, Browning and Chiappori (1995). In a simple interpretation, this restriction says that the distribution factors are valid instruments to account for the probable endogeneity of the demand for the conditioning good in the estimation of (5). This makes a particularly simple test of the collective approach.

2.3 An Illustrative Example

To clarify the preceding section, we present here a parametric example. We assume that individual demands can be represented by the Linear Expenditure System:

$$XQ = \alpha^{AX}Q + \beta^{AX} \cdot (Y^A - \alpha^{AX}Q -_{r=1}^{N} \alpha^{Ar}P_r) \qquad (6)$$

and

$$\begin{aligned} Z^nP_n &= \alpha^{An}P_n + \beta^{An} \cdot (Y^A - \alpha^{AX}Q -_{r=1}^{N} \alpha^{Ar}P_r) \\ &\quad + \alpha^{Bn}P_n + \beta^{Bn} \cdot (Y^B -_{r=1}^{N} \alpha^{Br}P_r) \qquad (7) \end{aligned}$$

where $Y^A = \rho$ and $Y^B = Y - \rho$ denote the endowment of each member. We invert (6) with respect to Y^A and, using $Y^B = Y - Y^A$, we introduce the solution in (7). We obtain:

$$\begin{aligned} Z^nP_n &= (\alpha^{An} + \alpha^{Bn})P_n + \beta^{Bn}(Y - \alpha^{AX}Q -_{r=1}^{N} (\alpha^{Ar} + \alpha^{Br})P_r) \\ &\quad + (\frac{\beta^{An} - \beta^{Bn}}{\beta^{AX}}) \cdot (XQ - \alpha^{AX}Q). \qquad (8) \end{aligned}$$

[5] This type of demand is called y – demand by Bourguignon, Browning and Chiappori (1995) because they denote the vector of distribution factors by y. This name is a little unfortunate, specially here, since it arbitrarily depends on the notation.

A few remarks are in order at this stage. First, it is straightforward, although somewhat tedious, to show that the Slutsky effects, computed from (8), are generally not symmetrical. Second, the parameters α^{An} and α^{Bn} which represent the price effects cannot be identified; only the sum $\alpha^{An} + \alpha^{Bn}$ can be retrieved from the estimation of (8). All the other parameters are identifiable. We will see below that these properties are much more general than it may seem at a first glance.

3 Main Results

3.1 Observation of One Conditional Demand

The next result says that some elements of the conditional sharing rule and the Marshallian demands can be identified from the observation of only one conditional demand.

Proposition 4 *Let us assume that $\tilde{Z}^n_{YY} \neq 0$ and $\tilde{Z}^n_{YX} \neq 0$. Then, the conditional sharing rule is identified up to a linearly-homogeneous function $\epsilon(P)$. In particular,*

$$\kappa_Q = -\frac{\tilde{Z}^n_{YQ}}{Z^n_{YY}} \qquad \textit{and} \qquad \kappa_X = -\frac{\tilde{Z}^n_{YX}}{\tilde{Z}^n_{YY}}.$$

Moreover, some elements of the Marshallian demands ζ^{An}, ζ^{Bn} and χ are identified as well.

P roof. If we use (5) and differentiate the conditional demand with respect to Y, we obtain the husband's income effect :

$$\tilde{Z}^n_Y = \zeta^{Bn}_Y. \tag{9}$$

If we differentiate again this expression (9) with respect to Y, we obtain the second order husband's income effect :

$$\tilde{Z}^n_{YY} = \zeta^{Bn}_{YY}. \tag{10}$$

Finally, if we differentiate again (9) with respect to Q and X, we obtain :

$$\tilde{Z}^n_{YQ} = -\zeta^{Bn}_{YY} \cdot \kappa_Q, \qquad \text{and} \qquad \tilde{Z}^n_{YX} = -\zeta^{Bn}_{YY} \cdot \kappa_X. \tag{11}$$

From these equations (10) and (11), the derivatives of the conditional sharing rule κ_Q and κ_X are identified and given by :

$$\kappa_Q = -\frac{\tilde{Z}^n_{YQ}}{\tilde{Z}^n_{YY}} \qquad \text{and} \qquad \kappa_X = -\frac{\tilde{Z}^n_{YX}}{\tilde{Z}^n_{YY}}.$$

To obtain other elements of the Marshallian demands, let us differentiate the conditional demand with respect to Q and X and replace ζ_Y^{Bn}, κ_Q and κ_X by their values. We have:

$$\zeta_Y^{An} = \tilde{Z}_Y^n - \tilde{Z}_X^n (\frac{\tilde{Z}_{YY}^n}{\tilde{Z}_{YX}^n}) \qquad \text{and} \qquad \zeta_Q^{An} = \tilde{Z}_Q^n - \tilde{Z}_X^n \frac{\tilde{Z}_{YQ}^n}{\tilde{Z}_{YX}^n}.$$

To obtain elements of the Marshallian demand for the exclusive good, let us use the implicit definition of the conditional sharing rule $X = \chi(P, Q, \kappa)$, apply the Theorem of Implicit Functions and replace κ_Q and κ_X by their values. We have:

$$\chi_Y = -\frac{\tilde{Z}_{YY}^n}{\tilde{Z}_{YX}^n} \qquad \text{and} \qquad \chi_Q = -\frac{\tilde{Z}_{YQ}^n}{\tilde{Z}_{YX}^n}. \qquad \blacksquare$$

This result can be interpreted in two distinct ways. In a first interpretation, it means that there is no need to model the determination of the conditioning good explicitly to obtain information on the household decision process. Some useful structural elements can be derived from the estimation of one conditional demand. Still, the interpretation of the conditional sharing rule is unclear. In a second interpretation, this result means that the modelling of the demand for one aggregate good and one exclusive good allows us to partially identify the sharing rule as a function of the usual exogenous variables P, Q, Y and S. Indeed, it suffices to introduce the demand for the exclusive good in the conditional sharing rule to obtain the traditional sharing rule since $\kappa(P, Q, \bar{X}(P, Q, Y, S)) = \rho(P, Q, Y, S)$.

The next basic result is that it is possible to derive from the collective setting a set of testable restrictions on observable behavior.

Proposition 5 *Let us assume that $\tilde{Z}_{YY}^n \neq 0, \tilde{Z}_{YQ}^n \neq 0$ and $\tilde{Z}_{YX}^n \neq 0$. Then, the conditional demand $\tilde{Z}^n(P, Q, Y, X)$ is homogeneous of degree zero in P, Q and Y. Moreover, it satisfies the following restrictions:*

1. $\dfrac{\tilde{Z}_{YY}^n}{\tilde{Z}_{YX}^n} \cdot X + \dfrac{\tilde{Z}_{YQ}^n}{\tilde{Z}_{YX}^n} > 0,$
2. $\left(\dfrac{\tilde{Z}_{YYP'}^n}{\tilde{Z}_{YY}^n} - \dfrac{\tilde{Z}_{YQP'}^n}{\tilde{Z}_{YQ}^n}\right) P + \left(\dfrac{\tilde{Z}_{YYQ}^n}{\tilde{Z}_{YY}^n} - \dfrac{\tilde{Z}_{YQQ}^n}{\tilde{Z}_{YQ}^n}\right) Q = 0,$
3. $\left(\dfrac{\tilde{Z}_{YXP'}^n}{\tilde{Z}_{YX}^n} - \dfrac{\tilde{Z}_{YYP'}^n}{\tilde{Z}_{YY}^n}\right) P + \left(\dfrac{\tilde{Z}_{YXQ}^n}{\tilde{Z}_{YX}^n} - \dfrac{\tilde{Z}_{YYQ}^n}{\tilde{Z}_{YY}^n}\right) Q = 1,$
4. $\dfrac{\tilde{Z}_{YYY}^n}{\tilde{Z}_{YY}^n} = \dfrac{\tilde{Z}_{YYQ}^n}{\tilde{Z}_{YQ}^n} = \dfrac{\tilde{Z}_{YYX}^n}{\tilde{Z}_{YX}^n}.$

P roof. See Appendix A1. ■

These conditions provide a simple test of collective rationality under specific assumptions (namely, egoistic agents with purely private consumption, one exclusive good and one (at least) distribution factor). They are not sufficient; in the next section, we present one further inequality that any conditional demand has to satisfy (see Proposition 6). One preliminary remark is that the homogeneity of the conditional demands is not a trivial property as it may seem. Indeed, we do not assume here that the function μ, which represents the distribution of power within the household, is homogeneous of degree zero. By comparison, the more usual demands (3) and (4) are not necessarily homogeneous. The other conditions (1)–(4) are completely different from those used by Browning and Chiappori (1998) in their empirical application and can be interpreted as follows. The first condition is a translation of the Slutsky negativity for the exclusive good in the collective context. The second and the third conditions correspond to the homogeneity of the demand for the exclusive good. The fourth condition results from the separable structure of the conditional demands (5). Concerning this condition, a related, although less general, result can be found in Bourguignon, Browning and Chiappori (1995) in the context of demand analysis with constant prices.

3.2 A Complete System of Conditional Demands

More can be obtained when several conditional demands, rather than a single one, are observed. To begin with, let us define $S^X = (\tilde{Z}^n_{YY}/\tilde{Z}^n_{YX}) \cdot X + (\tilde{Z}^n_{YQ}/\tilde{Z}^n_{YX})$. It is shown in the proof of Proposition 6 that this expression does not depend on n. We may now derive a generalized Slutsky matrix and another constraint. This is formally expressed in the next proposition.

Proposition 6 *Let us assume that* $\tilde{Z}^n_{YY} \neq 0, \tilde{Z}^n_{YQ} \neq 0$ *and* $\tilde{Z}^n_{YX} \neq 0$ *for any* n*. Then, the system of conditional demands* $\tilde{Z}(P,Q,Y,X)$ *satisfies the following restrictions:*

1. $\tilde{Z}_{YQ} \cdot \tilde{Z}'_{YY}$ *and* $\tilde{Z}_{YX} \cdot \tilde{Z}'_{YY}$ *are symmetrical matrices,*
2. $\tilde{Z}_{P'} + \tilde{Z}_Y \cdot \tilde{Z}' + \tilde{Z}_X \cdot (\tilde{Z}_Q + \tilde{Z}_Y \cdot X)' - \tilde{Z}_X \cdot S^X \cdot \tilde{Z}'_X$ *is a symmetrical and negative definite matrix.*

P roof. See Appendix A2. ■

A few remarks are in order. Firsly, the first condition results from the separable structure of the conditional demands (5). The main difference, with the fourth condition in Proposition 5, is that the constraints here are based on a second, rather than a third order partial differential equation, which is more restrictive. Secondly, the symmetry and negativity of the conditional demands here can be related to the main result in Browning and Chiappori (1998) which states that the Slutsky matrix has to be equal to the sum of a symmetric matrix and a rank one matrix. We obtain, in the present context, a similar result except that the rank one matrix is here observed. This condition is thus stronger. Thirdly, an implication of the second condition in Proposition 7 is:

$$\tilde{Z}^n_{P_n} + \tilde{Z}^n_Y \cdot \tilde{Z}^n + (\tilde{Z}^n_Q + \tilde{Z}^n_Y \cdot X) \cdot \tilde{Z}^n_X - \left(\frac{\tilde{Z}^n_{YQ}}{\tilde{Z}^n_{YX}} + \frac{\tilde{Z}^n_{YY}}{\tilde{Z}^n_{YX}} \cdot X \right) \cdot (\tilde{Z}^n_X)^2 < 0$$

This inequality, although intricate, can be tested with usual single-equation techniques. The left-hand-side of this expression corresponds, in fact, to the sum of the own-price substitution effects for each member. Fourthly, another implication of the symmetry condition in Proposition 6 is that

$$\tilde{Z}_{P'} + \tilde{Z}_Y \cdot \tilde{Z}' + (\tilde{Z}_Q + \tilde{Z}_Y \cdot X) \cdot \tilde{Z}'_X$$

is symmetrical since the matrix $\tilde{Z}_X \cdot S^X \cdot \tilde{Z}'_X$ is symmetrical.

3.3 An Extended Almost Ideal Demand System

It is not too difficult to implement these conditions on a functional form. As an illustration, we may consider a simple generalization of the AI Demand System of Deaton and Muellbauer (1980):

$$\frac{Z_n P_n}{Y - QX} = \alpha_n +{}_{s=1}^{N} \beta_{ns} \log P_s + \gamma_n \log \frac{Y - QX}{\Phi} + \delta_n \frac{QX \cdot \log X}{Y - QX}$$

where Φ is defined, as usual, by

$$\Phi = \alpha_0 +{}_{s=1}^{N} \alpha_s \log P_s +{}_{s=1}^{N} {}_{t=1}^{N} \beta_{st} \log P_s \log P_t.$$

By comparison with the traditional AIDS, the expenditure on the conditioning good here is subtracted from the exogenous income. Moreover, there is one more term in $QX \cdot (Y - QX)^{-1} \cdot \log X$.

To be consistent with collective rationality, the parameters have to satisfy constraints. It is straightforward, although somewhat tedious, to show that we must have:

$$\beta_{st} = \beta_{ts}$$

and

$${}_{s=1}^{N}\alpha_s = 1, \quad {}_{s=1}^{N}\beta_{st} = 0, \quad {}_{s=1}^{N}\gamma_s = 0, \quad {}_{s=1}^{N}\delta_s = 0.$$

Moreover, it is also possible to retrieve some elements of the decision process when these constraints are fulfilled. Using the results of Proposition 2, we obtain:

$$\begin{aligned} \zeta^{An}_Y &= \frac{\delta_n}{P_n} (\log X + 1) \\ \zeta^{Bn}_Y &= \frac{1}{P_n} \left(\alpha_n +{}_{s=1}^{N} \beta_{ns} \log P_s + \gamma_n \left(\log \frac{Y - QX}{\Phi} + 1 \right) \right) \end{aligned}$$

Some derivatives of the demand for the exclusive good and the conditional sharing rule can also be recovered but this is left to the reader.

4 Conclusion

In the present paper, we have presented a collective model of household demand based on Pareto-efficiency and three distinctive hypotheses: (i) each household member is egoistic and consumption is purely private, (ii) there is one exclusive good and (iii) there is a set of distribution factors.

The resulting restrictions on household behavior are especially simple when we use a conditional approach to specify household demands. They can be readily incorporated in an extended form of the well-known AI Demand System (which may be generalized, if necessary). This system of conditional demands must not be confused with the more usual notion of conditional demands whereby the quantity for one set of goods is conditioned on the price of these goods, total expenditure on these goods and the quantities of another set of goods. Browning and Meghir (1991) use these conditional demands, in the single-utility approach, to take into account labor supply as a conditioning good. A similar strategy can be followed here if we define the exclusive good as the wife's labor supply (the husband's labor supply is probably not sufficiently flexible for that purpose).

Appendix

A.1 Proof of Proposition 6

Homogeneity: In order to prove the homogeneity, we consider the individual Euler equations:

$$\zeta_{P'}^{An} \cdot P + \zeta_Q^{An} \cdot Q + \zeta_Y^{An} \cdot \kappa = 0 \qquad \text{and} \qquad \zeta_{P'}^{Bn} \cdot P + \zeta_Y^{Bn} \cdot (Y - \kappa) = 0.$$

If we sum up these matrices and use the identities $\zeta_{P'}^{An} + \zeta_{P'}^{Bn} = \tilde{Z}_{P'}^{n} - (\zeta_Y^{An} - \zeta_Y^{Bn}) \cdot \kappa_{P'}$ and $\zeta_Q^{An} = \tilde{Z}_Q^n - (\zeta_Y^{An} - \zeta_Y^{Bn}) \cdot \kappa_Q$, we obtain:

$$\tilde{Z}_{P'}^{n} \cdot P + \tilde{Z}_Q^n \cdot Q + \zeta_Y^{Bn} \cdot Y + (\zeta_Y^{An} - \zeta_Y^{Bn}) \cdot (\kappa - \kappa_{P'} \cdot P - \kappa_Q \cdot Q) = 0.$$

Since $\kappa(P, Q, Y, X)$ is homogeneous of degree one, the last term vanishes. If we replace ζ_Y^{Bn} by its value given in Proposition 4, we obtain:

$$\tilde{Z}_{P'}^{n} \cdot P + \tilde{Z}_Q^n \cdot Q + \tilde{Z}_Y^n \cdot Y = 0.$$

Condition 1: The first condition comes from the Slutsky negativity $\chi_Q + \chi_Y \cdot X \leqslant 0$ where χ_Q and χ_Y are replaced by their values given in Proposition 4.
Condition 2–3: The second and the third conditions result from the homogeneity of the demand for the exclusive commodity. This implies that κ_Q and κ_X are respectively homogeneous of degree zero and of degree one. We use the Euler equation and simplify.
Condition 4: The fourth condition results from the fact that the conditional

sharing rule, and consequently κ_X and κ_Q, does not depend on Y. We thus compute:

$$\frac{\partial}{\partial Y}\left(\frac{\tilde{Z}^n_{YQ}}{\tilde{Z}^n_{YY}}\right) \qquad \text{and} \qquad \frac{\partial}{\partial Y}\left(\frac{\tilde{Z}^n_{YX}}{\tilde{Z}^n_{YY}}\right),$$

where we use the values given in Proposition 4. We make equal to zero and simplify. ■

A.2 Proof of Proposition 6

Condition 1: The derivatives of the conditional sharing rule κ_Q and κ_X can be uniquely identified from any conditional demand n_1 or n_2. Whether we use the demand for the n_1th good or the demand for the n_2th good, however, we must retrieve the same expressions for the derivatives of the sharing rule κ_Q and κ_X. This yields

$$\frac{\tilde{Z}^{n_1}_{YQ}}{\tilde{Z}^{n_2}_{YQ}} = \frac{\tilde{Z}^{n_1}_{YX}}{\tilde{Z}^{n_2}_{YX}} = \frac{\tilde{Z}^{n_1}_{YY}}{\tilde{Z}^{n_2}_{YY}}$$

or, in matrix form, the conditions in Proposition 6.
Condition 2: To begin with, let us consider, for member A, the Slutsky symmetry between the exclusive good and the other goods and rearrange. We obtain:

$$Z^A - \kappa_P = \kappa_X \cdot (\zeta^A_Q + \zeta^A_Y \cdot X) \tag{12}$$

where we use $\chi_P = -\kappa_P/\kappa_X$ and $\chi_Y = 1/\kappa_X$. Let us now consider the individual substitution matrices:

$$\zeta^A_{P'} + \zeta^A_Y \cdot Z^{A\prime} \qquad \zeta^B_{P'} + \zeta^B_Y \cdot Z^{B\prime}.$$

If we sum up these matrices and use the identities $Z^B = \tilde{Z} - Z^A$ and $\zeta^A_{P'} + \zeta^B_{P'} = \tilde{Z}_{P'} - (\zeta^A_Y - \zeta^B_Y) \cdot \kappa_{P'}$, we obtain:

$$\tilde{Z}_{P'} + \zeta^B_Y \cdot \tilde{Z}' + (\zeta^A_Y - \zeta^B_Y) \cdot (Z^{A\prime} - \kappa_{P'}).$$

Of course, this matrix is symmetrical and definite negative. If we use (12) and replace $\zeta^A_Q, \zeta^A_Y, \zeta^B_Y$ and κ_X by their values, we obtain:

$$\tilde{Z}_{P'} + \tilde{Z}_Y \cdot \tilde{Z}' + \tilde{Z}_X \cdot (\tilde{Z}_Q + \tilde{Z}_Y \cdot X)' - \tilde{Z}_X \cdot S^X \cdot \tilde{Z}'_X,$$

where $S_X = (\tilde{Z}_{YQ}/\tilde{Z}_{YX}) + (\tilde{Z}_{YY}/\tilde{Z}_{YX}) \cdot X \geqslant 0$ is the opposite of the own-price substitution effect of the demand for the exclusive good. ■

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